

CS485 Spring 2007

Homework 4

Due Date: Feb 16 2007

NOTE: To speed up homework grading, please submit each homework problem on a separate sheet of paper, with you name and NetID on the top. Thank you!

1. Consider a set $N = \{1, \dots, n\}$, for a fixed n . We will be computing number of arithmetic progressions on such set. An arithmetic progression of length k is a set $\{a, a + b, \dots, a + (k - 1)b\} \subseteq N$ for some numbers $a, b \in N$. Please try to compute the following as exactly as you can:
 - (a) How many arithmetic progressions of length k are there in N ?
 - (b) How many pairs of arithmetic progressions are there in N that overlap on *at least* one element?
 - (c) How many are there that overlap on at least two elements?
 - (d) How many progressions overlap on *exactly* one element?
2. Describe in words what a phase transition is (i.e. what it means for a property to have a threshold). What is a *sharp* threshold? Imagine you were explaining it to a friend who has not taken the course. Clarity has higher priority than technicalities. Please make sure you describe what the “s-curve plot” means, with functions $p(n)$ on x-axis and probability of having a certain property on the y-axis.
3. Describe the second moment method. Say what is it useful for (what it says). Give an example of a random variable (on any probability space) where the method is useful, and another example where it does not work (what does it mean that it does not work?).
4. Let $N = \{1, \dots, n\}$ and let S_k be a random subset of N of size $k(n)$ (note that this model is different from the one we defined in class, very similar but different). Consider a property Q on S_k that says “ S_k contains a 3-term arithmetic progression”. Say whether Q has a threshold (consider the case of $n \rightarrow \infty$). If so, find what it is, and if not, say how does $P[S_k \in Q]$ change with $k(n)$ (i.e. how does the probability of property Q holding change as $k(n)$ grows)