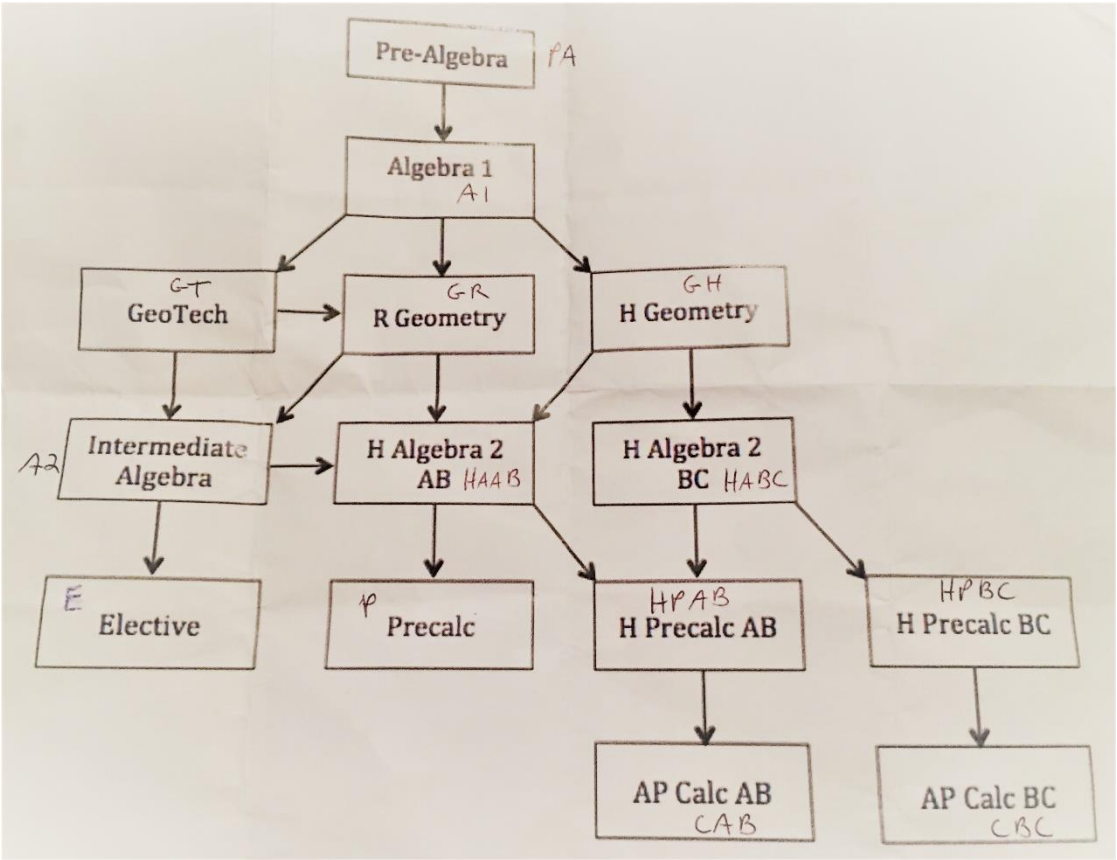
CS 4700: Foundations of Artificial Intelligence
Spring 2017
Homework 3
Due: Friday, April 14, at 12:00 noon

1. Recall the following figure from the ungraded second homework, which shows the prerequisite structure for math classes at Ithaca High School. It shows, for example that a student must have taken Pre-Algebra before taking Algebra 1. Two arrows coming into a course means *either* course can be taken as a prerequisite (as opposed to *both*).



Imagine we want to encode these prerequisites in first-order logic (*not* using propositional logic as in the ungraded homework assignment). We could use the predicate Prereq(x,y) to say that course x is a prerequisite of course y. Using the short labels written at the various nodes as the constants representing each course would let us write Prereq(PA,A1), namely that Pre-Algebra is a prerequisite of Algebra 1.

a. Write down the rest of the prerequisites in this fashion.
b. Use resolution to prove that there is a course that is a prerequisite of all three of the Geometry courses (GeoTech, R Geometry, and H Geometry).
c. How would you write the following in first-order logic?
    i. Pre-Algebra has no prerequisites.
    ii. No course is ever its own prerequisite.
    iii. If x is a prerequisite of y then y is not a prerequisite of x.

d. Consider the predicate Required(x,y), which means that x is an "ancestor" of y in the course requirements. Thus, for example, Required(PA,CBC) is true.
   i. Write a first-order logic definition of Required in terms of Prereq.
   ii. Convert your definition into CNF.
   iii. Prove Required(PA,CAB) using resolution.

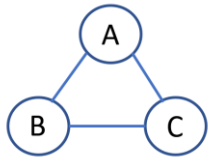
2. Consider the following facts in first-order logic:

$$\forall x\ [P(x,G(x)) \Rightarrow Q(x)]$$

$$\forall y\ [\neg P(F(y),y) \Rightarrow Q(y)]$$

Can you prove $\exists z\ Q(z)$ using resolution? Either provide a proof or explain why not if there is none.

3. A graph is *3-colorable* if each node can be assigned a color such that no two nodes with an edge in common have the same color. (In what follows we'll call the three colors simply 1, 2, and 3.) Consider the following 3-node graph:



The question of whether this graph is 3-colorable can be encoded in propositional logic as follows. Let A1 be true if A is assigned color 1, A2 if it is assigned color 2, and A3 if it is assigned color 3, and analogously so for B and C.

Encoding the problem using these variables involves creating the following sentences:

- There are a set of sentences that say that each node is assigned at least 1 of the 3 colors.
- There are a set of sentences that specify that no node is assigned more than 1 color.
- There are a set of sentences that specify that no two adjacent nodes have the same color.

Finding a solution to this graph coloring problem corresponds to finding a truth assignment to the 9 variables that satisfy the resulting sentences.

Imagine you want to find such a truth assignment using hillclimbing, in the following fashion:

- Convert the sentences to CNF.
- States correspond to truth assignments to the variables.
- The initial state is the assignment that sets all variables to True.
- The goal state is one where all of the clauses in the CNF evaluate to True given the state's truth assignments.
- Operators flip the assignment of one variable (if it is True it becomes False, and vice versa).
- The evaluation function is a count of the number of clauses that evaluate to False given the state's truth assignments. Lower numbers are better. If you reach a point where all successors states have a worse value, stop.

The goal of this problem is to use this hillclimbing approach to search for a 3-coloring of this graph.

a. Write down the set of clauses that encode this problem.
b. List the state that hillclimbing moves to at each step of the search.
c. Does it lead you to a valid 3-coloring of the graph?