Informed Search

CS472/CS473 — Fall 2005

Informed Methods: Heuristic Search

Idea : Informed search by using problem-specific knowledge.

Best-First Search : Nodes are selected for expansion based on an *evaluation function*, $f(n)$. Traditionally, f is a cost measure.

Heuristic : Problem specific knowledge that (tries to) lead the search algorithm faster towards a goal state.

→ Heuristic search is an attempt to search the most promising paths first. Uses heuristics, or rules of thumb, to find the best node to expand next.

Generic Best-First Search

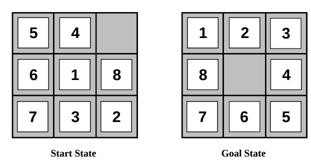
1. Set L to be the initial node(s) representing the initial state(s).
2. If L is empty, fail. Let n be the node on L that is "most promising" according to f. Remove n from L.
3. If n is a goal node, stop and return it (and the path from the initial node to n).
4. Otherwise, add *successors(n)* to L. Return to step 2.

Greedy Best-First Search

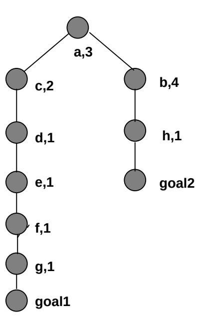
Heuristic function $h(n)$: estimated cost from node n to nearest goal node.

Greedy Search : Let $f(n) = h(n)$.

Example : 8-puzzle



Suboptimal Best-First Search



There exist strategies that enable optimal paths to be found without examining all possible paths.

A* Search

Idea: Use total estimated solution cost:

$g(n)$ Cost of reaching node n from initial node

$h(n)$ Estimated cost from node n to nearest goal

A* evaluation function: $f(n) = g(n) + h(n)$
→ $f(n)$ is estimated cost of cheapest solution through n.

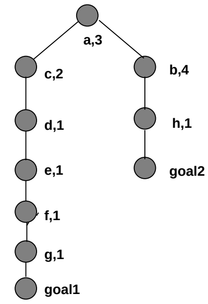
Comparison of Search Costs on 8-Puzzle

h1: number of misplaced tiles

h2: Manhattan distance

	Search Cost			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

Example



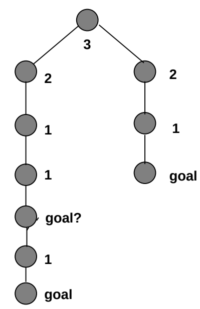
Admissibility

$h^*(n)$ *Actual* cost to reach a goal from n.

A heuristic function h is **optimistic** or **admissible** if $h(n) \leq h^*(n)$ for all nodes n.

If h is **admissible**, then the A* algorithm will never return a suboptimal goal node. (h **never overestimates** the cost of reaching the goal.)

Example



Example: Admissible Heuristic

What if $h(n) = h^*(n)$?

$$f(n) = g(n) + h^*(n)$$

The perfect heuristic function!

Example: Admissible Heuristic

What if $h(n) = 0$?

$$f(n) = g(n) + h(n)$$

8-puzzle

1. h_C = number of misplaced tiles
2. h_M = Manhattan distance

Which one should we use?

$$h_C \leq h_M \leq h^*$$

Comparison of Search Costs on 8-Puzzle

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Constructing Admissible Heuristics

- Use an admissible heuristic derived from a **relaxed** version of the problem.
- Use information from **pattern databases** that store exact solutions to subproblems of the problem.
- Use **inductive learning** methods.

Proving the optimality of A^*

Lemma: If h is admissible, then $f = g + h$ can be made non-decreasing.

1. g is non-decreasing since cost positive.
2. But h can be increasing, while still admissible. Example: Node p, with $f = 3 + 4 = 7$; child n, with $f = 4 + 2 = 6$.
3. But because any path through n is also a path through p, we can see that the value 6 is meaningless, because we already know the true cost is at least 7 (because h is admissible).
4. So, make $f = max(f(p), g(n) + h(n))$

Proof of the optimality of A^*

Assume: h admissible; f non-decreasing along any path.

Proof:
Let G be an optimal goal state, with path cost f^*.
Let G_2 be a suboptimal goal state, with path cost $g(G_2) > f^*$.
Let n is a node on an optimal path to G.

Because h is admissible, we must have

$$f^* \geq f(n).$$

Also, if n is not chosen over G_2, we must have

$$f(n) \geq f(G_2).$$

Gives us $f^* \geq f(G_2) = g(G_2)$. (Contradiction to G_2 suboptimal!)