1. Recall:
   - A TM is *total* if it halts on all inputs
   - A set is *r.e.* if it is $L(M)$ for some TM $M$
   - A set is *recursive* if it is $L(M)$ for some total TM $M$
   - The *halting problem* is the set

   $$\mathrm{HP} = \{M\#x \mid M \text{ is a TM, } x \text{ is a string over } M\text{'s input alphabet, } M \text{ halts on input } x\}.$$

   True or false?

   (i) Every CFL is recursive. **true**

   (ii) There exists a recursive set that is not a CFL. **true**

   (iii) All recursive sets are r.e. **true**

   (iv) $\{a^p \mid p \text{ is a prime number}\}$ is a recursive set. **true**

   (v) $L(M)$ is recursive if and only if $M$ is total. **false – If $M$ is total, then $L(M)$ is recursive, but not the converse. A machine can loop and still accept a recursive set. For example, a machine that loops on all inputs accepts $\varnothing$. For a set to be recursive, there must *exist* a total machine accepting it.**

   (vi) Nondeterministic TMs can accept non-r.e. sets. **false**

   (vii) TMs with two tapes accept more sets than TMs with one tape. **false**

   (viii) Every non-total Turing machine accepts a nonregular set. **false**

   (ix) It is decidable for a given TM $M$ and string $x$ whether $M$ rejects $x$. **false**

   (x) It is decidable for a given TM $M$ whether $L(M) = {\sim}\mathrm{HP}$. ($\sim$ denotes set complement.) **true – ${\sim}$HP is not r.e., so the answer is always "no".**

2. In the following TM, the input alphabet is $\{a, b\}$, the left endmarker is $\vdash$, and the blank symbol is $\sqcup$. The transitions are given in the following table.

| | | $\vdash$ | $a$ | $b$ | $\sqcup$ |
|---|---|---|---|---|---|
| start state $\rightarrow$ | $s$ | $s, \vdash, R$ | $t, b, L$ | $r, a, L$ | $s, \sqcup, L$ |
| accept state $\rightarrow$ | $t$ | $t, \vdash, R$ | $t, b, L$ | $t, a, L$ | $t, \sqcup, L$ |
| reject state $\rightarrow$ | $r$ | $r, \vdash, R$ | $r, b, L$ | $r, a, L$ | $r, \sqcup, L$ |

   What language does it accept?

   (a) strings beginning with $a$ ✓

   (b) strings containing only $a$'s

   (c) strings containing at least one $a$

3. True or false?

   (i) The machine of question 2 is total. **false – The machine loops on input $\varepsilon$.**

   (ii) The language accepted by the machine of question 2 is recursive. **true**