NetId _____ Name _____

150 minutes, open book and notes, no electronic devices. No collaboration allowed. Write answers in the exam book, not on this sheet. Indicate clearly which is your answer. Show all work for partial credit. Write your name and netId on this sheet and the exam book, and pass them in together. **Good luck!**

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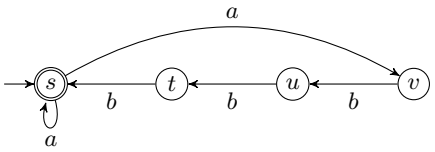
1. Minimize the following DFA:

	a	b
$\to 0F$	1	2
1	3	4
$2F$	4	5
3	5	6
$4F$	6	0
$5F$	7	6
$6F$	2	5
7	3	6
$8F$	5	4

   Show clearly the computation of the equivalence classes and which equivalence class corresponds to each state of the new automaton. (*Sanity check*: You should get four equivalence classes.)

2. Consider the following nondeterministic finite automaton.



   Construct an equivalent deterministic automaton using the subset construction. Show clearly which subset of $\{s, t, u, v\}$ corresponds to each state of the deterministic automaton. Omit inaccessible states.

More—go on to next page

3. Say whether the following sets are regular or nonregular. No proofs necessary.

	reg.	*nonreg.*	
(a)	□	□	$\{a^n b^m \mid n = 2m\}$
(b)	□	□	$\{a^n b^{2m} \mid n \geq 0 \text{ and } m \geq 0\}$
(c)	□	□	$\{a^n b^m c^n \mid n \geq 0 \text{ and } m \geq 0\}$
(d)	□	□	$\{x \in \{0,1\}^* \mid x \text{ contains more 0's than 1's}\}$
(e)	□	□	$\{a^n b^m \mid n \neq m\}$
(f)	□	□	$\{a^n b^{n+4810} \mid n \geq 0\}$

4. True or false? No proofs necessary.

	true	*false*	
(a)	□	□	Every CFL is recursive.
(b)	□	□	There exists a recursive set that is not a CFL.
(c)	□	□	All recursive sets are r.e.
(d)	□	□	$\{a^p \mid p \text{ is a prime number}\}$ is a recursive set.
(e)	□	□	If $L(M)$ is recursive, then M is total.
(f)	□	□	If M is total, then $L(M)$ is recursive.
(g)	□	□	TMs with two tapes accept more sets than TMs with one tape.
(h)	□	□	Every Turing machine accepts a nonregular set.
(i)	□	□	It is decidable for a given TM M and string x whether M accepts x.
(j)	□	□	It is decidable for a given TM M whether $L(M) = \sim\text{HP}$.

5. Say whether the following problems are decidable or undecidable. No proofs necessary.

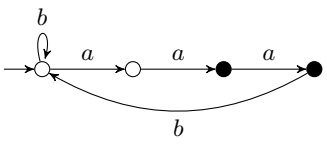
	decid.	*undecid.*	
(a)	□	□	whether a given a TM ever writes a nonblank symbol on its tape on some input
(b)	□	□	whether a given LBA halts on all inputs
(c)	□	□	whether a given NFA accepts Σ^*
(d)	□	□	whether a given PDA accepts Σ^*
(e)	□	□	whether a given LBA accepts $\varnothing$
(f)	□	□	whether a given TM runs for more than $4810^{4810^{4810}}$ steps on some input
(g)	□	□	whether a given TM accepts the set $\{a^p \mid p \text{ is prime}\}$
(h)	□	□	whether the intersection of two given CFLs is regular
(i)	□	□	whether the intersection of two given r.e. sets is r.e.
(j)	□	□	whether a given Java program will ever throw a null pointer exception
(k)	□	□	whether a given μ-recursive function is total
(l)	□	□	whether a given λ-term reduces to normal form

More—go on to next page

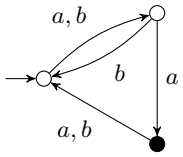
6. Give (a) a CFG, (b) a PDA for the set $\{a^m b^{m+n} a^n \mid m, n \geq 0\}$. In part (b), an informal description of the machine is sufficient.

7. Give regular expressions equivalent to the following finite automata.

(a)



(b)



8. Give deterministic finite automata equivalent to the following regular expressions.

   (a) $(aaa)^* + (aa)^*$

   (b) $a^*ab(a + b)^*$

9. The set of regular expressions over the alphabet $\{a, b\}$ is a context-free language. Here is a context-free grammar for it:

$$S \to M + S \mid M \qquad M \to ZM \mid Z \qquad Z \to a \mid b \mid 1 \mid 0 \mid Z^* \mid (S)$$

   The terminal symbols are $\{\boldsymbol{\varepsilon}, \boldsymbol{\emptyset}, a, b, {}^*, +, (,)\}$, the nonterminal symbols are $\{S, M, Z\}$, and the start symbol is S. Give a leftmost derivation of the string $(a + b)^*a$.

$$S \to M \to ZM \to$$

10. Prove that the set $\{M \mid L(M) = L(M)^*\}$ is not recursively enumerable. (*Hint.* Given M and x, construct a machine M' that accepts $\{\varepsilon\} \cup \text{VALCOMPS}(M, x)$.)

End of exam