Constraint Satisfaction

CS472/CS473 — Fall 2005

Moving to a different formalism...

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      SEND
  +   MORE
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     MONEY
```

Consider state space for cryptarithmetic (e.g. DFS).

Is this (DFS) how humans tackle the problem?

Human problem solving appears more **sophisticated!**

For example, we derive new constraints on the fly.

→ **little** or **no** search!

Constraint Satisfaction Problems (CSP)

A powerful representation for (discrete) search problems.

A **Constraint Satisfaction Problem (CSP)** is defined by:

- **X** is a set of n variables $X_1, X_2, \ldots, X_n$,
  each defined by a finite domain $D_1, D_2, \ldots, D_n$
  of possible values.
- **C** is a set of constraints $C_1, C_2, \ldots, C_m$.
  Each C_i involves a subset of the variables; specifies
  the allowable combinations of values for that subset.
- A **solution** is an assignment of values to the variables
  that satisfies all constraints.

Cryptarithmetic as a CSP

Problem: TWO + TWO = FOUR

Variables:

$T \in \{0, \ldots, 9\}$; $W \in \{0, \ldots, 9\}$; $O \in \{0, \ldots, 9\}$;

$F \in \{0, \ldots, 9\}$; $U \in \{0, \ldots, 9\}$; $R \in \{0, \ldots, 9\}$;

$X_1 \in \{0, \ldots, 1\}$; $X_2 \in \{0, \ldots, 1\}$; $X_3 \in \{0, \ldots, 1\}$;

Constraints:

$O + O = R + 10 * X_1$

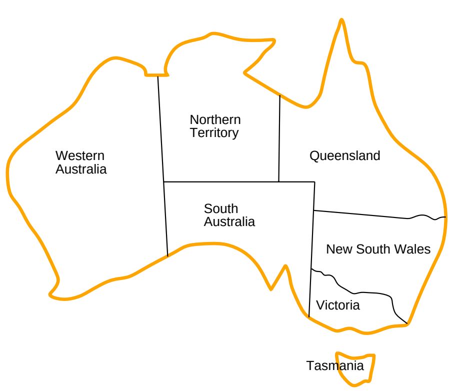
$X_1 + W + W = U + 10 * X_2$

$X_2 + T + T = O + 10 * X_3$

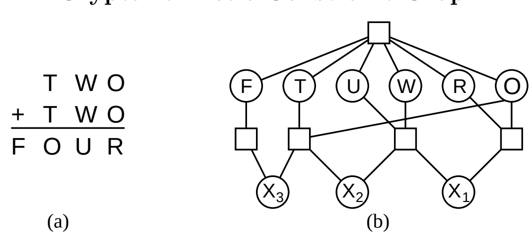
$X_3 = F$

each letter has a different digit (F ≠ T, F ≠ U, etc);

Map-Coloring Problem



Cryptarithmetic Constraint Graph



Types of Constraints

Unary Constraints: Restriction on single variable

Binary Constraints: Restriction on pairs of variables

Higher-Order Constraints: Restriction on more than two variables

Constraint Satisfaction Problems (CSP)

For a given CSP the problem is one of the following:

1. find all solutions
2. find one solution
   - just a feasible solution, or
   - a "reasonably good" feasible solution, or
   - the optimal solution given an objective function
3. determine if a solution exists

How to View a CSP as a Search Problem?

Initial State – state in which all the variables are unassigned.

Successor function – assign a value to a variable from a set of possible values.

Goal test – check if all the variables are assigned and all the constraints are satisfied.

Path cost – assumes constant cost for each step

Branching Factor

Approach 1 – any unassigned variable at a given state can be assigned a value by an operator: branching factor as high as sum of size of all domains.

Approach 2 – since order of variable assignment not relevant, consider as the successors of a node just the different values of a *single* unassigned variable: max branching factor = max size of domain.

Maximum Depth of Search Tree

n the number of variables → all solutions at depth n.

Prefer DFS or BFS?

CSP – Goal Decomposed into Constraints

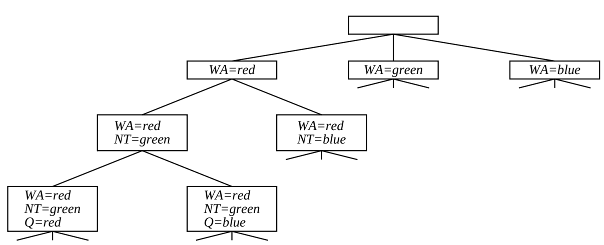
Backtracking Search : a DFS that

- chooses values for variables one at a time
- checks for *consistency* with the constraints.

Decisions during search :

- Which variable to choose next for assignment.
- Which value to choose next for the variable.

Example



General Purpose Heuristics

- **Variable and value ordering:**
  Minimum remaining values (MRV): choose the variable with the *fewest* possible values.

  Degree heuristic: assign a value to the variable that is involved in the largest number of constraints on other unassigned variables.

  Least-constraining value heuristic: choose a value that rules out the smallest number of values in variables connected to the current variable by constraints.