NetId _____ Name _____

Write your name and netId above. Do not begin until instructed.

The exam is 75 minutes, open book and notes, no electronic devices, no collaboration. There are 6 questions on 3 numbered pages. Make sure you have all the pages.

Write your answers directly on this exam. Indicate clearly which is your answer. Show all work for partial credit. Use the back of the pages for scratchpaper. Copies of the course textbook are available.

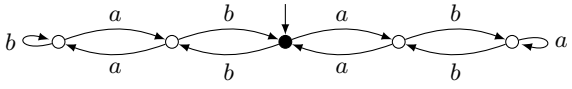
Good luck!

1. (10 points) Consider the following DFA:

	a	b
$\rightarrow 0$	1	5
1	5	2
2	6	4
3	4	5
$4F$	2	6
$5F$	0	1
6	4	0

Give an equivalent DFA with the minimum number of states. Show clearly the computation of the equivalence classes and which equivalence class corresponds to each state of the new automaton.

2. (10 points) Give a regular expression equivalent to the following DFA. Black states indicate accept states.



3. (10 points) Consider an NFA consisting of $k \geq 2$ states arranged in a circle. From each state, on input a, the automaton can move either one or two steps clockwise around the circle, and on input b, it can move only one step counterclockwise. There is a single start state, and all states except the start state are accept states.

   Formally, we can take the set of states to be $\{0, 1, \ldots, k-1\}$ and nondeterministic transition function

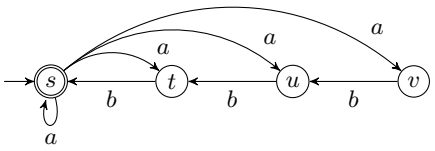
   $$\Delta(q, a) \stackrel{\text{def}}{=} \{q+1, q+2\} \qquad\qquad \Delta(q, b) \stackrel{\text{def}}{=} \{q-1\}$$

   (all arithmetic is modulo k). The only start state is 0 and the accept states are $\{1, \ldots, k-1\}$.

   (a) (3 points) When converting this NFA to a DFA using the subset construction, how many accessible states would there be in the resulting DFA as a function of k?

   (b) (7 points) Describe using set-theoretic notation or a regular expression the set of strings in $\{a, b\}^*$ accepted by this machine.

4. (10 points) Consider the following nondeterministic finite automaton.



   Construct an equivalent deterministic automaton using the subset construction. Show clearly which subset of $\{s, t, u, v\}$ corresponds to each state of the deterministic automaton. Omit inaccessible states.

5. (10 points) Let $L = \{a^n b^n \mid n \geq 0\}$. Give a context-free grammar for the complement of L, that is, the set $\{a, b\}^* - L$. (*Hint.* Note that this set consists of all strings containing the substring ba, all strings of the form $a^n x$ for $n \geq 1$ and $x \in L$, and all strings of the form xb^n for $n \geq 1$ and $x \in L$.)

6. (10 points) Using the grammar

$$S \to BC \mid a \qquad B \to CS \mid b \qquad C \to SB \mid b$$

run the CKY algorithm on the string bab. Fill in the table completely.