1 Kernel Method

Question 1: Say we have a linear dimensionality reduction technique. That is an algorithm that does whatever it does and in the end finds a projection matrix W of size $d \times K$ and computes $Y = XW$. Which of the following statements are true:

A. **For the right algorithm, each column of W can be written as linear combination of $\mathbf{x}_1, \ldots, \mathbf{x}_n$**

B. **Algorithm can always be equivalently rewritten with a W such that each column of W is a linear combination of $\mathbf{x}_1, \ldots, \mathbf{x}_n$**

C. **Y can sometimes be computed based on inner products between data points**

D. **Y can always be computed based on inner products between data points**

Question 2: Say $d = 1$ so that each data point is one dimensional. Now consider the function

$$k(x, y) = \exp(x \cdot y)$$

Can you figure out a feature mapping $x \mapsto \Phi(x)$ such that for any $x, y \in \mathbb{R}$, $\langle \Phi(x), \Phi(y) \rangle = k(x, y)$.

Hint: $\Phi(x)$ could be infinite dimensional and think of an expansion of the exponential function.