Introduction to Theory of Computing	**Problem Set 7**
CS 481 Fall 2004	**Due Friday, October 29, 2004**

Reading. To review the material for this week read Sections 19-23 of Kozen.

Please turn in Problem 1 and Problems 2-3 on separate papers with your name and cornell.edu email written on both.

(1) Which of the following languages are context free, and which are not? Give grammars for those that are context free, and proofs for those that are not.

(a) $\{a^n b^m | n, m \geq 0, n \neq m\}$ over the alphabet $\Sigma = \{a, b\}$.

(b) $\{a^n b^m | n, m \geq 0, n = 3m\}$ over the alphabet $\Sigma = \{a, b\}$.

(c) $\{a^n b^m | n, m \geq 0, n = m^2\}$ over the alphabet $\Sigma = \{a, b\}$.

(d) $\{a^n b^m c^k | n, m, k \geq 0, n = 2m \text{ or } m = 3k\}$.

(e) $\{a^n b^m c^k | n, m, k \geq 0, n = 2m \text{ and } m = 3k\}$.

(f) The complement of the language $\{a^n b^n c^n | n \geq 0\}$ over the alphabet $\Sigma = \{a, b, c\}$ in context free. (That is all words that are either NOT of the form $a^* b^* c^*$ or are $a^n b^m c^k$ for some $n, m, k \geq 0$ but not all of n, m, k are equal.)

The first version of the problem set asked for $\{a^n b^m c^k |$ not all of m, n and k are the same$\}$. This is not the same as the complement of $\{a^n b^n c^n | n \geq 0\}$. But you may solve either of the two.

Hint (for either version): you may use the closure properties of context free languages we proved in one of the first classes on the subject, namely that if A and B are context free, than so is $A \cup B$, AB and A^*.

(2) Assume that language A is accepted by a non-deterministic pushdown automata, and language B is regular. Give a nondeterministic pushdown automata that accepts $A \cap B$. (Hint: similar to why the intersection of regular languages is regular.)

(3) Solve Problem 4 on Page 306 of Kozen. Hint: you may want to also do Problem 3, but you only need to hand in problem 4.