Assessing Learning Results

CS4780 – Machine Learning
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Reading: Mitchell Chapter 5

Outline

- **What is the true error of classification rule h?**
- **Is rule h_1 more accurate than h_2?**
- **Is learning algorithm A1 better than A2?**
- **Cross Validation**

Learning as Prediction

Definition: *A particular* **instance of a learning problem** *is described by a probability distribution* $P(X,Y)$.

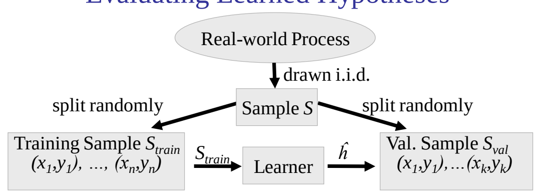
Definition: *A* **sample** $S = ((\vec{x}_1, y_1), ..., (\vec{x}_n, y_n))$ *is independently identically distributed (i.i.d.) according to* $P(X,Y)$.

Definition: *The* **error on sample** S $Err_S(h)$ *of a hypothesis* h *is* $Err_S(h) = \frac{1}{n}\sum_{i=1}^{n} \Delta(h(\vec{x}_i), y_i)$.

Definition: *The* **prediction/generalization/true error** $Err_P(h)$ *of a hypothesis* h *for a learning task* $P(X,Y)$ *is*

$$Err_P(h) = \sum_{\vec{x} \in X, y \in Y} \Delta(h(\vec{x}), y) P(X = \vec{x}, Y = y).$$

Evaluating Learned Hypotheses

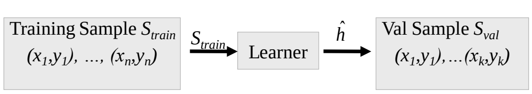


- **Goal:** Find h with small prediction error $Err_P(h)$ over $P(X,Y)$.
- **Question:** How good is $Err_P(\hat{h})$ of $\hat{h}$ found on training sample S_{train}.

- **Training Error:** Error $Err_{S_{train}}(\hat{h})$ on training sample.
- **Validation Error:** Error $Err_{S_{val}}(\hat{h})$ is an estimate of $Err_P(\hat{h})$.

What is the True Error of an Hypothesis?

- **Given**
  - Sample of labeled instances S
  - Learning Algorithm A
- **Setup**
  - Partition S randomly into S_{train} (70%) and S_{val} (30%)
  - Train learning algorithm A on S_{train}, result is $\hat{h}$.
  - Apply $\hat{h}$ to S_{val} and compare predictions against true labels.
- **Test**
  - Error on test sample $Err_{S_{val}}(\hat{h})$ is estimate of true error $Err_P(\hat{h})$.
  - Compute confidence interval.



Binomial Distribution

- The probability of observing x heads in a sample of n independent coin tosses, where in each toss the probability of heads is p, is

$$P(X = x | p, n) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

- **Normal approximation:** For $np(1-p)>=5$ the binomial can be approximated by the normal distribution with
  - Expected value: $E(X)=np$ Variance: $Var(X)=np(1-p)$
  - With probability δ, the observation x falls in the interval

$$E(X) \pm z_\delta \sqrt{Var(X)}$$

δ	50%	68%	80%	90%	95%	98%	99%
z_δ	0.67	1.00	1.28	1.64	1.96	2.33	2.58

Text Classification Example: Results

- **Data**
  - Training Sample: 2000 examples
  - Test Sample: 600 examples
- **Unpruned Tree:**
  - Size: 437 nodes Training Error: 0.0% Test Error: 11.0%
- **Early Stopping Tree:**
  - Size: 299 nodes Training Error: 2.6% Test Error: 9.8%
- **Post-Pruned Tree:**
  - Size: 167 nodes Training Error: 4.0% Test Error: 10.8%
- **Rule Post-Pruning:**
  - Size: 164 tests Training Error: 3.1% Test Error: 10.3%

Is Rule h_1 More Accurate than h_2? (Same Validation Sample)

- **Given**
  - Sample of labeled instances S
  - Learning Algorithms A_1 and A_2
- **Setup**
  - Partition S randomly into S_{train} (70%) and S_{val} (30%)
  - Train learning algorithms A_1 *and* A_2 on S_{train}, result are $\hat{h}_1$ and $\hat{h}_2$.
  - Apply $\hat{h}_1$ and $\hat{h}_2$ to S_{val} and compute $Err_{S_{val}}(\hat{h}_1)$ and $Err_{S_{val}}(\hat{h}_2)$.
- **Test**
  - Decide, if $Err_P(\hat{h}_1) \neq Err_P(\hat{h}_2)$?
  - Null Hypothesis: $Err_{S_{val}}(\hat{h}_1)$ and $Err_{S_{val}}(\hat{h}_2)$ come from binomial distributions with same p.

    → Binomial Sign Test (McNemar's Test)

Is Rule h_1 More Accurate than h_2? (Different Validation Samples)

- **Given**
  - Samples of labeled instances S_1 and S_2
  - Learning Algorithms A_1 and A_2
- **Setup**
  - Partition S_1 randomly into S_{train1} (70%) and S_{val1} (30%)
    Partition S_2 randomly into S_{train2} (70%) and S_{val2} (30%)
  - Train learning algorithm A_1 on S_{train1} and A_2 on S_{train2}, result are $\hat{h}_1$ and $\hat{h}_2$.
  - Apply $\hat{h}_1$ to S_{val1} and $\hat{h}_2$ to S_{val2} and get $Err_{S_{val1}}(\hat{h}_1)$ and $Err_{S_{val2}}(\hat{h}_2)$.
- **Test**
  - Decide, if $Err_P(\hat{h}_1) \neq Err_P(\hat{h}_2)$?
  - Null Hypothesis: $Err_{S_{val1}}(\hat{h}_1)$ and $Err_{S_{val2}}(\hat{h}_2)$ come from binomial distributions with same p.

    → t-Test (z-Test)

Is Learning Algorithm A_1 better than A_2?

- **Given**
  - k samples $S_1 \ldots S_k$ of labeled instances
  - Learning Algorithms A_1 and A_2
- **Setup**
  - For i from 1 to k
    - Partition S_i randomly into S_{train} (70%) and S_{val} (30%)
    - Train learning algorithms A_1 *and* A_2 on S_{train}, result are $\hat{h}_1$ and $\hat{h}_2$.
    - Apply $\hat{h}_1$ and $\hat{h}_2$ to S_{val} and compute $Err_{S_{val}}(\hat{h}_1)$ and $Err_{S_{val}}(\hat{h}_2)$.
- **Test**
  - Decide, if $E_S(Err_P(A_1(S_{train}))) \neq E_S(Err_P(A_2(S_{train})))$?
  - Null Hypothesis: $Err_{S_{val}}(A_1(S_{train}))$ and $Err_{S_{val}}(A_2(S_{train}))$ come from same distribution over samples S.

K-fold Cross Validation

- **Given**
  - Sample of labeled instances S
  - Learning Algorithms A_1 and A_2
- **Compute**
  - Randomly partition S into k equally sized subsets $S_1 \ldots S_k$
  - For i from 1 to k
    - Train A_1 and A_2 on $S_1 \ldots S_{i-1} S_{i+1} \ldots S_k$ and get $\hat{h}_1$ and $\hat{h}_2$.
    - Apply $\hat{h}_1$ and $\hat{h}_2$ to S_i and compute $Err_{S_i}(\hat{h}_1)$ and $Err_{S_i}(\hat{h}_2)$.
- **Estimate**
  - Average $Err_{S_i}(\hat{h}_1)$ is estimate of $E_S(Err_P(A_1(S_{train})))$
  - Average $Err_{S_i}(\hat{h}_2)$ is estimate of $E_S(Err_P(A_2(S_{train})))$
  - Count how often $Err_{S_i}(\hat{h}_1) > Err_{S_i}(\hat{h}_2)$ and $Err_{S_i}(\hat{h}_1) < Err_{S_i}(\hat{h}_2)$