

Parametric surfaces and solid modeling

CS 465 Lecture 12

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From curves to surfaces

- So far have discussed spline curves in 2D
 - it turns out that this already provides of the mathematical machinery for several ways of building curved surfaces
- Building surfaces from 2D curves
 - extrusions and surfaces of revolution
- Building surfaces from 2D and 3D curves
 - generalized swept surfaces
- Building surfaces from spline patches
 - generalizing spline curves to spline patches
- Also to think about: generating triangles

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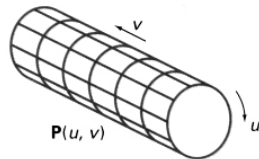
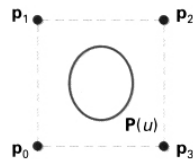
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Extrusions

- Given a spline curve $C \in \mathbb{R}^2$ define $S \in \mathbb{R}^3$ by

$$S = C \times [a, b]$$
- This produces a “tube” with the given cross section
 - Circle: cylinder; “L”: shelf bracket; “I”: I beam
- It is parameterized by the spline t and the interval $[a, b]$

$$s(t, s) = [c_x(t), c_y(t), s]^T$$



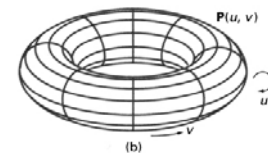
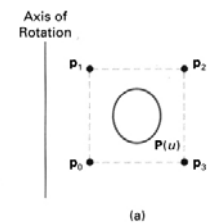
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Surfaces of revolution

- Take a 2D curve and spin it around an axis
- Given curve $c(t)$ in the plane, the surface is defined easily in cylindrical coordinates:

$$s(t, s) = (r, \phi, z) = (c_x(t), s, c_y(t))$$
 - the torus is an example in which the curve c is a circle



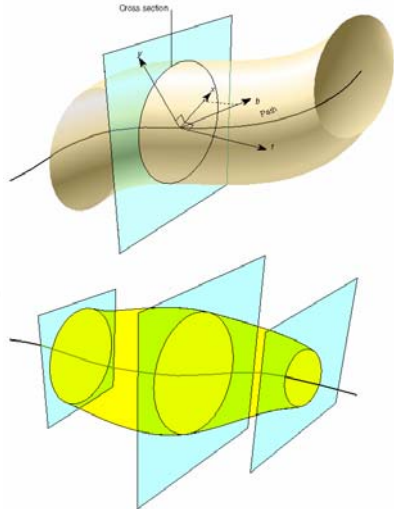
[Hearn & Baker.]

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Swept surfaces

- Surface defined by a *cross section* moving along a *spine*
- Simple version: a single 3D curve for spine and a single 2D curve for the cross section



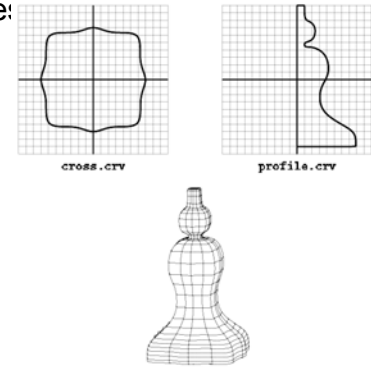
(S01) OpenGL: Optimizer Programmer's Guide: An Open API for Large-Model Visualization

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Generalized cylinders

- General swept surface:
 - varying radius
 - varying cross-section
 - curved axis



[Snyder 1992]

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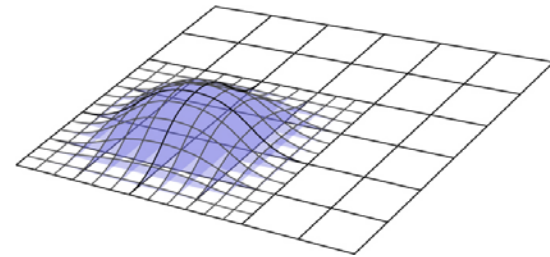
From curves to surface patches

- Curve was sum of weighted 1D basis functions
- Surface is sum of weighted 2D basis functions
 - construct them as separable products of 1D fns.
 - choice of different splines
 - spline type
 - order
 - closed/open (B-spline)

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Separable product construction

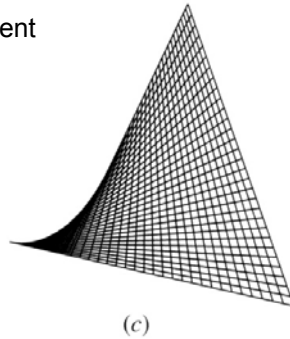


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Bilinear patch

- Simplest case: 4 points, cross product of two linear segments
 - basis function is a 3D tent



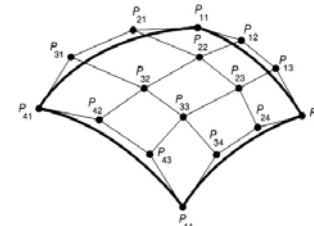
[Rogers]

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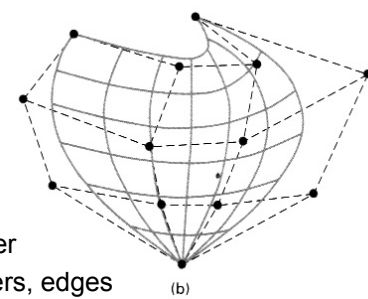
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Bicubic Bézier patch

- Cross product of two cubic Bézier segments



[Foley et al.]



[Hearn & Baker]

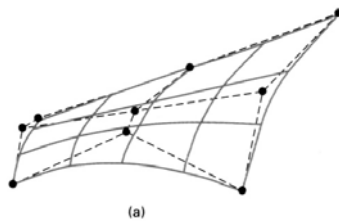
- properties that carry over
 - interpolation at corners, edges
 - tangency at corners, edges
 - convex hull

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Biquadratic Bézier patch

- Cross product of quadratic Bézier curves



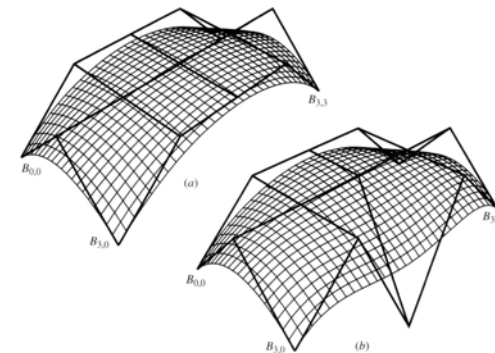
[Hearn & Baker]

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3x5 Bézier patch

- Cross product of quadratic and quartic Béziers



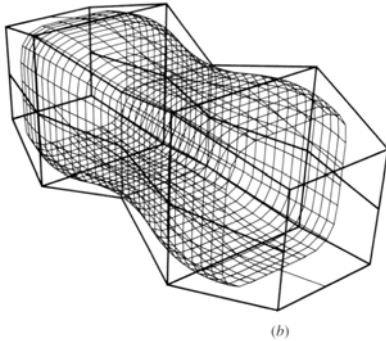
[Rogers]

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Cylindrical B-spline surfaces

- Cross product of closed and open cubic B-splines



[Rogers]

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Approximating spline surfaces

- Similarly to curves, approximate with simple primitives
 - in surface case, triangles or quads
 - quads widely used because they fit in parameter space
 - generally eventually rendered as pairs of triangles
- adaptive subdivision
 - basic approach: recursively test flatness
 - if the patch is not flat enough, subdivide into four using curve subdivision twice, and recursively process each piece
 - as with curves, convex hull property is useful for termination testing (and is inherited from the curves)

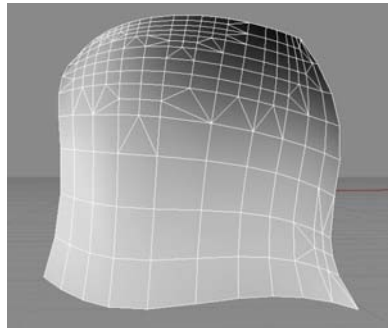
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Approximating spline surfaces

With adaptive subdivision, must take care with cracks

- (at the boundaries between degrees of subdivision)
- So-called “T-vertices”



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Modeling in 3D

- Representing subsets of 3D space
 - volumes (3D subsets)
 - surfaces (2D subsets)
 - curves (1D subsets)
 - points (0D subsets)

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Representing geometry

- In order of dimension...
- Points: trivial case
- Curves
 - normally use parametric representation
 - line—just a point and a vector (like ray in ray tracer)
 - polylines (approximation scheme for drawing)
 - more general curves: usually use splines
 - $\mathbf{p}(t)$ is from $\mathbb{R}$ to $\mathbb{R}^3$
 - $\mathbf{p}$ is defined by piecewise polynomial functions

Representing geometry

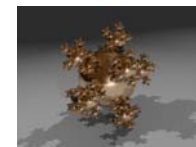
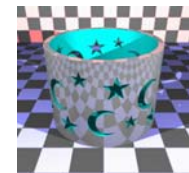
- Surfaces
 - this case starts to get interesting
 - implicit and parametric representations both useful
 - example: plane
 - implicit: vector from point perpendicular to normal
 - parametric: point plus scaled tangent
 - example: sphere
 - implicit: distance from center equals r
 - parametric: write out in spherical coordinates
 - messiness of parametric form not unusual

Representing geometry

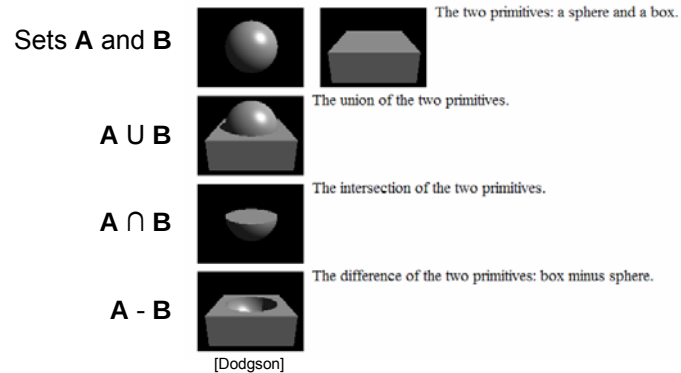
- Volumes
 - boundary representations (B-reps)
 - just represent the boundary surface
 - convenient for many applications
 - must be closed (watertight) to be meaningful
 - an important constraint to maintain in many applications

Representing geometry

- Volumes
 - CSG (Constructive Solid Geometry)
 - apply boolean operations on solids
 - simple to define
 - simple to compute in some cases
 - [e.g. ray tracing]
 - difficult to compute stably with B-reps
 - [e.g. coincident surfaces]



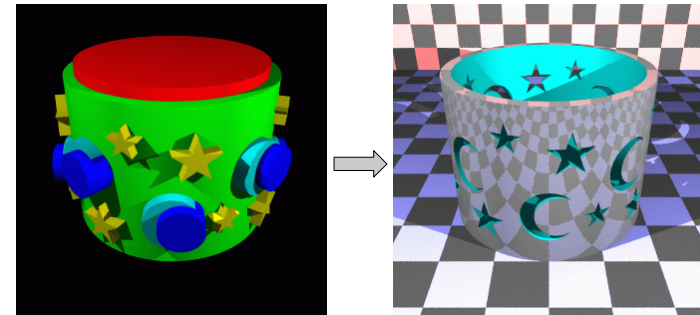
Constructive Solid Geometry (CSG)



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Constructive Solid Geometry



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Specific surface representations

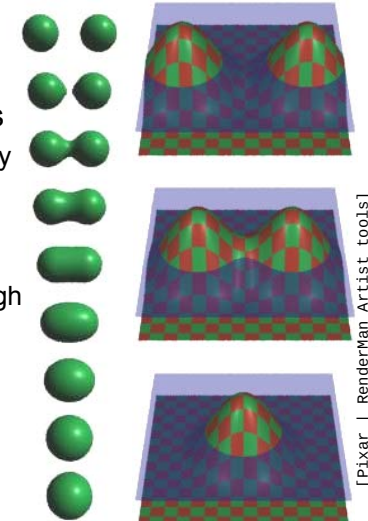
- Parametric surfaces
 - extrusions
 - surfaces of revolution
 - generalized cylinders
 - spline patches

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Specific surface reps.

- Algebraic implicit surfaces
 - defined as zero sets of fairly arbitrary functions
 - good news: CSG is easy using min/max
 - bad news: rendering is tough
 - ray tracing: intersect arbitrary zero sets w/ray
 - pipeline: need to convert to triangles
 - e.g. “blobby” modeling



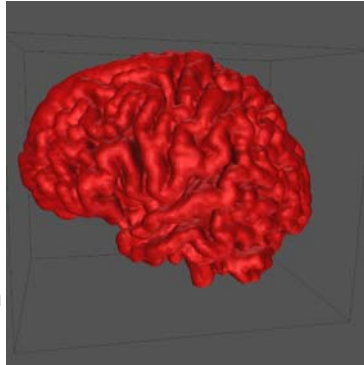
[Pixar | RenderMan Artist tools]

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Specific surface representations

- Isosurface of volume data
 - implicit representation
 - function defined by regular samples on a 3D grid
 - (like an image but in 3D)
 - example uses:
 - medical imaging
 - numerical simulation



[source unknown]

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Specific surface representations

- Triangle or polygon meshes
 - parametric (per face)
 - very widely used
 - final representation for pipeline rendering
 - these days restricting to triangles is common
 - rather unstructured
 - need to be careful to enforce necessary constraints
 - to bound a volume need a watertight *manifold* mesh
 - Not necessary for mere renderings

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Polygons in Unreal Engine 3



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Polygons in Unreal Engine 3



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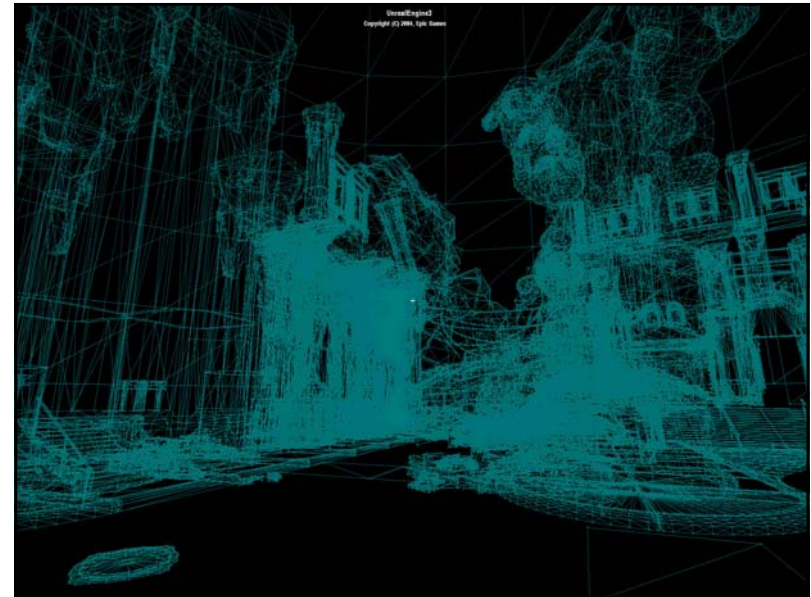
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“Normal Mapped” Mesh



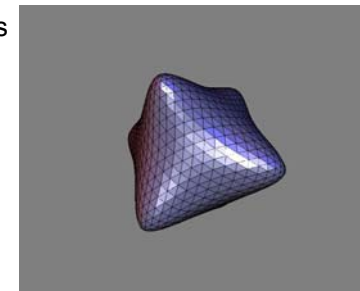
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Specific surface representations

- Subdivision surfaces
 - based on polygon meshes (quads or triangles)
 - rules for subdividing surface by adding new vertices
 - converges to continuous limit surface

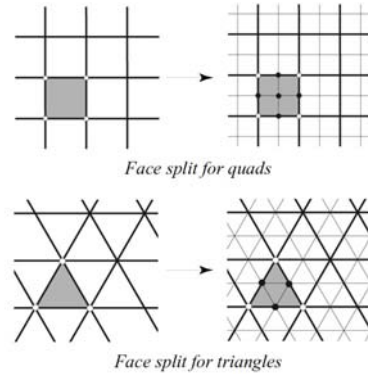


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Subdivision of meshes

- Quadrilaterals
 - Catmull-Clark 1978
- Triangles
 - Loop 1987
- Quadrisection
 - Splitting into four
 - C.f. bisection for lines

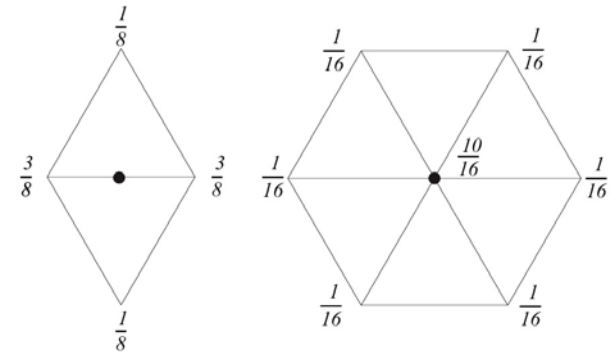


[Schröder & Zorin SIGGRAPH 2000 course 23]

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Loop Subdivision Rules (regular case)



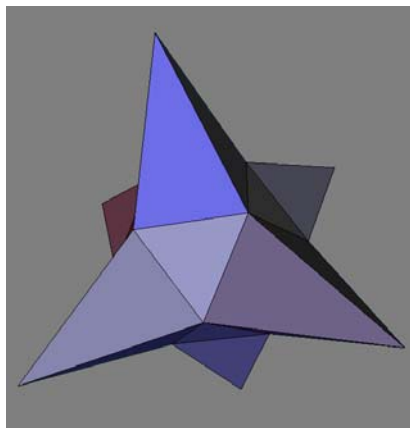
[Schröder & Zorin SIGGRAPH 2000 course 23]

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Loop Subdivision Example

control polyhedron

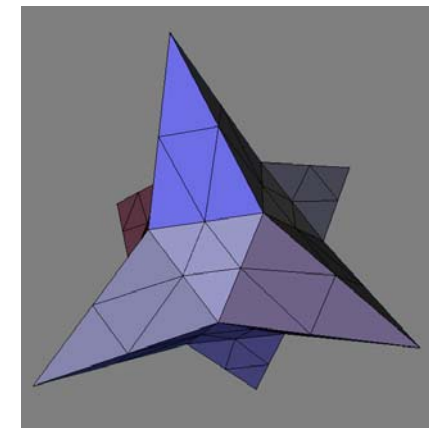


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Loop Subdivision Example

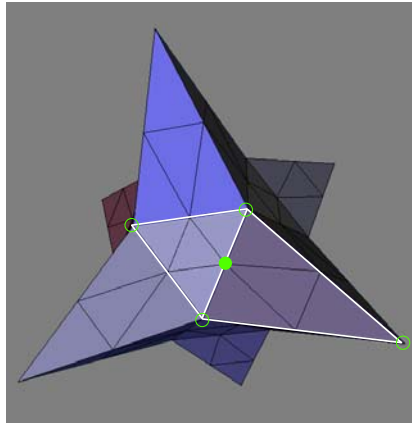
refined
control polyhedron



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Loop Subdivision Example

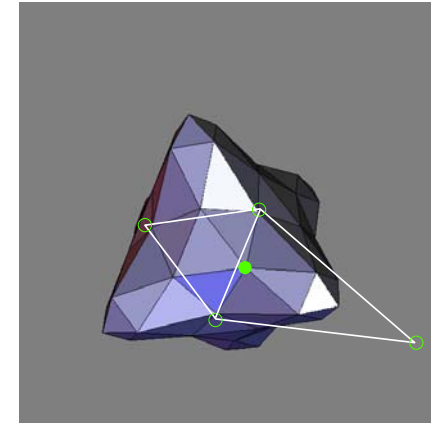


odd
subdivision mask

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Loop Subdivision Example

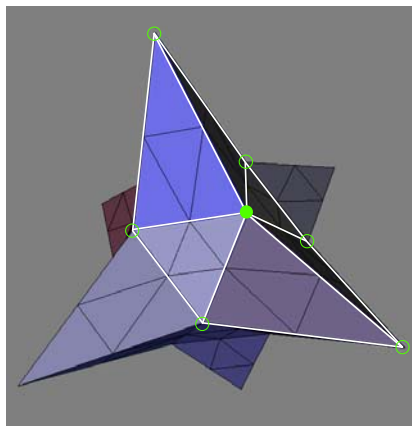


subdivision level 1

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Loop Subdivision Example

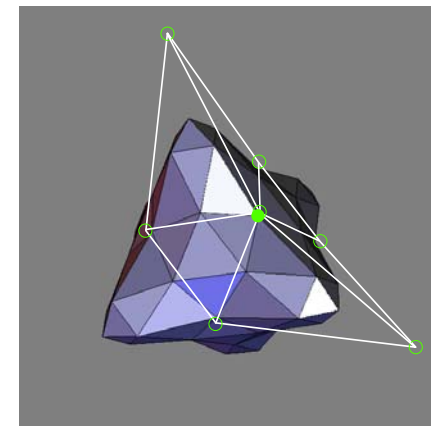


even
subdivision mask
(ordinary vertex)

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Loop Subdivision Example

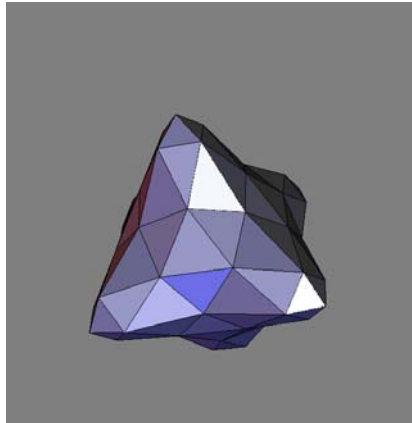


subdivision level 1

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Loop Subdivision Example

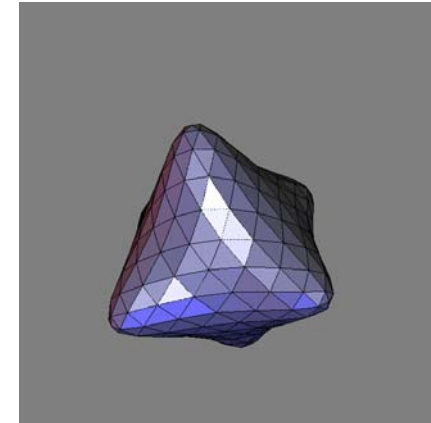


subdivision level 1

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Loop Subdivision Example

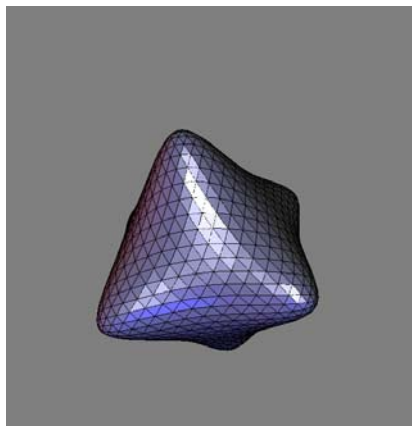


subdivision level 2

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Loop Subdivision Example

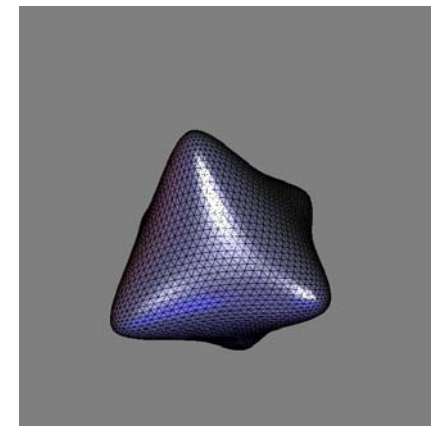


subdivision level 3

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Loop Subdivision Example

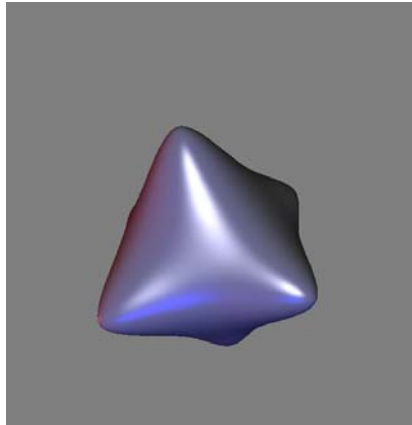


subdivision level 4

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Loop Subdivision Example



limit surface