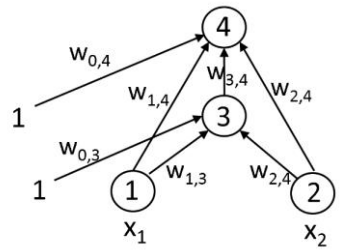
CS 4700: Foundations of Artificial Intelligence
Spring 2017
Homework 4
Due: Friday, May 5, at 12:00 noon
Revised: May 1

1. Consider the following neural network:




   a. For this part of this question let the activation function for nodes 3 and 4 be the logistic function $\frac{1}{1+e^{-x}}$.

      i. Write down the formula for unit 3's output as a function solely of $x_1$, $x_2$, and the relevant $w_{i,j}$'s.

      ii. Write down the formula for unit 4's output as a function solely of $x_1$, $x_2$, and the relevant $w_{i,j}$'s.

   b. For this part of the question let nodes 3 and 4 be perceptrons with $w_{1,4} = w_{2,4} = w_{3,4} = w_{1,3} = w_{2,3} = 1.0$, $w_{0,4} = -1.5$, and $w_{0,3} = -3.0$.

      If $x_1$ and $x_2$ are constrained to be 0 (= false) or 1 (= true) the neural net can be viewed as a Boolean function.

      i. Write the truth table for the function on $x_1$ and $x_2$ computed at node 4.

      ii. Can this function be represented by a single perceptron? If yes, give such a perceptron. If not, explain why not.

2. Consider the following data set for classification learning that contains only two items, each described by two real-valued attributes: ((0,0),1), ((1,1),0). Give a perceptron that correctly labels this data.

3. Consider the following data set for clustering that contains six items, each described by two real-valued attributes: (0,0), (0,1), (1,0), (2,3), (3,2), and (3,3). Simulate k-means clustering on this data set with k=2. Assume the first two points used as the initial centroids are (2,3) and (3,3). For each iteration show the two centroids that are generated and to which of the centroids each instance is assigned to.

4. Consider the following data set for classification learning:

| Instance # | Outlook | Temp | Humidity | Wind | Play Tennis? |
|---|---|---|---|---|---|
| 1 | No rain | Hot | High | Weak | No |
| 2 | No rain | Hot | High | Strong | No |
| 3 | Rain | Mild | High | Weak | Yes |
| 4 | Rain | Mild | High | Weak | Yes |
| 5 | Rain | Mild | Normal | Weak | Yes |
| 6 | Rain | Mild | Normal | Strong | No |
| 7 | Rain | Mild | Normal | Strong | Yes |
| 8 | No rain | Mild | High | Weak | No |
| 9 | No rain | Mild | Normal | Weak | Yes |
| 10 | Rain | Mild | Normal | Weak | Yes |
| 11 | No rain | Mild | Normal | Strong | Yes |
| 12 | Rain | Mild | High | Strong | Yes |
| 13 | Rain | Hot | Normal | Weak | Yes |
| 14 | Rain | Mild | High | Strong | No |

The last column is the binary classification we are using learning to predict – whether to play tennis that day (either Yes or No) – and columns 2-5 are binary attributes describing the given day.

a. Use a naïve Bayes classifier to compute which of Yes and No has higher probability for Play Tennis? on a day whose Outlook is Rain, Temp is Hot, Humidity is High, and Wind is Strong if you do no Laplace smoothing.
b. Do the same with Laplace smoothing using k=1.