CS485 Spring 2007
Homework 12

Due Date: April 20 2007

NOTE: To speed up homework grading, please submit each homework problem on a separate sheet of paper, with you name and NetID on the top. Thank you!

1. (a) For what value of d is the volume of a d-dimensional hypersphere with radius 1, $V(d)$, maximized?

   (b) How does the volume of a hyper sphere of radius two behave as the dimension of the space increases? What if the radius was larger than two but constant independent of d? What function of d would the radius need to be for a hyper sphere of radius r to have approximately constant volume as the dimension increases?

2. Write a computer program that generates n points uniformly distributed over the surface of a unit d dimensional sphere (for e.g. $d=1, 3, 900$). Create a random line through the origin and project the points onto the line. Plot the distribution of points on the line. What does your result say about the surface area of the sphere in relation to the line, i.e., where is the surface area concentrated relative to the line?

3. Generate a 1000 pairs of random points on the surface of a hyper sphere in 900 dimensions. Make sure that the points are uniformly distributed on the sphere. What is the distribution of distances between the two points in each pair? What the distribution of the angle between the pairs of vectors (from origin to the points on the unit sphere). Plot both distributions.

   The angle between vectors $\bar{x}$ and $\bar{y}$ is defined as an angle θ such that $\cos(\theta) = \frac{\bar{x}^T \bar{y}}{||\bar{x}|| \, ||\bar{y}||}$, where $\bar{x}^T \bar{y}$ is the dot-product and $||.||$ is the length of the vector.

4. Explain what is going on with the "non-intersecting bands around a unit sphere". Consider a unit hypersphere for large enough dimension d, with many points scattered uniformly on its surface. In class, we showed that fixing an arbitrary point (and calling it "north pole"), all other points are concentrated in a narrow band around the equator. But if we take 3 such "north pole" points perpendicular to each other, the bands seemed not to intersect. And yet the intersection is supposed to have the most mass! Explain what is wrong with this reasoning.