Introduction to Theory of Computing **Prelim 1 Info and Practice Questions**
CS 481 Fall 2004

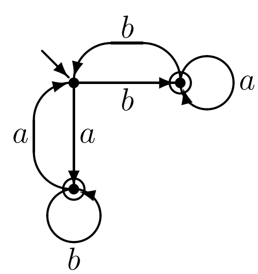
Prelim 1 will be in class, Wednesday October 20 (9:05-9:55 in HO 110). This is a closed book, closed notes test. On the test you may use any fact that was proved in the text, lectures, homeworks, or this practice sheet, without proving them again. We will not ask you to state any theorem (such as the pumping lemma), but you should be able to use them. Typical construction questions will not require proof (but you have to make sure they are correct). For proving a language not regular, you may use any of the methods we discussed (pumping lemma, intersection property, or homomorphism).

Problem set 6 solutions will be available on the Web during the weekend, and in Upson 303 starting Monday.

Material. Prelim 1 will cover problem sets 1-6. The question on the topic of the last problem set will be only at a quiz level. The rest of the questions will be easier than most homeworks, but harder than a quiz. The topics covered are roughly Sections 3-12 (not including section A), state equivalence and equivalence of automata from Sections 13-14, and context free grammars from sections 19-21 (only Chomsky normal from from 21).

Practice Questions. The following questions all have been on past prelims (though there are too many of them here). We will spend Monday's class reviewing some of these questions, and answering any other questions that come up. We may also add an additional office hour early in the week.

(1) Give a regular expression equivalent to the following DFA.



Here $\rightarrow$ indicates the start state (rather than our usual s).

(2) Give a DFA equivalent to the regular expression $(00+11)^*\,(01+10)\,(00+11)^*$.

(3) Match the following grammars with the sets they generate. In all cases, S is the start symbol.

(a)	$S \rightarrow aSb \mid T$	$T \rightarrow bTa \mid \varepsilon$		(i)	$\{a^n b^{n+m} a^m \mid n, m \geq 0\}$
(b)	$S \rightarrow TT$	$T \rightarrow aTb \mid \varepsilon$		(ii)	$\{a^n b^m a^m b^n \mid n, m \geq 0\}$
(c)	$S \rightarrow TU$	$T \rightarrow aTb \mid \varepsilon$	$U \rightarrow bUa \mid \varepsilon$	(iii)	$\{a^n b^n a^m b^m \mid n, m \geq 0\}$

(4) The following grammar generates the set of (non-empty) balanced parentheses. The start symbol is S.

$$S \to \texttt{[}B \qquad\qquad B \to \texttt{]} \mid \texttt{]}S \mid \texttt{[}BB$$

Give a derivation of the string `[[][]][]`.

$$S \to [B \to \qquad \ldots \qquad \to \texttt{[[][]][]}$$

(5) Suppose the language $L = L(M)$ is accepted by a DFA $M = (Q, \Sigma, \delta, s, F)$. Define the language of words that have a suffix in L:

$$\{w \in \Sigma^* | \exists x \in \Sigma^*, y \in L \text{ such that } w = xy.\}$$

Show that this new language is regular by giving an NFA that accepts it.

(6) Let $M = (Q, \Sigma, \delta, s, F)$ be a DFA with $k = |Q|$ states. Prove that if there is a word $w \in \Sigma^*$ of length $|w| > k$ that is $w \notin L(M)$, then there are infinitely many words in $\Sigma^* \setminus L(M)$.

(7) Are the following true or false. Give a quick justification for your answer in either case:

(a) $A \subset B \subset \Sigma^*$ and B is regular, then A is also regular.

(b) $A \subset (a + b)^*$, and there exists a homomorphism h such that $h(A) = a^*$ then A is regular.

(8) Are the following languages regular? Prove your answer.

(a) $\{a^n b^m c^n | n, m \geq 0\}$

(d) $\{a^n b^n | 0 \leq n \leq 20\}$

(c) $\{a^n b^m | n \geq m, m \leq 20\}$

(d) $\{a^n b^m | n \geq m, m \geq 20\}$