NetId ____________ Name ______________________________

150 minutes, open book and notes, no electronic devices. No collaboration allowed. Write answers in the exam book, not on this sheet. Indicate clearly which is your answer. Show all work for partial credit. Write your name and netId on this sheet and the exam book, and pass them in together. **Good luck!**

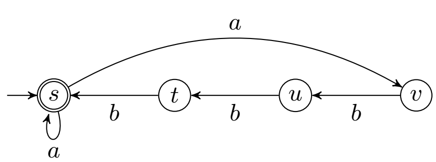
1. Minimize the following DFA:

	a	b
$\rightarrow 0F$	1	2
1	3	4
$2F$	4	5
3	5	6
$4F$	6	0
$5F$	7	6
$6F$	2	5
7	3	6
$8F$	5	4

Show clearly the computation of the equivalence classes and which equivalence class corresponds to each state of the new automaton. (*Sanity check*: You should get four equivalence classes.)

	a	b
$\rightarrow \{0,5\}F$	$\{1,7\}$	$\{2,4,6\}$
$\{1,7\}$	$\{3\}$	$\{2,4,6\}$
$\{2,4,6\}F$	$\{2,4,6\}$	$\{0,5\}$
$\{3\}$	$\{0,5\}$	$\{2,4,6\}$

2. Consider the following nondeterministic finite automaton.



Construct an equivalent deterministic automaton using the subset construction. Show clearly which subset of $\{s,t,u,v\}$ corresponds to each state of the deterministic automaton. Omit inaccessible states.

	a	b
$\rightarrow \{s\}F$	$\{s,v\}$	$\varnothing$
$\{s,v\}F$	$\{s,v\}$	$\{u\}$
$\{u\}$	$\varnothing$	$\{t\}$
$\{t\}$	$\varnothing$	$\{s\}$
$\varnothing$	$\varnothing$	$\varnothing$

3. Say whether the following sets are regular or nonregular. No proofs necessary.

	reg.	*nonreg.*	
(a)	☐	☑	$\{a^n b^m \mid n = 2m\}$
(b)	☑	☐	$\{a^n b^{2m} \mid n \geq 0 \text{ and } m \geq 0\}$
(c)	☐	☑	$\{a^n b^m c^n \mid n \geq 0 \text{ and } m \geq 0\}$
(d)	☐	☑	$\{x \in \{0,1\}^* \mid x \text{ contains more 0's than 1's}\}$
(e)	☐	☑	$\{a^n b^m \mid n \neq m\}$
(f)	☐	☑	$\{a^n b^{n+4810} \mid n \geq 0\}$

4. True or false? No proofs necessary.

	true	*false*	
(a)	☑	☐	Every CFL is recursive.
(b)	☑	☐	There exists a recursive set that is not a CFL.
(c)	☑	☐	All recursive sets are r.e.
(d)	☑	☐	$\{a^p \mid p$ is a prime number$\}$ is a recursive set.
(e)	☐	☑	If $L(M)$ is recursive, then M is total.
(f)	☑	☐	If M is total, then $L(M)$ is recursive.
(g)	☐	☑	TMs with two tapes accept more sets than TMs with one tape.
(h)	☐	☑	Every Turing machine accepts a nonregular set.
(i)	☐	☑	It is decidable for a given TM M and string x whether M accepts x.
(j)	☑	☐	It is decidable for a given TM M whether $L(M) = {\sim}\mathrm{HP}$.

5. Say whether the following problems are decidable or undecidable. No proofs necessary.

	decid.	*undecid.*	
(a)	☑	☐	whether a given a TM ever writes a nonblank symbol on its tape on some input
(b)	☐	☑	whether a given LBA halts on all inputs
(c)	☑	☐	whether a given NFA accepts Σ^*
(d)	☐	☑	whether a given PDA accepts Σ^*
(e)	☐	☑	whether a given LBA accepts $\varnothing$
(f)	☑	☐	whether a given TM runs for more than $4810^{4810^{4810}}$ steps on some input
(g)	☐	☑	whether a given TM accepts the set $\{a^p \mid p \text{ is prime}\}$
(h)	☐	☑	whether the intersection of two given CFLs is regular
(i)	☑	☐	whether the intersection of two given r.e. sets is r.e.
(j)	☐	☑	whether a given Java program will ever throw a null pointer exception
(k)	☐	☑	whether a given μ-recursive function is total
(l)	☐	☑	whether a given λ-term reduces to normal form

More—go on to next page

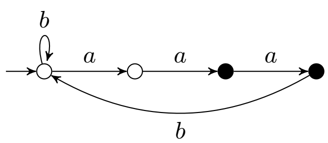
6. Give (a) a CFG, (b) a PDA for the set $\{a^m b^{m+n} a^n \mid m, n \geq 0\}$. In part (b), an informal description of the machine is sufficient.

   (a) $S \to XY \quad X \to aXb \mid \varepsilon \quad Y \to bYa \mid \varepsilon$

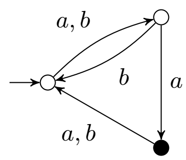
   (b) Note that this set is $\{a^m b^m \mid m \geq 0\} \cdot \{b^n a^n \mid n \geq 0\}$. If it sees an a as the first letter of the input, it scans and pushes a's until it sees the first b, then it reads b's and pops the a's until the stack is empty again. At that point it has scanned $a^m b^m$ for some m. It now does the same thing with b's followed by a's, pushing the b's and popping the b's while reading the a's. If the stack is empty when it reaches the end of the input, it accepts. If at any time the stack is emptied prematurely or if it sees an unexpected letter, it rejects.

7. Give regular expressions equivalent to the following finite automata.

   (a)



   (b)



   (a) $(b + aaab)^*(aa + aaa)$ (b) $((a + b)(b + a(a + b)))^*(a + b)a$

8. Give deterministic finite automata equivalent to the following regular expressions.

   (a) $(aaa)^* + (aa)^*$

   (b) $a^* ab(a + b)^*$

   (a) States $\{0, 1, 2, 3, 4, 5\}$, accept states $\{0, 2, 3, 4\}$, transitions $\delta(i, a) = i + 1 \bmod 6$, start state 0.

   (b)

	a	b
$\to 0$	1	d
1	1	2
$2F$	2	2
d	d	d

9. The set of regular expressions over the alphabet $\{a, b\}$ is a context-free language. Here is a context-free grammar for it:

$$S \to M + S \mid M \qquad M \to ZM \mid Z \qquad Z \to a \mid b \mid 1 \mid 0 \mid Z^* \mid (S)$$

   The terminal symbols are $\{\boldsymbol{\varepsilon}, \boldsymbol{\emptyset}, a, b, {}^*, +, (,)\}$, the nonterminal symbols are $\{S, M, Z\}$, and the start symbol is S. Give a leftmost derivation of the string $(a + b)^* a$.

$$S \to M \to ZM \to Z^*M \to (S)^*M \to (M + S)^*M \to (Z + S)^*M \to (a + S)^*M \to (a + M)^*M$$
$$\to (a + Z)^*M \to (a + b)^*M \to (a + b)^*Z \to (a + b)^*a$$

MORE—GO ON TO NEXT PAGE

10. Prove that the set $\{M \mid L(M) = L(M)^*\}$ is not recursively enumerable. (*Hint.* Given M and x, construct a machine M' that accepts $\{\varepsilon\} \cup \mathrm{VALCOMPS}(M, x)$.)

We reduce $\sim$HP to $\{M \mid L(M) = L(M)^*\}$. Given M and x, construct a machine M' that accepts its input y provided either $y = \varepsilon$ or y is a valid accepting computation history of M on input x. Then $L(M') = \{\varepsilon\} \cup \mathrm{VALCOMPS}(M, x)$. Either

- M halts on x, in which case $\mathrm{VALCOMPS}(M, x) = \{v\}$, where v is the unique valid accepting computation history of M on x, thus $L(M') = \{\varepsilon, v\}$; or
- M does not halt on x, in which case $\mathrm{VALCOMPS}(M, x) = \varnothing$, thus $L(M') = \{\varepsilon\}$.

In the former case, $L(M')^* = \{\varepsilon, v\}^* = \{v^n \mid n \geq 0\} \neq \{\varepsilon, v\} = L(M')$, whereas in the latter case, $L(M')^* = \{\varepsilon\}^* = \{\varepsilon\} = L(M')$. Thus $M\#x \in {\sim}\mathrm{HP}$ iff M does not halt on x iff $L(M')^* = L(M')$. The map $M\#x \mapsto M'$ is easily computable by a total Turing machine with output and constitutes a reduction from $\sim$HP to $\{M \mid L(M) = L(M)^*\}$.

End of exam