Adversarial Search

CS472/CS473 — Fall 2005

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Game Playing

An AI Favorite

- structured task
- clear definition of success and failure
- does not require large amounts of knowledge (at first glance)
- focus on games of perfect information

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Game Playing

Initial State is the initial board/position

Successor Function defines the set of legal moves from any position

Terminal Test determines when the game is over

Utility Function gives a numeric outcome for the game

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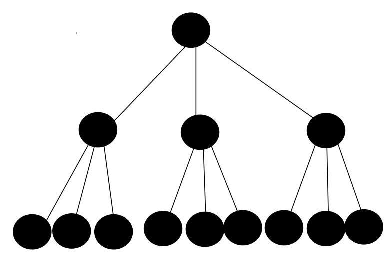
Game Playing as Search



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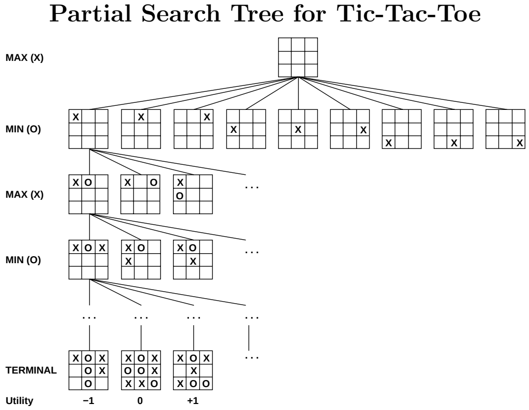
Partial Search Tree for Tic-Tac-Toe



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Simplified Minimax Algorithm

1. Expand the entire tree below the root.
2. Evaluate the terminal nodes as wins for the minimizer or maximizer (i.e. utility).
3. Select an unlabeled node, n, all of whose children have been assigned values. If there is no such node, we're done — return the value assigned to the root.
4. If n is a minimizer move, assign it a value that is the minimum of the values of its children. If n is a maximizer move, assign it a value that is the maximum of the values of its children. Return to Step 3.

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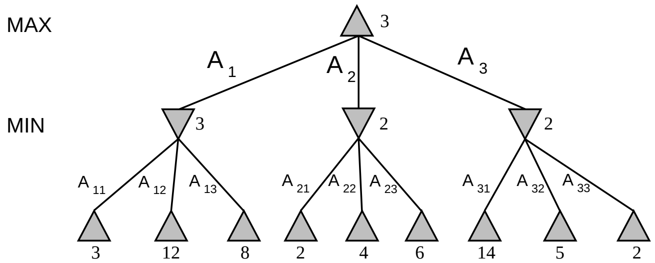
Another Example



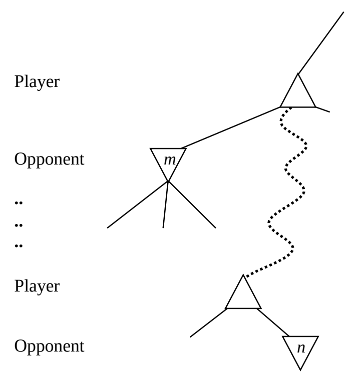
Minimax

```
function MINIMAX-DECISION(game) returns an operator

  for each op in OPERATORS[game] do
      VALUE[op] ← MINIMAX-VALUE(APPLY(op, game), game)
  end
  return the op with the highest VALUE[op]

function MINIMAX-VALUE(state, game) returns a utility value

  if TERMINAL-TEST[game](state) then
      return UTILITY[game](state)
  else if MAX is to move in state then
      return the highest MINIMAX-VALUE of SUCCESSORS(state)
  else
      return the lowest MINIMAX-VALUE of SUCCESSORS(state)
```


Improving Minimax — $\alpha - \beta$ pruning

Idea: Avoid generating the whole search tree

Approach: Analyze which subtrees have no influence on the solution



If m is better than n for Player, never get to n in play.

$\alpha - \beta$ Search

α = lower bound on Max's outcome; initially set to $-\infty$
β = upper bound on Min's outcome ; initially set to $+\infty$

We'll call $\alpha - \beta$ procedure recursively with a narrowing range between α and β.

Maximizing levels may reset α to a higher value; Minimizing levels may reset β to a lower value.

$\alpha - \beta$ Search Algorithm

1. If terminal state, compute $e(n)$ and return the result.
2. Otherwise, if the level is a **minimizing** level,
   - Until no more children or $\beta \leq \alpha$,
     - $v_i \leftarrow \alpha - \beta$ search on child.
     - If $v_i < \beta$, $\beta \leftarrow v_i$.
   - Return $min(v_i)$.
3. Otherwise, the level is a **maximizing** level:
   - Until no more children or $\alpha \geq \beta$,
     - $v_i \leftarrow \alpha - \beta$ search on child.
     - If $v_i > \alpha$, set $\alpha \leftarrow v_i$.
   - Return $max(v_i)$.