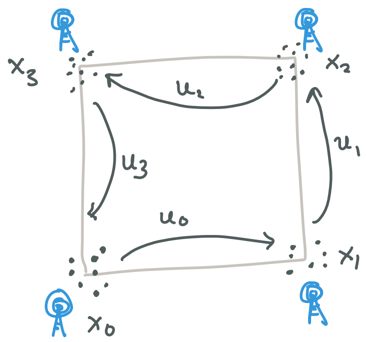
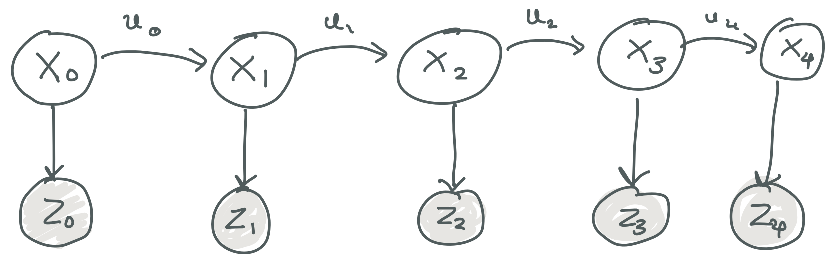
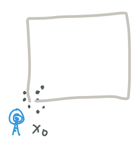




ASSUME

* LOCATION OF LANDMARK AT TIME t, l_t, IS KNOWN
* OBSERVE Z_t WHICH IS THE DELTA FROM
* OBSERVATION MODEL: $P(Z_t \mid X_t) \propto \exp\left(-\frac{\|Z_t-(l_t-X_t)\|^2}{\Sigma_t^{obs}}\right)$
* PRIOR AT $t=0$: $P(X_0) \propto \exp\left(-\frac{\|X_0-\mu_0\|^2}{\Sigma_0}\right)$



T = 0

GIVEN OBSERVATION Z_0, WHAT IS THE MOST LIKELY STATE X_0?

$$\underset{X_0}{\operatorname{argmax}} \log P(X_0 \mid Z_0)$$

$$= \underset{X_0}{\operatorname{argmax}} \log \frac{P(Z_0 \mid X_0)\, P(X_0)}{P(Z_0)}$$

[$P(Z_0)$ DOES NOT DEPEND ON X_0]

$$= \underset{X_0}{\operatorname{argmax}} \log P(Z_0 \mid X_0) + \log P(X_0)$$

SUBSTITUTE PRIOR & OBSERVATION MODEL

$$= \underset{x_0}{\arg\max} \; \log\left[\frac{1}{\eta}\exp\left(-\frac{\|x_0-\mu_0\|^2}{\Sigma_0}\right)\right] +$$

$$\log\left[\frac{1}{\eta}\exp\left(-\frac{\|z_0-(l_0-x_0)\|^2}{\Sigma_0^{obs}}\right)\right]$$

$$= \underset{x_0}{\arg\min}\left[\frac{\|x_0-\mu_0\|^2}{\Sigma_0} + \frac{\|z_0-(l_0-x_0)\|^2}{\Sigma_0^{obs}}\right]$$

TAKE GRAD $\frac{\partial}{\partial x_0}(\cdot) = 0$ AND SET TO ZERO

$$\cancel{2}\frac{(x_0-\mu_0)}{\Sigma_0} + \cancel{2}\frac{(z_0-(l_0-x_0))}{\Sigma_0^{obs}} = 0$$

$$x_0\left(\frac{1}{\Sigma_0}+\frac{1}{\Sigma_0^{obs}}\right) = \frac{\mu_0}{\Sigma_0} + \frac{(l_0-z_0)}{\Sigma_0^{obs}}$$

CONTRIBUTION BY PRIOR

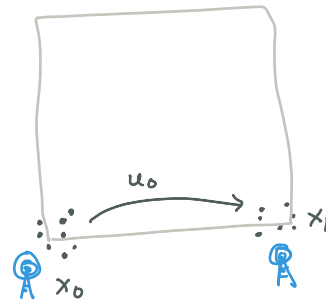
CONTRIBUTION BY OBSERVATION

$$\boxed{x_0 = \frac{\boxed{\frac{\mu_0}{\Sigma_0}} + \boxed{\frac{(l_0-z_0)}{\Sigma_0^{obs}}}}{\left(\frac{1}{\Sigma_0}+\frac{1}{\Sigma_0^{obs}}\right)}}$$

$T=1$

GIVEN OBSERVATION Z_0, Z_1 AND ACTION u_0 FIND THE MOST LIKELY $X_0, X_1, \ldots$

ASSUME DYNAMICS MODEL: $P(x_1 | x_0, u_0) \propto \exp\left(-\frac{\|x_1 - (x_0+u_0)\|^2}{\Sigma_0^{dyn}}\right)$



$$\arg\max_{x_0, x_1} P(x_1, x_0 \,|\, z_1, u_0, z_0)$$

(APPLY BASE RULE)

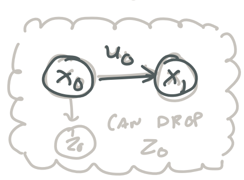
$$\arg\max_{x_0, x_1} \frac{P(z_1 | x_1, x_0, u_0, z_0)\, P(x_1, x_0 | u_0, z_0)}{P(z_1)}$$

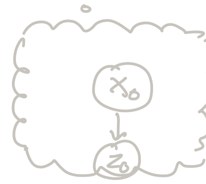
DOES NOT DEPEND ON x_0, x_1

$$\arg\max_{x_0, x_1} P(z_1 | x_1)\, P(x_1, x_0 | u_0, z_0)$$

$P(A,B) \stackrel{?}{=} P(A|B)P(B)$

$$\arg\max_{x_0, x_1} P(z_1 | x_1)\, P(x_1 | x_0, u_0, z_0)\, P(x_0 | u_0, z_0)$$





CAN DROP u_0

$$\arg\max_{x_0, x_1} P(z_1 | x_1)\, P(x_1 | x_0, u_0)\, P(x_0 | z_0)$$

BAYES THEOREM

$$\arg\max_{x_0, x_1} P(z_1 \mid x_1) P(x_1 \mid x_0, u_0) P(z_0 \mid x_0) P(x_0)$$

$$\arg\max_{x_0, x_1} \log P(z_1 \mid x_1) + \log P(x_1 \mid x_0, u_0) + \log P(z_0 \mid x_0) + \log P(x_0)$$

$$\arg\max_{x_0, x_1} \log\left(\frac{1}{\eta}\exp\left(-\frac{\|z_1-(l_1-x_1)\|^2}{\Sigma_1^{obs}}\right)\right) + \log\left(\frac{1}{\eta}\exp\left(-\frac{\|x_1-(x_0+u_0)\|^2}{\Sigma_1}\right)\right)$$

$$+\log\left(\frac{1}{\eta}\exp\left(-\frac{\|z_0-(l_0-x_0)\|^2}{\Sigma_0^{obs}}\right)\right) + \log\left(\frac{1}{\eta}\exp\left(-\frac{\|x_0-\mu_0\|^2}{\Sigma_0}\right)\right)$$

$$\arg\min_{x_0, x_1} \frac{\|z_1-(l_1-x_1)\|^2}{\Sigma_1^{obs}} + \frac{\|x_1-(x_0+u_0)\|^2}{\Sigma_1} + \frac{\|z_0-(l_0-x_0)\|^2}{\Sigma_0^{obs}} + \frac{\|x_0-\mu_0\|^2}{\Sigma_0}$$

$$\nabla_{x_0}(\cdot) = 0 \Rightarrow -\frac{x_1-(x_0+u_0)}{\Sigma_1} + \frac{z_0-(l_0-x_0)}{\Sigma_0^{obs}} + \frac{x_0-\mu_0}{\Sigma_0} = 0$$

$$\nabla_{x_1}(\cdot) = 0 \Rightarrow \frac{z_1-(l_1-x_1)}{\Sigma_1^{obs}} + \frac{x_1-(x_0+u_0)}{\Sigma_1} = 0$$

AT THIS POINT, IT IS HARD TO ANALYTICALLY MAKE PROGRESS

AND WE SOLVE NUMERICALLY

$$\begin{bmatrix} A \end{bmatrix} \underbrace{\begin{bmatrix} x_0 \\ x_1 \end{bmatrix}}_{\theta} = \begin{bmatrix} B \end{bmatrix}$$

$$\theta = A^{-1} b$$

x_0: BALANCE PRIOR, OBS, MOTION

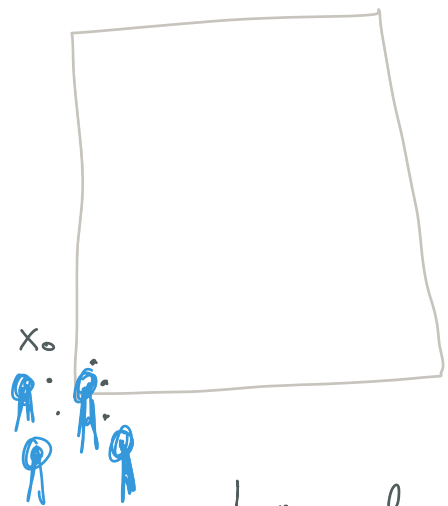
$$\frac{\mu_0}{\Sigma}, \quad \frac{(l_0 - z_0)}{\Sigma^{obs}}, \quad \frac{(x_1 - u_0)}{\Sigma^{dyn}}$$

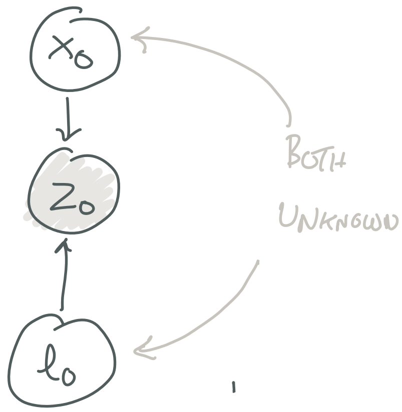
IN GENERAL

$$P(x_0, x_1, \dots \mid z_0, u_0, z_1, \dots)$$

IS A LEAST SQUARES OPTIMIZATION

Now what if Landmarks are Unknown





Let l_0 be landmark which is unknown.

$$\underset{x_0,\, l_0}{\arg\max}\; P(x_0, l_0 \mid z_0)$$

$$\underset{x_0,\, l_0}{\arg\max}\; \frac{P(z_0 \mid x_0, l_0)\, P(x_0, l_0)}{P(z_0)}$$

$$\underset{x_0,\, l_0}{\arg\max}\; \underbrace{\log P(z_0 \mid x_0, l_0)}_{=\text{ same obs model.}} + \log P(x_0) + \log P(l_0)$$

= prior over x_0

= prior over landmark

$$\underset{x_0,\, l_0}{\arg\min} \; \frac{\| z_0 - (x_0 - l_0) \|^2}{\Sigma_0^{obs}} + \frac{(x_0 - \mu_0)^2}{\Sigma_0} + \frac{(l_0 - \mu_0^l)^2}{\Sigma_0}$$

$$\frac{\partial}{\partial}(\cdot) = 0 \Rightarrow \quad x_0 = \quad \ldots$$

$$l_0 = \quad \ldots$$

LANDMARKS BECOME EXTRA VARIABLES YOU SOLVE FOR!