CS 465 Final Exam (Fall 2007, Prof. James)

Thursday 6 December 2007—2.5 hours (150 minutes)

The exam contains *7 questions* worth a total of *100 points.*

Explain your reasoning in words and equations for full marks.

Problem 1: Image Filtering (15 pts)

Consider the following 4-by-4 image:

0	1	1	0
1	4	9	1
1	4	4	1
0	1	1	0

(a) Convolve the image with a 3-by-3 box blur filter. As in the assignment, renormalize the filter to account for missing values at the edges.

(b) Convolving an image with a box filter computes local mean values. Similarly, convolving an image with a median filter computes local median values–a nonlinear operation.

Recall that given an odd number of numbers, the median is the middle number. Given an even number of numbers, the median is the mean of the two middle numbers. Thus, the median of the numbers 2, 4, 7, 12 is (4+7)/2=11/2.

Analogous to image filtering using a 3-by-3 blur filter, **apply a 3-by-3 median filter to the 4-by-4 image**. Ignore missing values at the edges.

(c) The 3-by-3 box blur filter is a separable filter, but is the 3-by-3 median filter a separable filter?

Problem 2: Ray-Quadrilateral Intersection (10 pts)

Consider a nonconvex planar quadrilateral whose four vertices are given by

$$\mathbf{p}_0 = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad \mathbf{p}_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad \mathbf{p}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad \mathbf{p}_3 = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

and a ray $\mathbf{r}(t) = \mathbf{e} + \mathbf{v}t,\ t > 0$, with

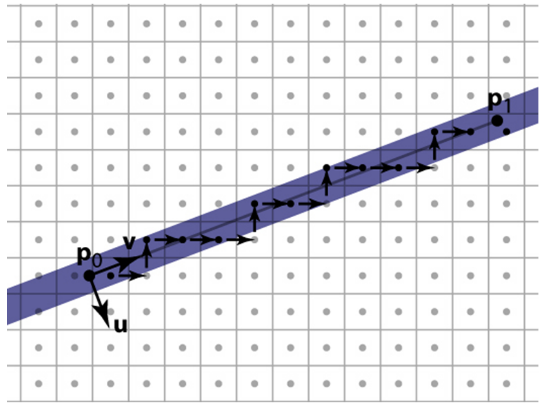
$$\mathbf{e} = \begin{pmatrix} 2 \\ 1 \\ 10 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} -0.15 \\ -0.01 \\ -1 \end{pmatrix}.$$

Does the ray hit the planar quadrilateral? You can draw a calibrated diagram (with labels) to support your claim. [Hint: Look at the particular geometry.]

Problem 3: Curve Rasterization (20 pts)

You have rasterized lines such as $y = mx + b$, $m \in [0, 1]$ using a digital differential analyzer (DDA) that exploits cheap additions and comparisons to update quantities. For example, recall the Pixel-walk line rasterizer on an integer grid ($\Delta x = \Delta y = 1$):

```
x = ceil(x0)
y = round(m*x + b)
d = m*x + b - y
while x < floor(x1)
    if d > 0.5
        y += 1; d -= 1;
    else
        x += 1; d += m;
    if -0.5 < d ≤ 0.5
        output(x, y)
```



In this question you will explore how quadratic splines can be rasterized using a DDA-type algorithm. Consider rasterizing a section of the quadratic curve

$$y = ax^2 + mx + b.$$

For simplicity, let us assume "shallow curves" with $\frac{dy}{dx} \in [0, 1]$, analogous to the $m \in [0, 1]$ condition for lines.

(a) First, derive *addition-only updates* of the implicit difference value

$$d = d(x, y) = ax^2 + mx + b - y$$

for both

(i) unit increments of y: $d \mathrel{+}= d(x, y + 1) - d(x, y)$;

(ii) unit increments of x: $d \mathrel{+}= d(x + 1, y) - d(x, y)$.
(Hint: You may need to additively update more than just x, y, and d quantities.)

(b) Second, rewrite the aforementioned Pixel-walk rasterizer for the quadratic case using addition-only updates in the inner `while` loop.
(Hint: You should only need to modify the parts indicated by gray boxes.)

<*Intermission!*>

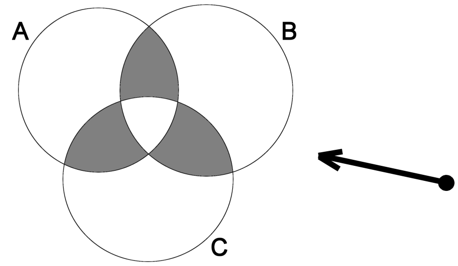
Question: Why doesn't $\mathbf{p} = \mathbf{N}\mathbf{p}$ iff $\mathbf{N}$ is the identity matrix?

Answer: Because $\mathbf{p}$ could be the zero vector. <*Ha ha ha. Boo...*>

<*Exeunt*>

Problem 4: Constructive Solid Geometry (CSG) (15 pts)

(a) CSG allows you to build objects from simple point sets using set operations (op): intersection ($\cap$), union ($\cup$), and difference ($-$). **Give a CSG expression for the model M (shown in solid gray)** in terms of the circular point sets A, B, and C.



(b) Consider intersecting a ray with your CSG model, M. Assume that your implementation supports intersecting a ray ($\mathbf{r}(t) = \mathbf{e} + \mathbf{v}t,\ t > 0$) with any solid circle, X, to generate a t interval, $\mathcal{I}_X = [t_X^{min}, t_X^{max}]$ (or possibly the empty set). Denote the ray-intersection operator by $\bullet\!\!\to$, so that

$$\bullet\!\!\to A = \mathcal{I}_A,$$

and use the fact that $\bullet\!\!\to$ passes through CSG operands,

$$\bullet\!\!\to(A \text{ op } B) = (\bullet\!\!\to A) \text{ op } (\bullet\!\!\to B).$$

Given the CSG model M from part (a), derive an expression for $\bullet\!\!\to M$, the set of intersected t intervals.

Problem 5: FOX News Moon Hoax (10 pts)

FOX Television Network recently[a] advanced the claim that NASA's Apollo Moon missions were a hoax, largely due to alleged "anomalies" in Apollo photos. Here is an excerpt:

FOX narrator: *"In this picture the Sun is directly behind the astronaut, his figure should be a silhouette, yet even the smallest characteristics of his suit are recognizable."* A photographic expert then says: *"He seems like **he's** standing in the spotlight. I can't explain that."*

Can *you* defend NASA's image? What factors are contributing to the brightness of the astronaut's figure?

[a] FOX Television Network, "Conspiracy Theory: Did We Land on The Moon?" February 15, 2001. (see http://users.erols.com/igoddard/moon01.htm)

<*Intermission!*>

A font walks into a bar.

The bartender says, "Welcome! We don't see many of your *type* around these parts." <*Ha ha ha. Boo...*>

<*Exeunt*>

Problem 6: Triangle Meshes (20 pts)

Consider the 12-triangle mesh of a cube, with indexed vertices, shown here unfolded and flattened–analogous to a cube map texture. Look at it for a moment to reconstruct it in your mind.

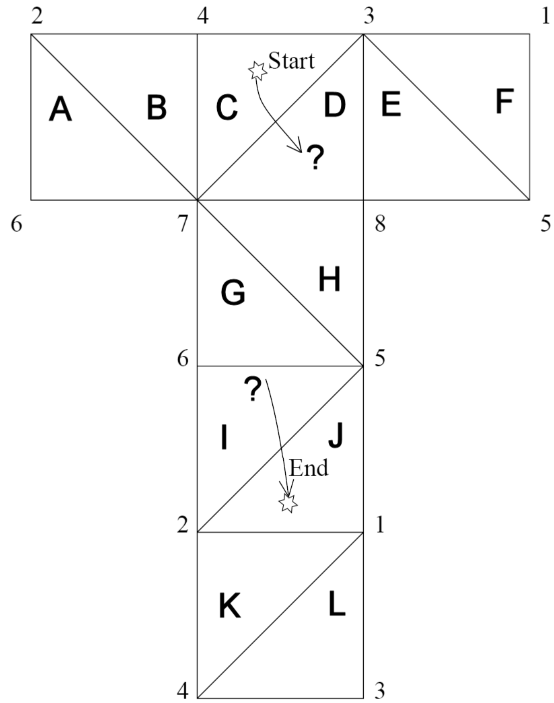
(a) **Find a triangle strip** containing all triangles that starts with triangles C, D,... and ends with ... I, J. State your strip as a sequence of 12 triangles:

C, D, ?, ?, ?, ?, ?, ?, ?, ?, I, J.

Remember adjacent strip triangles must share an edge. (If you get really stuck, state another strip, else you can state two strips.)

(b) **Find three nonoverlapping triangle fans** that span the surface. State each fan as the vertex followed by a sequence of edge-sharing triangles, e.g.,

3: C, D, E, F.



Problem 7: Quaternion Splines (10 pts)

Spherical linear interpolation, or SLERP, is widely used to interpolate key-framed rotations represented as quaternions. Recall that a unit quaternion can be represented as a four-vector $\mathbf{q} = (\cos\theta, \hat{\mathbf{v}}\sin\theta)$, and that the SLERP of two unit quaternions, $\mathbf{q}_0$ and $\mathbf{q}_1$, is

$$\mathsf{SLERP}(\mathbf{q}_0, \mathbf{q}_1; \alpha) = \mathbf{q}_0 \frac{\sin(\Omega(1-\alpha))}{\sin\Omega} + \mathbf{q}_1 \frac{\sin(\Omega\alpha)}{\sin\Omega}, \qquad \alpha \in [0,1],$$

where $\cos\Omega = \mathbf{q}_0 \cdot \mathbf{q}_1$.

One criticism of SLERP is that interpolating a temporal sequence of quaternions, $\mathbf{q}_0, \mathbf{q}_1, \mathbf{q}_2, \ldots$, sampled at equally spaced times $t = 0, 1, 2, \ldots$, can produce nonsmooth rotations at quaternion nodes.

(a) Consider the case of interpolating the quaternion pair $(\mathbf{q}_0, \mathbf{q}_1)$ for $t \in [0,1)$, followed by $(\mathbf{q}_1, \mathbf{q}_2)$ for $t \in [1,2]$. State the function for this piecewise-SLERP quaternion curve,

$$\mathbf{Q}(t) = \begin{cases} ?, & t \in [0,1), \\ ?, & t \in [1,2]. \end{cases}$$

(b) Show that this quaternion spline has a derivative discontinuity at $t = 1$.

<Intermission!?!>

A freshly shaded "exam pixel" walks onto the frame buffer.

"Whew! I'm sure glad I got by that Early-Z test." *<Ha ha ha. Boo...>*

<Exeunt (for good)>