# Foundations of Artificial Intelligence

CS472/3 — Fall 1998

Lecture #3

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## Today's Lecture

**Problem Solving as Search, cont.**

**Uninfomed search, wrap-up.**

**Informed Search**

Readings: R&N, Chapter 3&4.

## Iterative Deepening

**Idea:**

Use an *artificial* depth cutoff, $c$.

If search to depth $c$ succeeds, we're done. If not, increase $c$ by 1 and start over.

Each iteration searches using DFS.

Space requirements? Same as DFS. Each search is just a DFS.

Time requirements. Would seem very expensive!! **BUT** not much different from single BFS or DFS to depth $d$.

**Reason:** Almost all work is in the final couple of layers.

E.g., binary tree: 1/2 the nodes are in the bottom layer.

With $b = 10$, 9/10th of the nodes in final layer!

So, repeated runs are on much smaller trees (become exponentially smaller).

**Example:** b=10, d=5, the number of nodes expanded in DFS

$$1 + 10 + 100 + 1000 + 10,000 + 100,000 = 111,111$$

bottom level is expanded once, second to bottom twice...

$$(d+1)1 + (d)b + (d-1)b^2 + ... + 2b^{d-1} + 1b^d \text{ i.e.,:}$$

$$6 + 50 + 400 + 3,000 + 20,000 + 100,000 = 123,456$$

only about 11% more!

Ratio of ID to DFS: (b+1)/(b-1).

Cost of repeating the work is not prohibitive.

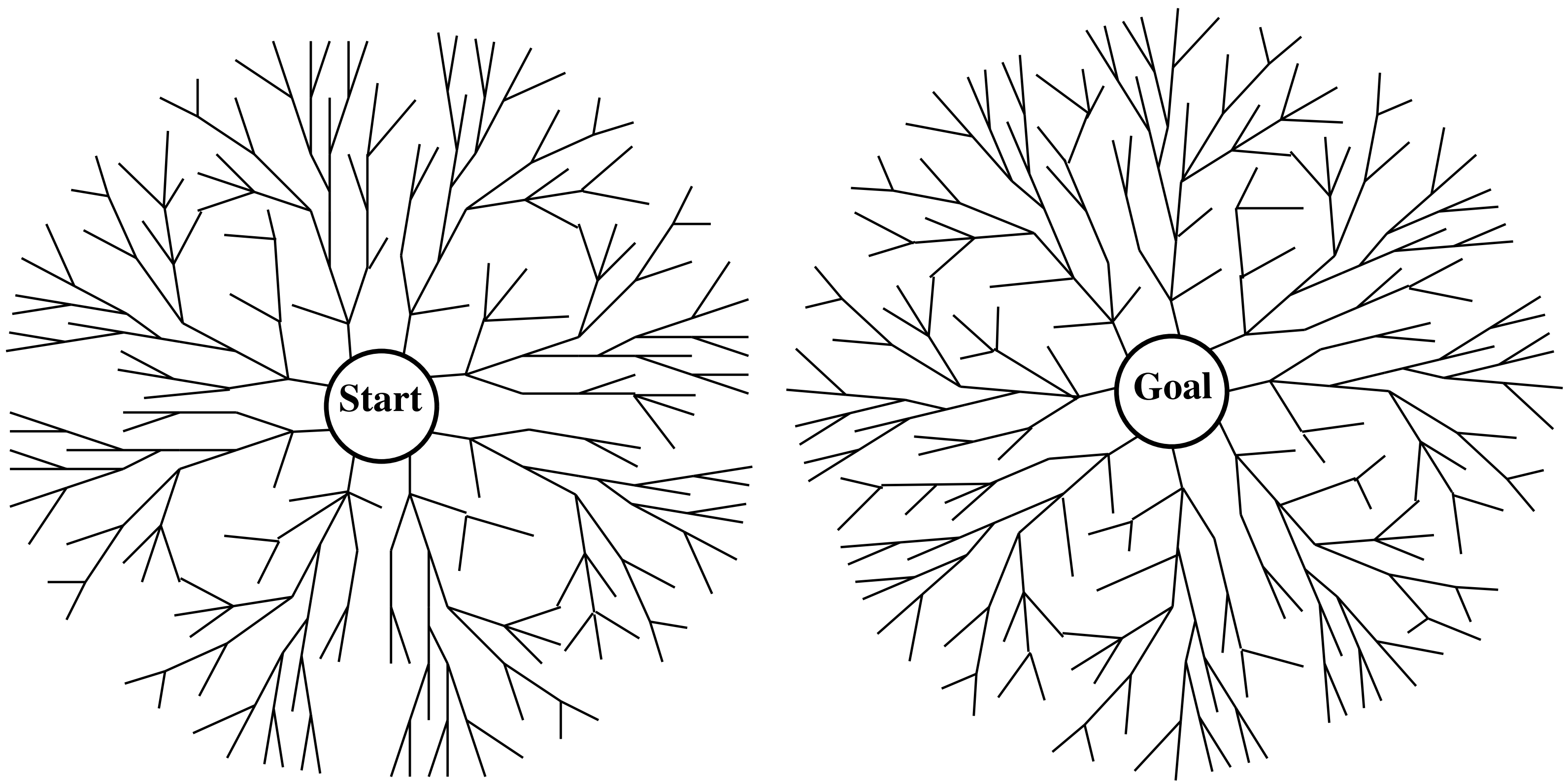
(Note: quite a clever insight.)

## Cost of Iterative Deepening

**space:** $O(bd)$ (as DFS); **time:** $O(b^d)$

| b | ratio of ID to DFS |
|---|---|
| 2 | 3 |
| 3 | 2 |
| 5 | 1.5 |
| 10 | 1.2 |
| 25 | 1.08 |
| 100 | 1.02 |

# Bidirectional Search



- Search forward from the start state and backward from the goal state simultaneously and stop when the two searches meet in the middle
- If branching factor = b from both directions, and solution exists at depth d, then need only $O(2b^{d/2}) = O(b^{d/2})$ steps.
- Example b = 10, d = 6 then BFS needs 1,111,111 nodes and bidirectional search needs only 2,222.
- Issues: what does it mean to search backwards from a goal? What if there is more than one goal state? (chess).

## Uniform-cost Search

Use BFS, but always expand the lowest-cost node on the fringe as measured by path cost $g(n)$ to find optimal solution.

See p. 75 R&N.

# Comparing Search Strategies

| Criterion | Breadth-First | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening | Bidirectional (if applicable) |
|---|---|---|---|---|---|---|
| Time | $b^d$ | $b^d$ | $b^m$ | $b^l$ | $b^d$ | $b^{d/2}$ |
| Space | $b^d$ | $b^d$ | $bm$ | $bl$ | $bd$ | $b^{d/2}$ |
| Optimal? | Yes | Yes | No | No | Yes | Yes |
| Complete? | Yes | Yes | No | Yes, if $l \geq d$ | Yes | Yes |

# Constraint Satisfaction Problems (CSP)

A powerful representation for (discrete) search problems.

Led to "constraint programming".

A **Constraint Satisfaction Problem (CSP)** is defined by:

**X** is a set of n variables $X_1, X_2, \ldots, X_n$,

each defined by its finite domain $D_1, D_2, \ldots, D_n$.

**C** is a set of constraints $C_1, C_2, \ldots, C_m$.

# Constraints

A **constraint** $C_i$ restricts the set of possible values that can be assigned to the variables in the constraint.

In other words, a constraint specifies which values are compatible for the variables in the constraint.

A **solution** is an assignment of values to the variables that satisfies all constraints.

Example: Graph Coloring

**Any NP-complete** problem can be efficiently formulated as a finite CSP problem.

**Why?** — . . .

# Constraint Satisfaction Problems (CSP)

For a given CSP the problem is one of the following:

- find all solutions
- find one solution
  - just a feasible solution, or
  - a "reasonably good" feasible solution, or
  - the optimal solution given an objective
- determine if a solution exists

## How to View a CSP as a Search Problem?

**Initial State** – state in which all the variables are unassigned.

**Operators** – assign a value to a variable from a set of possible values.

**Goal test** – check if all the variables are assigned and all the constraints are satisfied.

**Solving CSPs**

figure backtrack search

**Branching Factor**

**Hypothesis 1** – any unassigned variable at a given state can be assigned a value by an operator: branching factor as high as sum of all domains.

**Better approach** – since order of variable assignment not relevant, consider as the successors of a node just the different values of a *single* unassigned variable: max branching factor = max domain.

**Maximum Depth of Search Tree**

$n$ the number of variables; all the solutions are at depth $n$. What are the implications in terms of using DFS vs. BFS?

## CSP – Goal Decomposed into Constraints

**How to exploit it?**

**Backtracking** only insert successors if *consistent* with constraints.

**Constraint propagation** "looking ahead" to remove inconsistencies.

**Forward Checking** — each time variable is instantiated, remove other inconsistent values

**Consistency** — state is arc-consistent, if every variable has some value that is consistent with each of its constraints (consider pairs of variables)

**K-Consistency** generalizes arc-consistency. Consistency of groups of K variables.

**Branching:**

Most-constrained variable first; Most-constraining variable; Least-constraining value. (p. 105 R&N)

Example forward-checking / arc consistency.

## Send More Money as a CSP

**Variables:**

$S = \{0, \ldots, 9\}$; $E = \{0, \ldots, 9\}$;

$N = \{0, \ldots, 9\}$; $D = \{0, \ldots, 9\}$; $M = \{0, \ldots, 9\}$;

$O = \{0, \ldots, 9\}$; $R = \{0, \ldots, 9\}$; $Y = \{0, \ldots, 9\}$;

**Constraints:**

$\text{send} = 1000 \times S + 100 \times E + 10 \times N + D$;

$\text{more} = 1000 \times M + 100 \times O + 10 \times R + E$;

$\text{money} = 10000 \times M + 1000 \times O + 100 \times N + 10 \times E + Y$;

send + more = money;

each letter has a different digit (S $\neq$ E, S $\neq$ N, etc);

Dramatic recent progress in **Constraint Satisfaction.**

For example, we can now handle problems with **10,000** to **100,000** variables, and up to **1,000,000** constraints.

## Informed Methods: Heuristic Search

*Informed Methods* use problem-specific knowledge.

*Heuristic* search is an attempt to search the most promising paths first. Uses heuristics, or rules of thumb, to find the best node to expand next.

Relies on an *evaluation function* — indicates the desirability of expanding a node. E.g., *path cost*.

$h(n)$ = estimated cost of the cheapest path from the state at node $n$ to a goal state (*Heuristic Function*: e.g., straight line on a map)

Given a list of nodes to be expanded, choose the one that the heuristic function estimates as the most promising.

## Best-First Search

1. Set $L$ to be the initial node(s).
2. Let $n$ be the node on $L$ that is "most promising" according to eval function ($h(n)$ / $f(n)$). If $L$ is empty, fail.
3. If $n$ is a goal node, stop and return it (and the path from the initial node to $n$).
4. Otherwise, remove $n$ from $L$ and add all of $n$'s children to $L$ (labeling each with its path from the initial node). Return to step 2.