Machine Learning for Data Science (CS4786)
Lecture 19

Graphical Models

Course Webpage :
http://www.cs.cornell.edu/Courses/cs4786/2016sp/

Graphical Models

- A graph whose nodes are variables $X_1, \ldots, X_N$
- Graphs are an intuitive way of representing relationships between large number of variables
- Allows us to abstract out the parametric form that depends on θ and the basic relationship between the random variables.

Conditional and Marginal Independence

- Conditional independence
  - X_i is conditionally independent of X_j given $A \subset \{X_1, \ldots, X_N\}$:

$$\begin{aligned} X_i \perp X_j | A &\Leftrightarrow P_\theta(X_i, X_j | A) = P_\theta(X_i | A) \times P_\theta(X_j | A) \\ &\Leftrightarrow P_\theta(X_i | X_j, A) = P_\theta(X_i | A) \end{aligned}$$

- Marginal independence:

$$X_i \perp X_j | \varnothing \Leftrightarrow P_\theta(X_i, X_j) = P_\theta(X_i) P_\theta(X_j)$$

Bayesian Networks

- Directed acyclic graph (DAG): $G = (V, E)$
- Joint distribution P_θ over $X_1, \ldots, X_n$ that factorizes over G:

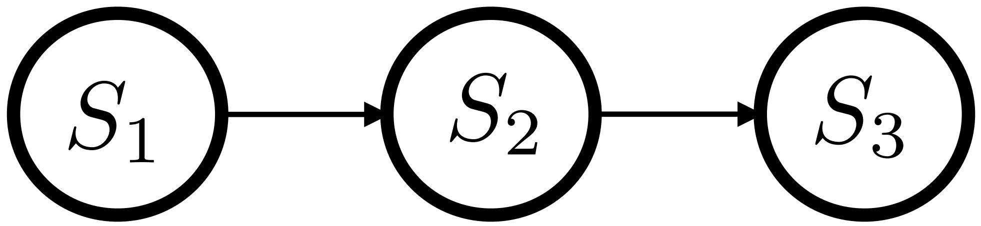
$$P_\theta(X_1, \ldots, X_n) = \prod_{i=1}^{N} P_\theta(X_i | \mathrm{Parent}(X_i))$$

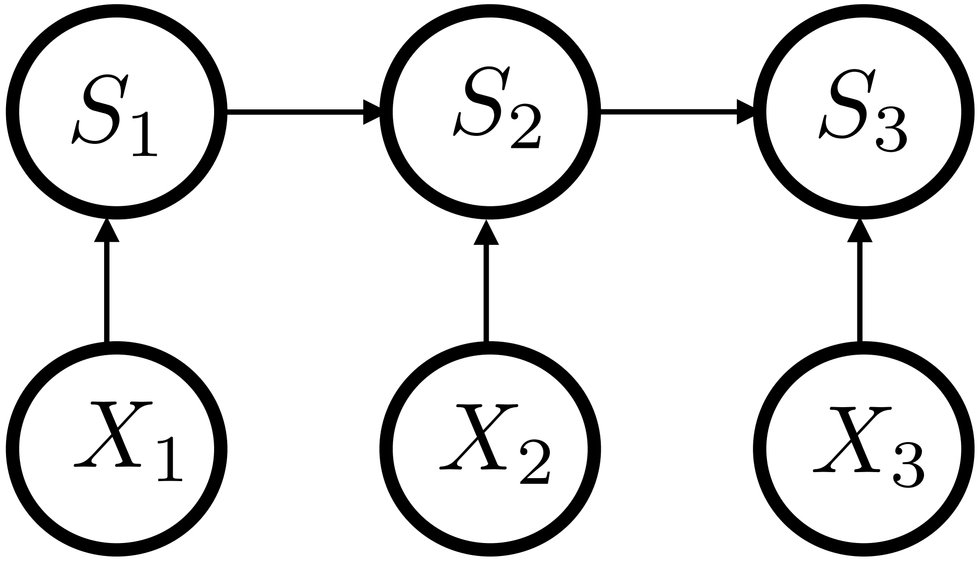
- Hence Bayesian Networks are specified by G along with CPD's over the variables (given their parents)

Local Markov Property

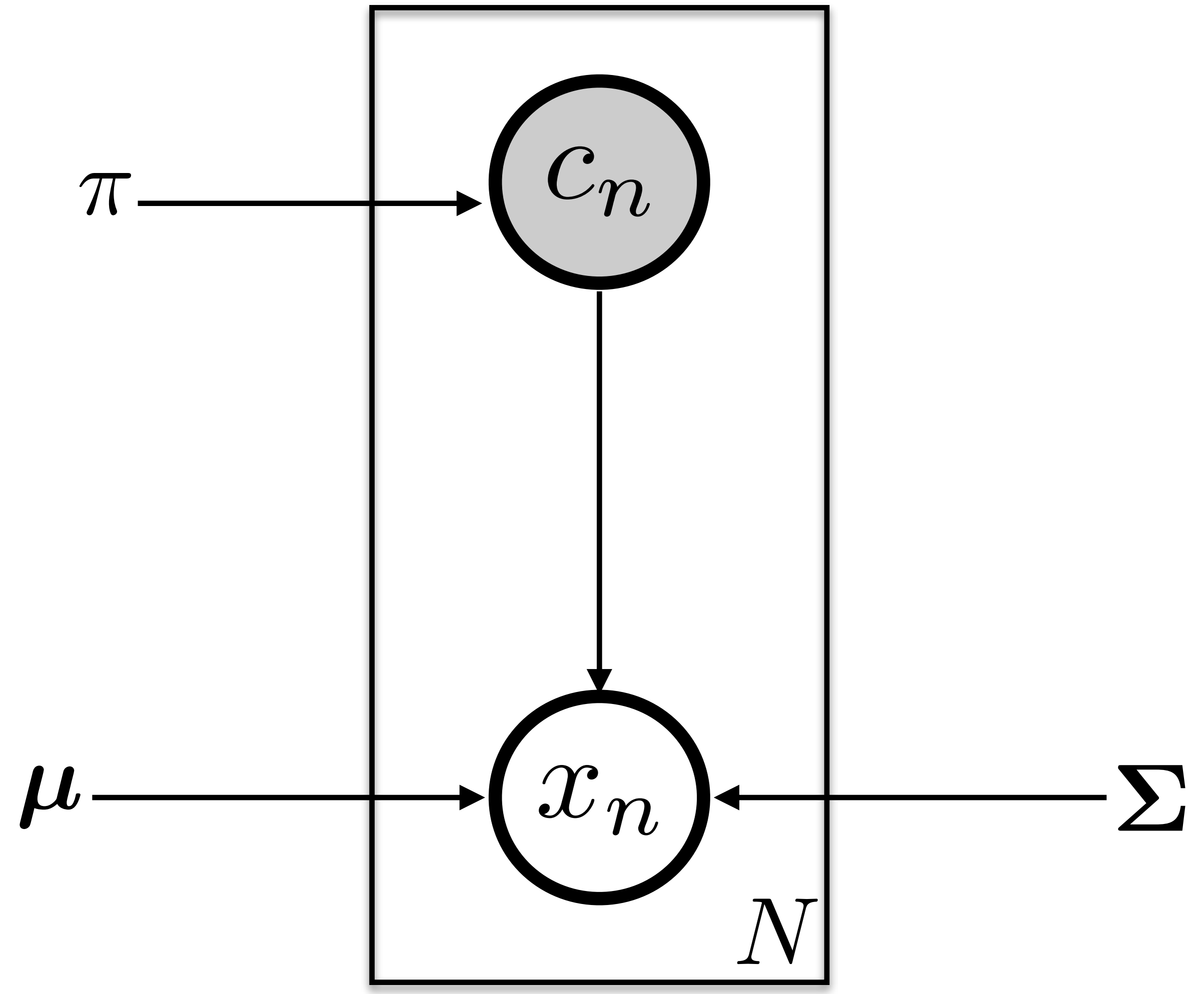
- Each variable is conditionally independent of its non-descendants given its parents
- Any joint distribution satisfying the local markov property w.r.t. graph factorizes over the graph

Example: Sum of Coin Flips

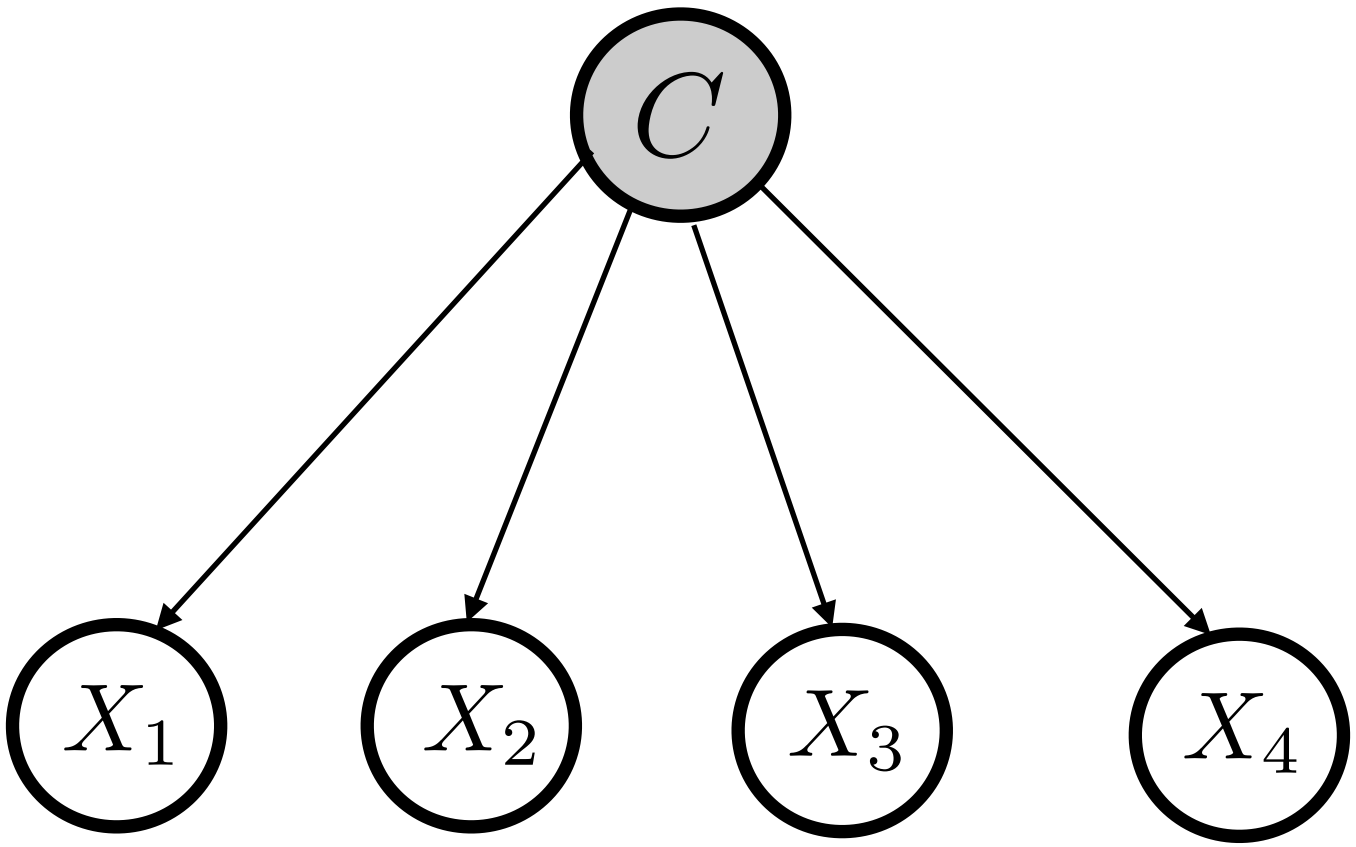




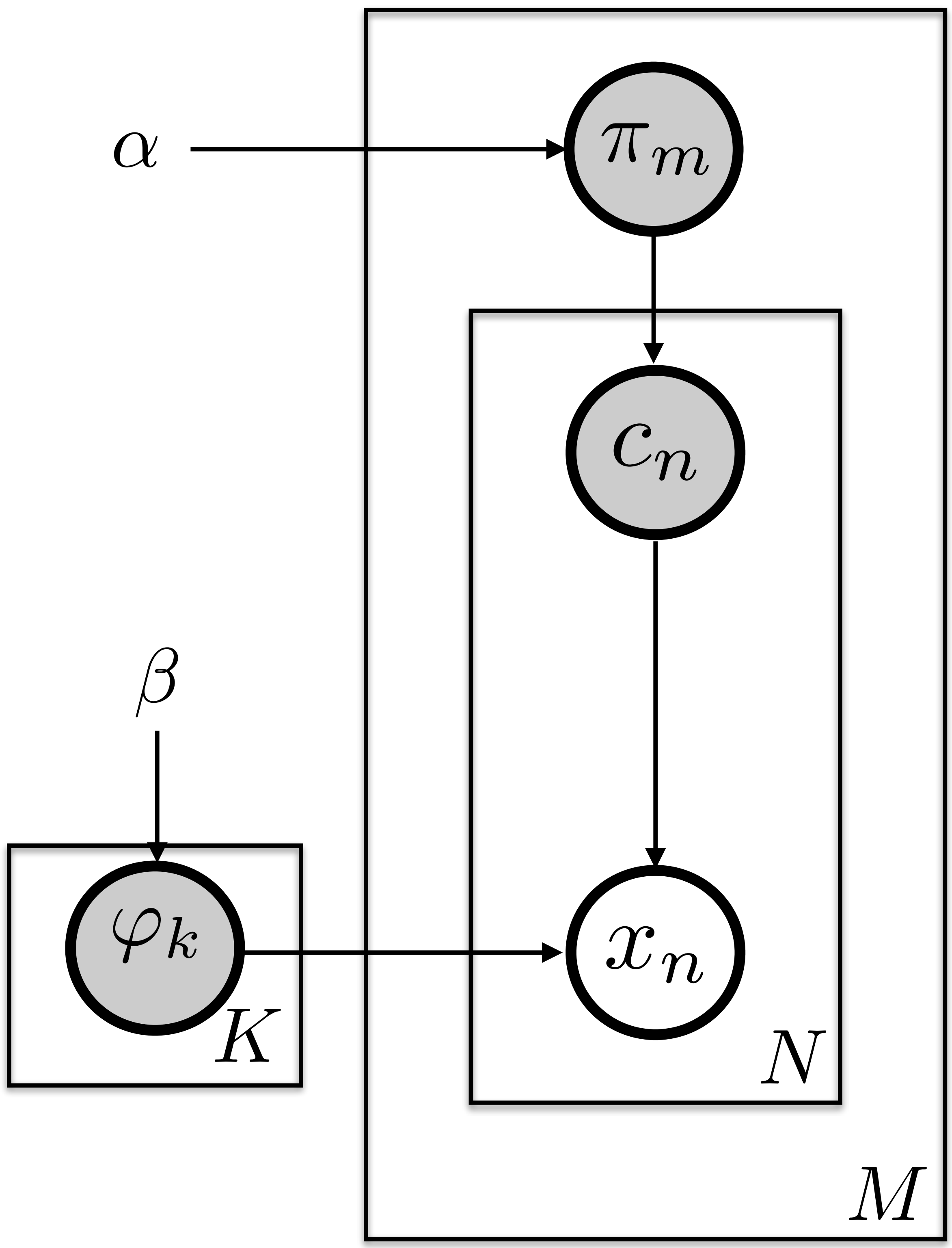
EXAMPLE: MIXTURE MODELS



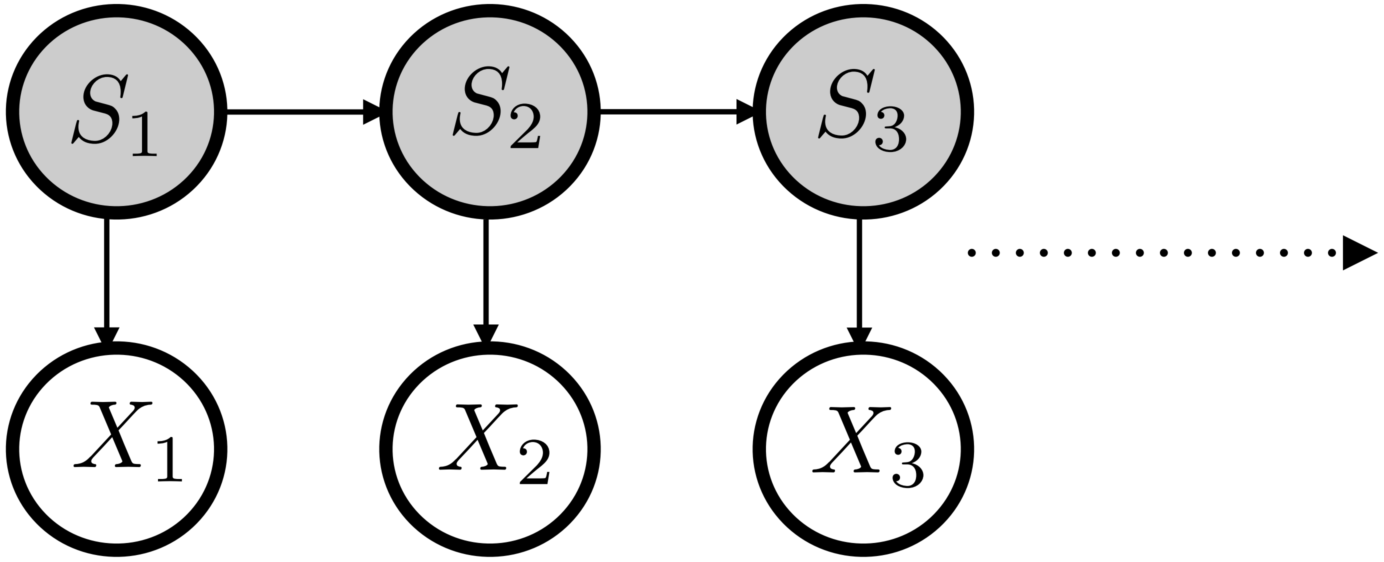
Example: Naive Bayes Classifier



Example: Latent Dirichlet Allocation



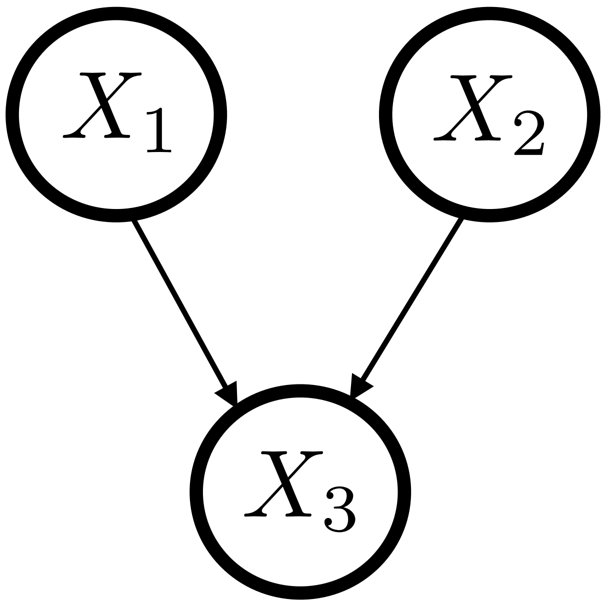
Example: Hidden Markov Model



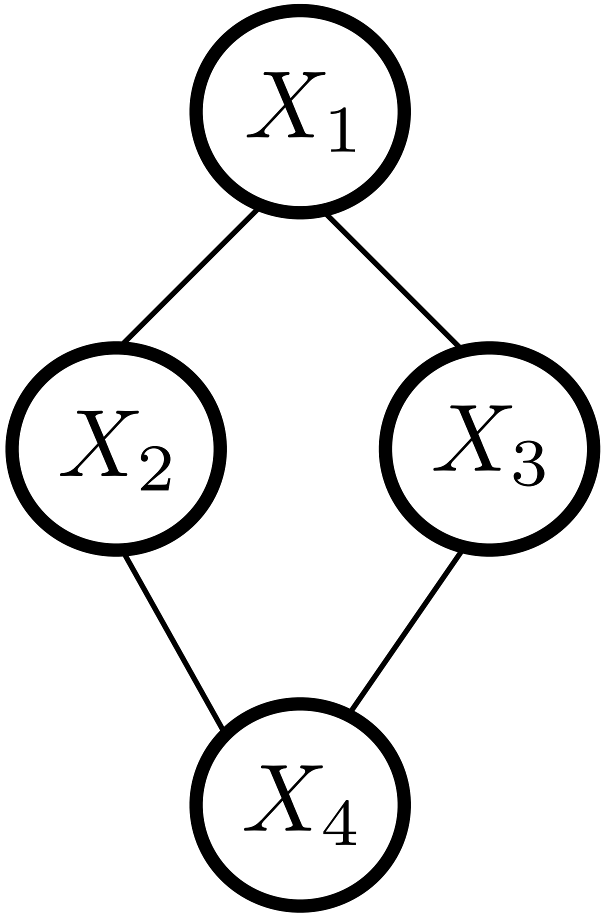
Markov Networks

- Not all distributions can be represented by Bayesian networks
- We also have undirected graphical models.
- Undirected graph $G = (V, E)$ and a set of RV's $X_1, \ldots, X_N$ form a markov network if
    - Any two non adjacent variables are conditionally independent given all other variables
    - Given its neighbors a variable is conditionally independent of all other variables
    - Any two sets of variables are conditionally independent given a separating set

Representational Power: BN Vs MN



No undirected graph can capture the above dependence



No directed graph can capture the above dependence

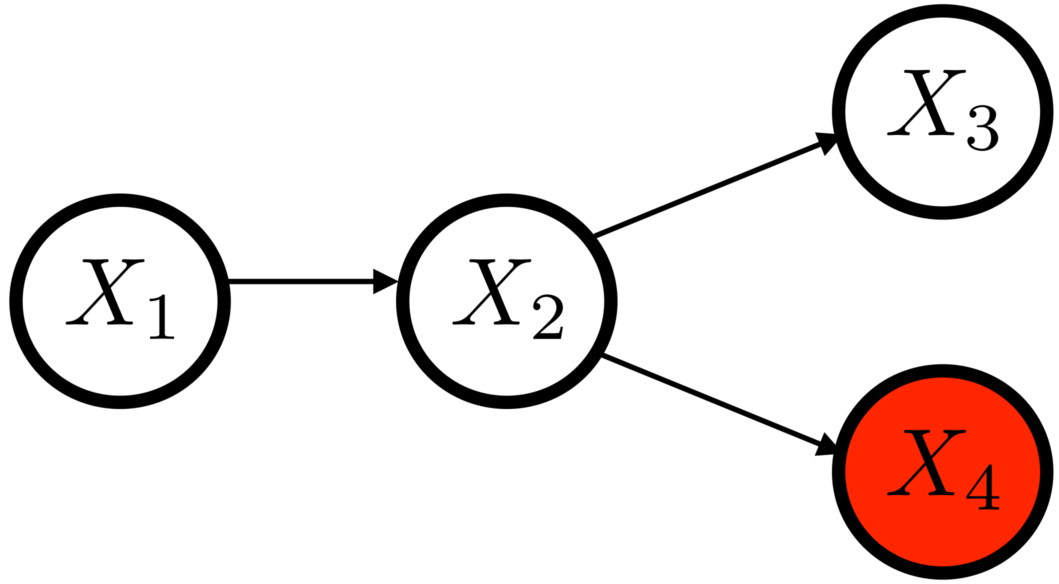
Inference in Graphical Models

Given parameters of a graphical model, we can answer any questions about distributions of variables in the model
Example queries:

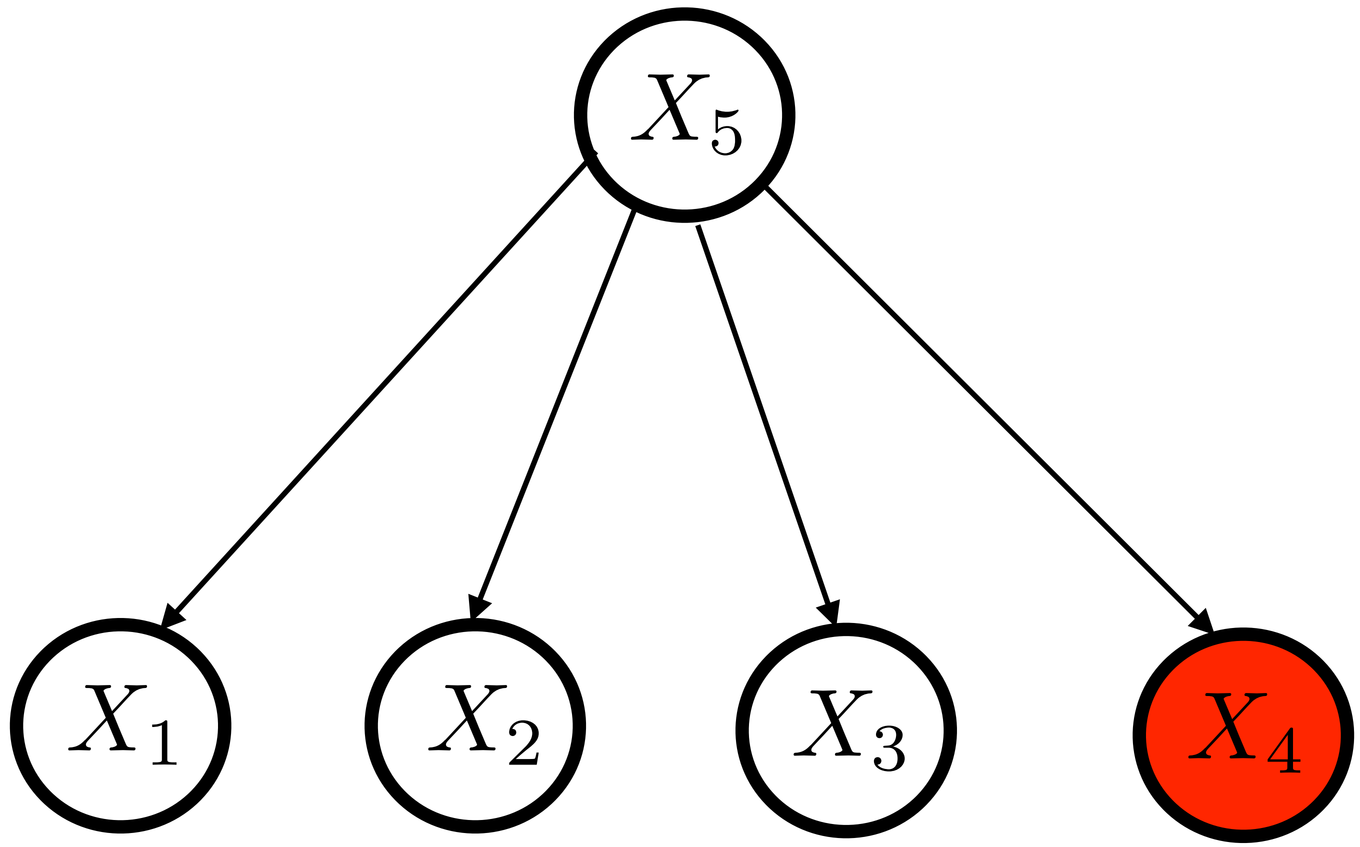
1. What is the probability of a given assignment for a subset of variables (marginal)?
2. What is the probability of a particular assignment of a subset of variables given observed values (evidence) of some subset of the variables (conditional)?
3. Given observed values (evidence) of some subset of variables what is the most likely assignment for a given subset of variables?

Suffices to calculate marginals.

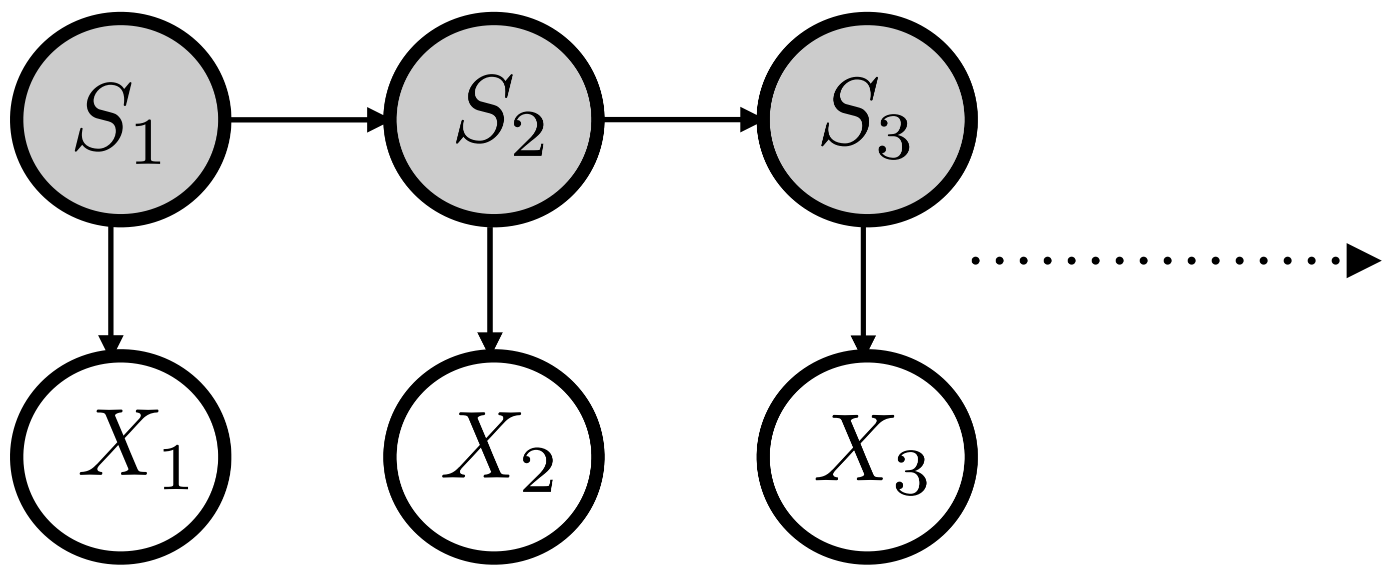
Variable Elimination: Examples



Variable Elimination: Order Matters



Variable Elimination: Examples



Variable Elimination: Bayesian Network

Initialize List with conditional probability distributions

Pick an order of elimination I for remaining variables

For each $X_i \in I$

Find distributions in List containing variable X_i and remove them

Define new distribution as the sum (over values of X_i) of the product of these distributions

Place the new distribution on List

End

Return List

Message Passing

- Often we need more than one marginal computation
- Over variables we need marginals for, there are many common distributions/potentials in the list
- Can we exploit structure and compute these intermediate terms that can be reused?