

Homework Assignment 9 is due Friday April 16

**Exercise 1:** (Boolean OR has a linear separator.) Take as examples all the  $2^d$  elements of  $\{0,1\}^d$ . Label the example by +1 if there is at least one coordinate with a +1 and label it by -1 if all its coordinates are 0. This is like taking the Boolean OR, except we look upon the coordinates as real numbers. Show that there is a linear separator for these labeled examples. Show that we can achieve a margin of  $\Omega(1/d)$  for this problem.

**Exercise 2:** Show that the parity function, the Boolean function that is 1 if and only if an odd number of inputs is 1, cannot be represented as a threshold function.

**Exercise 3:** Suppose we have  $n$  points in the plane and  $C$  is a circle containing at least three points. Prove or disprove the following statement: there is a circle  $C'$  so that (i) there are 3 points lying on  $C'$  or two points lying on a diameter of  $C'$  and (ii) the set of points in  $C$  is the same as the set of points in  $C'$ .

**Exercise 4:** Given  $n$  points in the plane define two circles as equivalent if they enclose the same set of points. Prove that there are only  $O(n^3)$  equivalence classes of points defined by circles and thus only  $n^3$  subsets out of the  $2^n$  subsets can be enclosed by circles.

**Exercise 5:** Prove that the VC dimension of convex polygons is infinite.

**Exercise 6:** If a class contains only convex sets prove that no set in which some point is in the convex hull of other points can be shattered.

**Exercise 7:** Show that there is a set of 3 points which can be shattered by axis-parallel squares. Show that the system of axis-parallel squares cannot shatter any set of 4 points.