

Reading. To review the material for this week read Sections 32-35 of Kozen.

Please turn in Problems 1-2 and Problem 3-4 on separate papers with your name and cornell.edu email written on both.

(1) The language TOTAL is defined as follows

$$TOTAL = \{M \mid M \text{ halts on all inputs.}\}$$

Show that neither TOTAL nor its complement is r.e.

(2) Solve Exercise 2 on p. 310.

(3) Solve Exercise 120 on p. 345.

(4) Let Σ be an alphabet that does not contain #.

(a) Show that the following language is context free

$$L_1 = \{w_1\#w_2\#\dots\#w_n\# \mid n \geq 0, w_i \in \Sigma^* \text{ for all } i, \text{ and } w_{2i} = \text{rev}(w_{2i-1}) \text{ for all } i = 1, \dots, n\},$$

where $\text{rev}(w)$ is the reverse of string w .

(b) Show that the following language can be written as intersection of two context free languages.

$$L = \{w_1\#w_2\#\dots\#w_n\# \mid n \geq 0, w_i \in \Sigma^* \text{ for all } i, \text{ and } w_{i+1} = \text{rev}(w_i) \text{ for all } i = 1, \dots, n-1\}.$$

(c) Show that it is not decidable if the intersection of two context free languages (given by their grammars) is nonempty.

Hint: Use ideas from parts (a) and (b), for a variant of the language *VALCOMPS* defined in Lecture 35, where we write every second state backwards.