Moving to a different formalism...

```
    SEND
+   MORE
--------
   MONEY
```

Consider search space for cryptarithmetic.
DFS (depth-first search)

Is this (DFS) how humans tackle the problem?

And if not, what do humans do?

Human problem solving appears much more **sophisticated!**

For example, we derive new constraints on the fly.
In a sense, we try to solve problems with **little** or **no** search!

In example, we can immediately derive that $M = 1$.
It then follows that $S = 8$ or $S = 9$. Etc. (derive more!)

Constraint Satisfaction Problems (CSP)

A powerful representation for (discrete) search problems.
A **Constraint Satisfaction Problem (CSP)** is defined by:

- **X** is a set of n variables $X_1, X_2, \ldots, X_n$, each defined by a finite domain $D_1, D_2, \ldots, D_n$ of possible values.
- **C** is a set of constraints $C_1, C_2, \ldots, C_m$. Each C_i involves some subset of the variables; specifies the allowable combinations of values for that subset.
- A **solution** is an assignment of values to the variables that satisfies all constraints.

Cryptarithmetic as a CSP

```
     TWO
+    TWO
--------
    FOUR
```

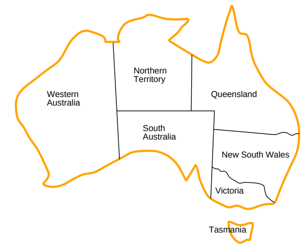
Variables:

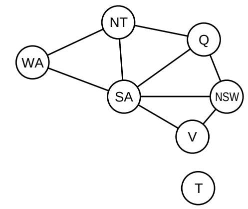
$T = \{0, \ldots, 9\}$; $W = \{0, \ldots, 9\}$; $O = \{0, \ldots, 9\}$;
$F = \{0, \ldots, 9\}$; $U = \{0, \ldots, 9\}$; $R = \{0, \ldots, 9\}$;

Constraints:

$O + O = R + 10 * X_1$
$X_1 + W + W = U + 10 * X_2$
$X_2 + T + T = O + 10 * X_3$
$X_3 = F$
TWO + TWO = FOUR;
each letter has a different digit (F $\neq$ T, F $\neq$ U, etc);

Map-Coloring Problem

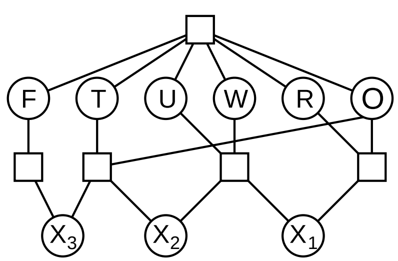




Cryptarithmetic Constraint Graph

```
   T W O
 + T W O
 -------
 F O U R
```

(a)



(b)

Constraint Satisfaction Problems (CSP)

For a given CSP the problem is one of the following:

- find all solutions
- find one solution
  - just a feasible solution, or
  - a "reasonably good" feasible solution, or
  - the optimal solution given an objective function
- determine if a solution exists

How to View a CSP as a Search Problem?

Initial State – state in which all the variables are unassigned.

Successor function – assign a value to a variable from a set of possible values.

Goal test – check if all the variables are assigned and all the constraints are satisfied.

Path cost – assumes constant cost for each step

Branching Factor

Hypothesis 1 – any unassigned variable at a given state can be assigned a value by an operator: branching factor as high as sum of size of all domains.

Better approach – since order of variable assignment not relevant, consider as the successors of a node just the different values of a *single* unassigned variable: max branching factor = max size of domain.

Maximum Depth of Search Tree

n the number of variables; all the solutions are at depth n.

What are the implications in terms of using DFS vs. BFS?

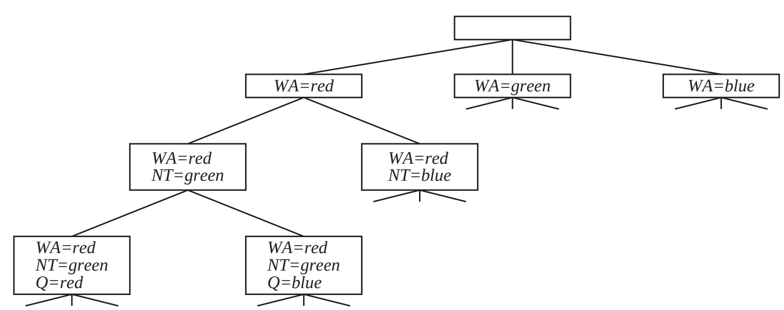
CSP – Goal Decomposed into Constraints

How to exploit it?

Backtracking search: a DFS that chooses values for variables one at a time, checking for *consistency* with the constraints.

An uninformed search

Example



Constraint propagation: "looking ahead"

- **Forward Checking** — each time variable is instantiated, delete from domains of the uninstantiated variables all of those values that conflict with current variable assignment.
- **Arc Consistency** — state is arc-consistent, if every variable has some value that is consistent with each of its constraints (consider pairs of variables)
- **K-Consistency** generalizes arc-consistency. Consistency of groups of K variables.

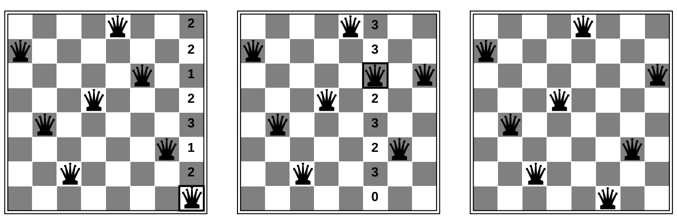
Constraint propagation: "looking ahead"

- **Variable and value ordering:**
  Minimum remaining values (MRV): choose the variable with the *fewest* possible values.

  Degree heuristic: assign a value to the variable that is involved in the largest number of constraints on other unassigned variables.

  Least-constraining value heuristic: choose a value that rules out the smallest number of values in variables connected to the current variable by constraints.

Local Search for CSPs



Comparison of CSP Algorithms

Problem	BT	BT+MRV	BT+FC	BT+FC+	Min-Conf
USA	(>1,000K)	(>1,000K)	2K	60	64
n-queens	(>40,000K)	13,500K	(>40,000K)	817K	4K

Dramatic recent progress in **Constraint Satisfaction.**

For example, methods can now handle problems with **10,000** to **100,000** variables, and up to **1,000,000** constraints.