Lecture 24: Low-Precision Machine Learning

CS4787/5777 — Principles of Large-Scale Machine Learning Systems

In this lecture, we'll talk about the relatively recent trend of using number formats with a small number of bits to reduce the computational cost of machine learning training. But first, we need to understand the baseline: what do traditional machine learning applications do? Usually, we reason about machine learning algorithms as if they were computing with infinite-precision real numbers, but of course this isn't actually the case. Traditional machine learning systems use **32-bit "single-precision" floating point numbers** (or occasionally 64-bit "double-precision" floating point numbers). How are these numbers represented?

31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
sign	8-bit exponent								23-bit mantissa																						

$$\text{represented number} = (-1)^{\text{sign}} \cdot 2^{\text{exponent}-127} \cdot 1.b_{22}b_{21}b_{20}\dots b_0$$

Note the implicit leading 1-bit before the mantissa. This is how floating point numbers are represented, except for three special cases. The first special case occurs for the smallest representable numbers, when the exponent $= 0$. In this case,

$$\text{represented number} = (-1)^{\text{sign}} \cdot 2^{-126} \cdot 0.b_{22}b_{21}b_{20}\dots b_0.$$

When the mantissa is also zero, this represents 0 (note the possibility of negative zero). When the mantissa is non-zero, these are so-called *denormal numbers*. Denormal numbers can cause performance issues in some code, since they require separate computational pathways. As a result, denormal numbers are often *flushed to zero*—we just round them to zero when they occur. Typically this has minimal numerical impact on machine learning code.

The second special case occurs wfor the largest exponent value, when exponent $= 255 = 2^8 - 1$. Here, there are two possibilities. If the mantissa is zero, then the floating point value represents either positive infinity ($+\infty$) or negative infinity ($-\infty$), depending on the sign bit. If the mantissa is nonzero, then the floating point value represents something that is *not a number*, called a NaN value. This usually indicates some sort of error. Here, the bits of the mantissa can contain a message that indicates how the error occurred.

Floating-point operations are usually equivalent to doing the following.

- Interpret the ordinary floating-point number inputs as real numbers.
- Perform the desired operation on those real numbers, producing a real-number result.
- Find the floating-point number that is closest to that result, and return that floating-point number.

In practice this is done with a circuit that is equivalent to this procedure, since obviously we can't actually compute with real numbers. This last rounding step has bounded relative error, such that for some constant $\epsilon_{\text{machine}}$ (called the machine epsilon)

$$|\mathsf{FloatingPointOp}(x, y) - \mathsf{RealNumberOp}(x, y)| \le |\mathsf{RealNumberOp}(x, y)| \cdot \epsilon_{\text{machine}}.$$

The machine epsilon $\epsilon_{\text{machine}}$ measures the numerical error caused by the floating point format. For single-precision floats, $\epsilon_{\text{machine}} \approx 1.2 \times 10^{-7}$.

There are a few special cases here as well.

- If the real-number result of the operation is larger in magnitude than the largest non-infinite representable floating point number (about $\pm 3.4 \times 10^{38}$ for single-precision floats), then the operation returns positive or negative infinity, as appropriate. This is called *overflow*.
- If the real-number result of the operation is smaller in magnitude than $1/2$ the smallest non-zero representable floating point number (about 1.4×10^{-45} for single-precision floats, assuming subnormal numbers are not flushed) then the result of the operation is zero. This is called *underflow*.
- If the operation does not make sense, such as division by zero or something like $(+\infty) - (+\infty)$, then the result is NaN.

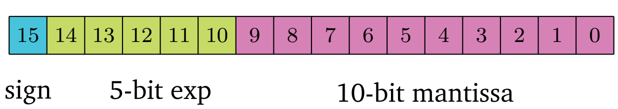
What is the cost of 32-bit floating-point computation?

- Need to store 32 bits in memory for each number used in the algorithm.
- Need to have specialized hardware to support the 32-bit FP computation (this is almost always the case, but may not hold for some embedded devices).

How can we reduce this cost? **Reduce the number of bits!**

Motivation: *machine learning computations are already noisy* due both to random sampling of examples in SGD and measurement imprecision when gathering data from the real world. As long as the numerical error from a reduced-bit-count representation is small compared with the noise already observable in SGD, we can expect that the performance won't be impacted much.

Half-precision floating point numbers. 16-bit "half-precision" floating-point numbers have recently become popular for machine learning tasks, particularly for deep learning. These numbers are represented much like single-precision floats, except with fewer bits.



In comparison to single-precision floating point numbers, half-precision floats have:

- a larger machine epsilon (meaning more numerical errors due to rounding), $\epsilon_{\text{machine}} \approx 9.8 \times 10^4$
- a smaller overflow threshold (meaning more overflow could happen) of about 6.5×10^4
- a larger underflow threshold of about 6.0×10^{-8}.

As long as the numbers we compute in the course of a learning algorithm stay far away from these thresholds, half-precision floating point numbers can do a pretty good job of approximating real numbers.

What benefits can we expect to get from computing in lower precision?

Pros of low-precision computing.

- Can fit more numbers (and therefore more training examples, activations, etc.) in memory
- Can store more numbers (and therefore larger models) in the cache
- Can transmit more numbers per second
- Can compute faster by extracting more parallelism in a fixed-width SIMD register
- Uses less energy

Cons of low-precision computing.

- Limits the range of numbers we can represent (the range between the overflow and underflow thresholds)
- Need specialized support from the hardware
- Introduces **quantization error** when we store a full-precision number in a low-precision representation

Let's try to address these cons.

One way to address limited range: use more exponent bits. Nothing (apart from the IEEE standard) forces us to size the exponent and mantissa of our 16-bit floating-point numbers as a 5-bit exponent and 10-bit mantissa. One popular alternate format (used in many ML processors including the TPU) is the **bfloat16** format. This splits the exponent and mantissa into 8/7 as follows

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
sign	8-bit exp								7-bit mantissa						

What can we say about the range of bfloat16 numbers as compared with IEEE half-precision floats and single-precision floats? How does their machine epsilon compare?

One way to address the need for hardware support: use fixed-point arithmetic instead. Fixed-point arithmetic represents a number as an integer times a *fixed* exponent. That is, we have something like this:

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
sign	fixed-point number														

and the number that it represents is

$$\text{represented number} = 2^{\text{fixed exponent} - 127} \cdot (\text{number as signed integer}) \, .$$

Most operations with fixed-point numbers can be done with integer arithmetic, which is supported on most processors already. It's also much cheaper to compute in terms of hardware costs. People have even used 8-bit fixed-point numbers for machine learning tasks!

What are the downsides of using fixed-point numbers for ML? Can you think of a place where you've already used something like fixed-point numbers in a programming assignment?

One way to address quantization error: use stochastic rounding. The standard mode of floating-point rounding is to round to the nearest representable number. This is the ordinary form of rounding that you probably learned in school.

However, for a lot of ML applications, especially where we are computing *averages*, this is not necessarily the best approach. The problem with round-to-nearest is that it is *biased* in a statistical sense. After rounding, the number no longer in expectation the same as the number before rounding.

To address this, people often use *stochastic rounding*, also known as *randomized rounding*. Randomized rounding rounds a number either up or down at random in such a way that the expected value of the output after rounding is equal to the input.

- This can be great for computing sums and averages where the law of large numbers will eventually kick in.
- Example: Hoeffding's inequality.