CS 465 Homework 3

out: Friday 21 September 2007
due: Friday 28 September 2007

Problem 1: [2D Transformations] Consider a 2D transformation that transforms point $\mathbf{p}_1 \rightarrow \mathbf{p}'_1$, $\mathbf{p}_2 \rightarrow \mathbf{p}'_2$, $\mathbf{p}_3 \rightarrow \mathbf{p}'_3$.

1. Express the affine transformation solution to this general case using a matrix representation.
2. If point (0,1) is tranformed to (3,4), (1,1) to (7,1), and (1,0) to (4,-3), where will point (2,1) be transformed to?

Problem 2: [3D Transformations] Rodrigues' rotation formula gives an efficient method for computing the rotation matrix corresponding to a rotation by an angle $\theta \in \mathbb{R}$ about a fixed axis specified by the *unit* vector $\tilde{\omega} = (\omega_x, \omega_y, \omega_z) \in \mathbb{R}^3$. The rotation matrix is given by

$$
\begin{aligned}
e^{\tilde{\omega}\theta} &= I + \tilde{\omega}\sin\theta + \tilde{\omega}^2(1-\cos\theta) \\
&= \begin{bmatrix} \cos\theta + \omega_x^2(1-\cos\theta) & \omega_x\omega_y(1-\cos\theta) - \omega_z\sin\theta & \omega_y\sin\theta + \omega_x\omega_z(1-\cos\theta) \\ \omega_z\sin\theta + \omega_x\omega_y(1-\cos\theta) & \cos\theta + \omega_y^2(1-\cos\theta) & -\omega_x\sin\theta + \omega_y\omega_z(1-\cos\theta) \\ -\omega_y\sin\theta + \omega_x\omega_z(1-\cos\theta) & \omega_x\sin\theta + \omega_y\omega_z(1-\cos\theta) & \cos\theta + \omega_z^2(1-\cos\theta) \end{bmatrix}
\end{aligned}
$$

where I is the 3×3 identity matrix and $\tilde{\omega}$ denotes the antisymmetric matrix with entries

$$
\tilde{\omega} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}.
$$

1. State the three familiar canonical rotations, $R_x(\theta)$, $R_y(\theta)$ and $R_z(\theta)$, and verify that Rodrigues' formula reproduces them.
2. Compute the matrix for a rotation around the axis vector $[1, 2, 3]^T$ for $\theta = 60°$. Verify that $det(R) = 1$.

Problem 3: [Splines] Clearly state the formula for a cubic Bezier curve, and show that the curve is affine invariant for a general affine transformation.

Problem 4: [Spline Conversion] Show explicitly how to convert from a Catmull-Rom representation for a cubic spline segment with control points $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$, $\mathbf{p}_4$ to a Bezier spline with control points $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{q}_3$ and $\mathbf{q}_4$. Provide a geometric illustration of the quantities involved.