

If a course is easy, some students are happy.

Put in first-order form:

$$\forall c, \text{easy}(c) \Rightarrow \exists s, \text{happy}(s)$$

Skolemize:

$$\forall c, \text{easy}(c) \Rightarrow \text{happy}(f(c))$$

Drop universal quantifier:

$$\text{easy}(c) \Rightarrow \text{happy}(f(c))$$

Eliminate implications:

$$\neg \text{easy}(c) \vee \text{happy}(f(c))$$

If a course has a final, no students are happy.

Put in first-order form:

$$\forall c, \text{final}(c) \Rightarrow \neg \exists s, \text{happy}(s)$$

Move negation inward:

$$\forall c, \text{final}(c) \Rightarrow \forall s, \neg \text{happy}(s)$$

Move quantifiers outward:

$$\forall c, \forall s, \text{final}(c) \Rightarrow \neg \text{happy}(s)$$

Drop universal quantifiers:

$$\text{final}(c) \Rightarrow \neg \text{happy}(s)$$

Eliminate implications:

$$\neg \text{final}(c) \vee \neg \text{happy}(s)$$

So, we have two clauses (with variables renamed to get rid of duplicates):

$$\neg \text{easy}(c) \vee \text{happy}(f(c))$$

$$\neg \text{final}(k) \vee \neg \text{happy}(s)$$

Now, we want to show *If a course has a final, the course isn't easy.*:

Put in first-order form:

$$\forall c, \text{final}(c) \Rightarrow \neg \text{easy}(c)$$

Drop universal quantifier:

$$\text{final}(c) \Rightarrow \neg \text{easy}(c)$$

Eliminate implications:

$$\neg \text{final}(c) \vee \neg \text{easy}(c)$$

Rename variables to avoid conflicts with other clauses:

$$\neg \text{final}(j) \vee \neg \text{easy}(j)$$

For resolution, we want to negate the clause we're trying to prove:

$$\text{final}(j) \wedge \text{easy}(j)$$

Now, our complete clause set at the start of resolution looks like:

A. $\neg \text{easy}(c) \vee \text{happy}(f(c))$

B. $\neg \text{final}(k) \vee \neg \text{happy}(s)$

C. $\text{final}(j)$

D. $\text{easy}(j)$

Resolution:

Combine A and D with the resolution rule to get:

E. $\text{happy}(f(c))$

Combine B and C with the resolution rule to get:

F. $\neg \text{happy}(s)$

Combine E and F with the resolution rule to get:

\emptyset

Intuitively, E and F contradict each other (E states that a particular student, $f(c)$, is happy, while F states that no student is happy), so we've derived a contradiction by assuming the negation of $\neg \text{final}(j) \vee \neg \text{easy}(j)$.