1 Kernel PCA

Question 1: Given datapoints $\mathbf{x}_1, \ldots, \mathbf{x}_n$, let $\mu = \frac{1}{n}\sum_{t=1}^{n} \mathbf{x}_t$. Let $\tilde{\mathbf{x}}_t = \mathbf{x}_t - \mu$. Show that for any $t, s \in [n]$, $\tilde{\mathbf{x}}_t^\top \tilde{\mathbf{x}}_s$ can be rewritten only in terms of inner products of x's, ie. in terms of inner products $\mathbf{x}_i^\top \mathbf{x}_j$ for $i, j \in [n]$

2 ISOMAP

Question 2: For the step 3 of ISOMAP, what is the problem if we use longest distance on the graph instead of shortest path?