CS485 Spring 2007
Homework 8

Due Date: March 16 2007

NOTE: To speed up homework grading, please submit each homework problem on a separate sheet of paper, with you name and NetID on the top. Thank you!

The two problems are related, and it might help you to do them in parallel.

1. Consider a graph that is a chain of n nodes $1, \ldots, n$. We'll be performing a random walk on the chain, starting at 1, where the probabilities of going from i to $i-1$ or $i+1$ are both 0.5. At the boundary, we always go from 1 to 2, and we never leave n (self loop with probability 1). Write a program to perform the walk.

   (a) Fix $n = 50$. Plot the average number of times the walk leaves each node i before reaching n. You might need to average over a very large (10,000) number of walks to get a nice result.

   (b) Fix $n = 50$. Plot the average number of steps required to get from i to $i+1$ for the first time, for every node i. You might need to average across many walks again. What would you expect to see for different i's? If it is different from what you see, can you explain it?

   (c) Plot the average number of steps that the walk needs in order to reach n from 1 for the first time, against n (for several values of n).

2. Consider again a graph that is a chain of n nodes $1, \ldots, n$, and a random walk as in the previous problem. Answer the same questions as above, but analytically this time:

   (a) What is the expected number of steps required to get from i to $i+1$ for the first time, for every i? *Hint:* use $h_{i,i+1} =$ "expected number of steps ...", show its boundary condition ($h_{1,2}$), write down a recurrence equation for it, and solve it.

   (b) What is the expected number of steps required to get from 1 to n for the first time? *Hint:* use previous result.

   (c) What is the expected number of times the walk leaves each node i before reaching n? *Hint:* use $c^j_{i,i+1} =$"expected number of times walk leaves j on the way from i to $i+1$ for the first time". Be sure you get the boundary conditions right.