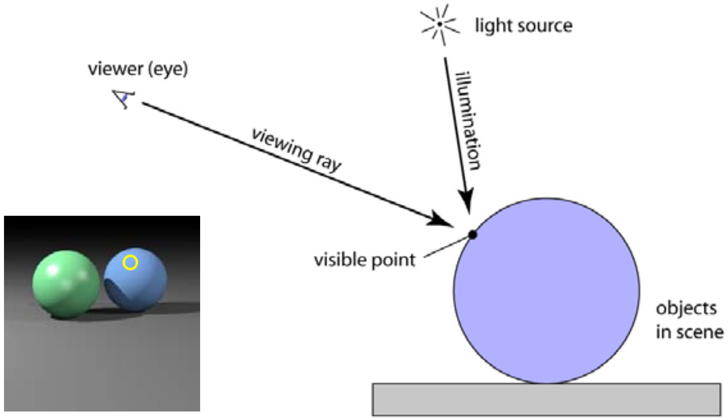
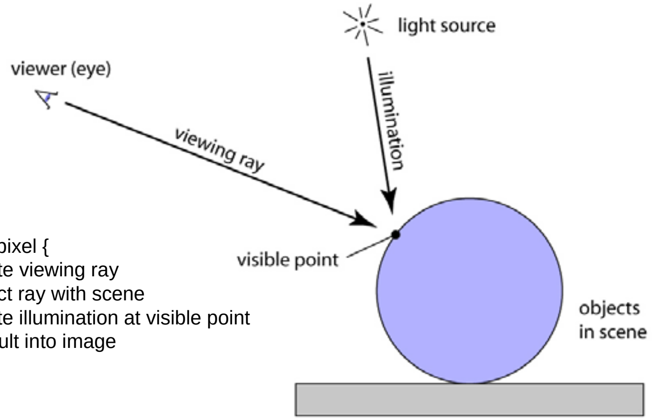
Ray Tracing

CS 465 Lecture 3

Ray tracing idea

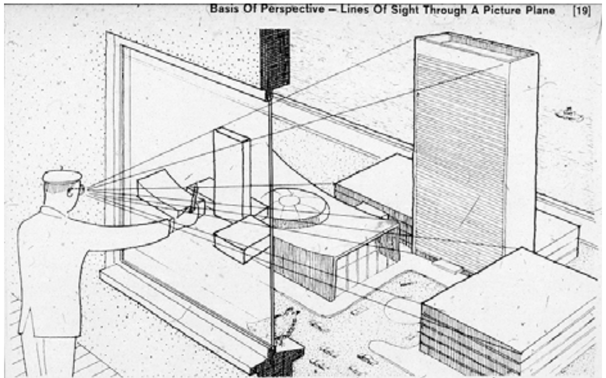


Ray tracing algorithm



for each pixel {
compute viewing ray
intersect ray with scene
compute illumination at visible point
put result into image
}

Plane projection in drawing



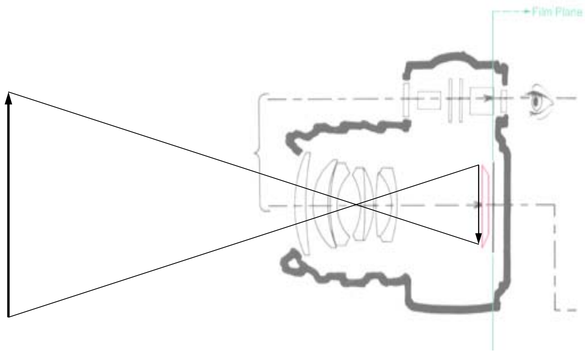
Basis Of Perspective – Lines Of Sight Through A Picture Plane [19]

The concept of the picture plane may be better understood by looking through a window or other transparent plane from a fixed viewpoint. Your lines of sight, the multitude of straight lines leading from your eye to the subject, will all intersect this plane. Therefore, if you were to reach out with a grease pencil and draw the image of the subject on this plane you would be "tracing out" the infinite number of points of intersection of sight rays and plane. The result would be that you would have "transferred" a real three-dimensional object to a two-dimensional plane.

[CS 417 Spring 2002]

Plane projection in photography

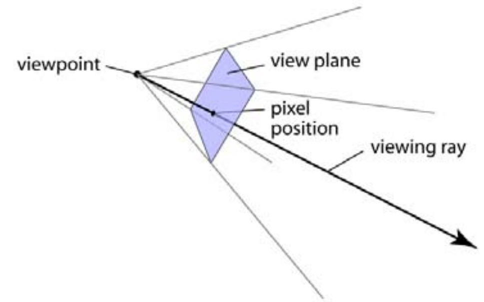
- This is another model for what we are doing
  - applies more directly in realistic rendering



[CS 417 Spring 2002]

Generating eye rays

- Use window analogy directly



Vector math review

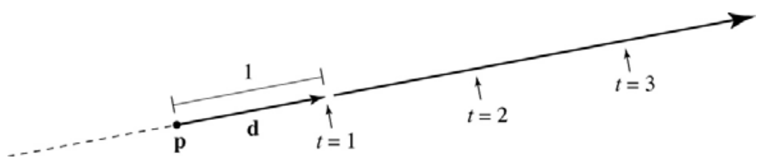
- Vectors and points
- Vector operations
  - addition
  - scalar product
- More products
  - dot product
  - cross product
- Bases and orthogonality

Ray: a half line

- Standard representation: point **p** and direction **d**

  $\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$

  - this is a *parametric equation* for the line
  - lets us directly generate the points on the line
  - if we restrict to $t > 0$ then we have a ray
  - note replacing **d** with a**d** doesn't change ray ($a > 0$)



Ray-sphere intersection: algebraic

- Condition 1: point is on ray

$$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$

- Condition 2: point is on sphere
  - assume unit sphere; see Shirley or notes for general

$$\|\mathbf{x}\| = 1 \Leftrightarrow \|\mathbf{x}\|^2 = 1$$
$$f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{x} - 1 = 0$$

- Substitute:

$$(\mathbf{p} + t\mathbf{d}) \cdot (\mathbf{p} + t\mathbf{d}) - 1 = 0$$

  - this is a quadratic equation in *t*

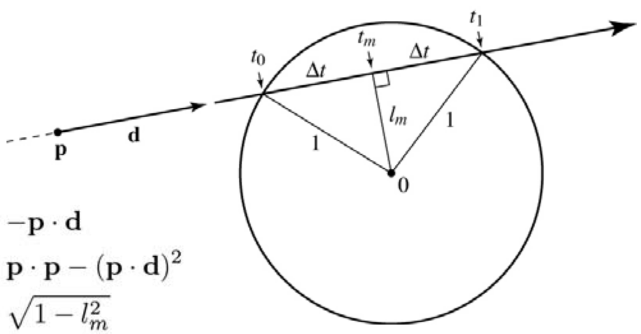
Ray-sphere intersection: algebraic

- Solution for *t* by quadratic formula:

$$t = \frac{-\mathbf{d} \cdot \mathbf{p} \pm \sqrt{(\mathbf{d} \cdot \mathbf{p})^2 - (\mathbf{d} \cdot \mathbf{d})(\mathbf{p} \cdot \mathbf{p} - 1)}}{\mathbf{d} \cdot \mathbf{d}}$$
$$t = -\mathbf{d} \cdot \mathbf{p} \pm \sqrt{(\mathbf{d} \cdot \mathbf{p})^2 - \mathbf{p} \cdot \mathbf{p} + 1}$$

  - simpler form holds when **d** is a unit vector
    but we won't assume this in practice (reason later)
  - I'll use the unit-vector form to make the geometric interpretation

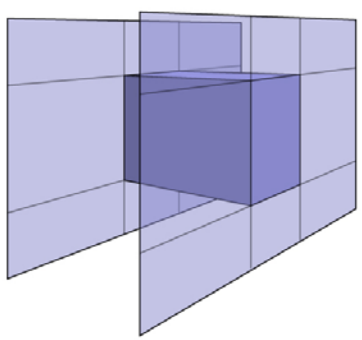
Ray-sphere intersection: geometric



$$t_m = -\mathbf{p} \cdot \mathbf{d}$$
$$l_m^2 = \mathbf{p} \cdot \mathbf{p} - (\mathbf{p} \cdot \mathbf{d})^2$$
$$\Delta t = \sqrt{1 - l_m^2}$$
$$= \sqrt{(\mathbf{p} \cdot \mathbf{d})^2 - \mathbf{p} \cdot \mathbf{p} + 1}$$
$$t_{0,1} = t_m \pm \Delta t = -\mathbf{p} \cdot \mathbf{d} \pm \sqrt{(\mathbf{p} \cdot \mathbf{d})^2 - \mathbf{p} \cdot \mathbf{p} + 1}$$

Ray-box intersection

- Could intersect with 6 faces individually
- Better way: box is the intersection of 3 slabs

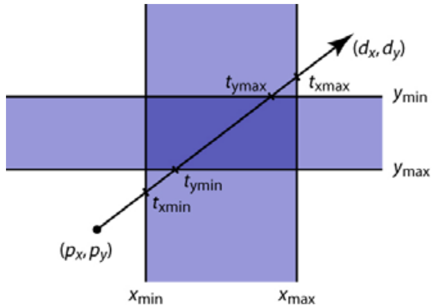


Ray-slab intersection

- 2D example
- 3D is the same!

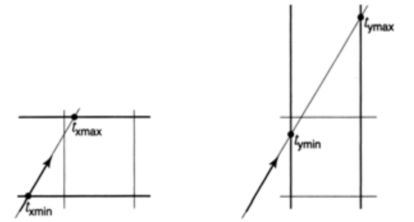
$$p_x + t_{x\min}\, d_x = x_{\min}$$
$$t_{x\min} = (x_{\min} - p_x)/d_x$$

$$p_y + t_{y\min}\, d_y = y_{\min}$$
$$t_{y\min} = (y_{\min} - p_y)/d_y$$

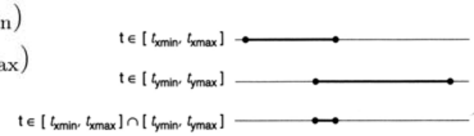


Intersecting intersections

- Each intersection is an interval
- Want last entry point and first exit point



$$t_{\min} = \max(t_{x\min}, t_{y\min})$$
$$t_{\max} = \min(t_{x\max}, t_{y\max})$$



Shirley fig. 10.16

Ray-triangle intersection

- Condition 1: point is on ray

  $$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d}$$

- Condition 2: point is on plane

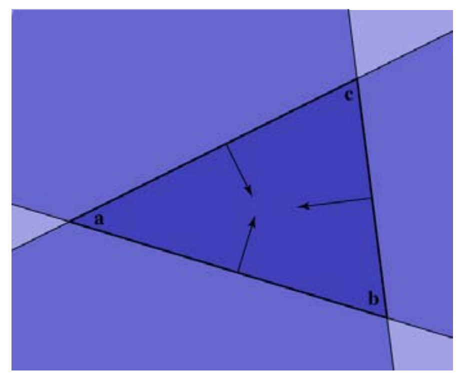
  $$(\mathbf{x} - \mathbf{a}) \cdot \mathbf{n} = 0$$

- Condition 3: point is on the inside of all three edges
- First solve 1&2 (ray–plane intersection)
  - substitute and solve for *t*:

$$(\mathbf{p} + t\mathbf{d} - \mathbf{a}) \cdot \mathbf{n} = 0$$
$$t = \frac{(\mathbf{a} - \mathbf{p}) \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$

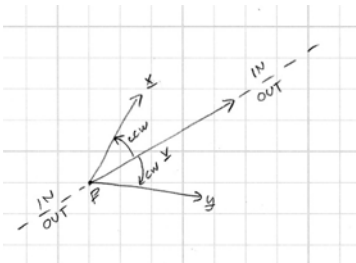
Ray-triangle intersection

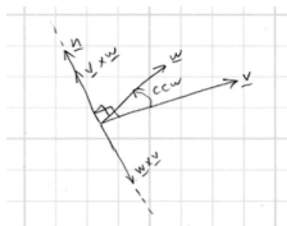
- In plane, triangle is the intersection of 3 half spaces



Inside-edge test

- Need outside vs. inside
- Reduce to clockwise vs. counterclockwise
  - vector of edge to vector to **x**
- Use cross product to decide





Ray-triangle intersection

$$(\mathbf{b}-\mathbf{a})\times(\mathbf{x}-\mathbf{a})\cdot\mathbf{n}>0$$

$$(\mathbf{c}-\mathbf{b})\times(\mathbf{x}-\mathbf{b})\cdot\mathbf{n}>0$$

$$(\mathbf{a}-\mathbf{c})\times(\mathbf{x}-\mathbf{c})\cdot\mathbf{n}>0$$

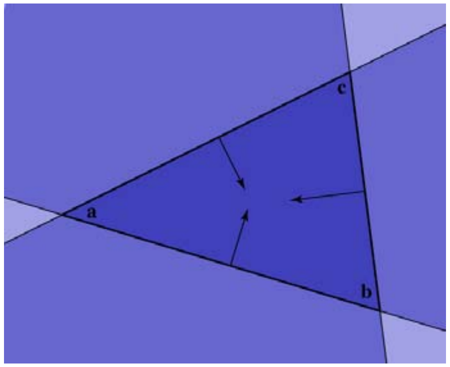
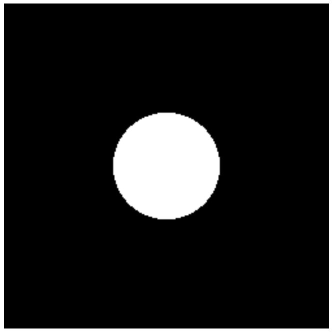


Image so far

- With eye ray generation and sphere intersection

```
Surface s = new Sphere((0.0, 0.0, 0.0), 1.0);
for 0 <= iy < ny
    for 0 <= ix < nx {
        ray = camera.getRay(ix, iy);
        if (s.intersect(ray, 0, +inf) < +inf)
            image.set(ix, iy, white);
    }
```



Intersection against many shapes

- The basic idea is:

```
hit (ray, tMin, tMax) {
    tBest = +inf; hitSurface = null;
    for surface in surfaceList {
        t = surface.intersect(ray, tMin, tMax);
        if t < tBest {
            tBest = t;
            hitSurface = surface;
        }
    }
    return hitSurface, t;
}
```

  - this is linear in the number of shapes
    but there are sublinear methods (acceleration structures)

Image so far

- With eye ray generation and scene intersection

```
for 0 <= iy < ny
   for 0 <= ix < nx {
      ray = camera.getRay(ix, iy);
      c = scene.trace(ray, 0, +inf);
      image.set(ix, iy, c);
   }

…

trace(ray, tMin, tMax) {
   surface, t = hit(ray, tMin, tMax);
   if (surface != null) return surface.color();
   else return black;
}
```

Shading

- Compute light reflected toward camera
- Inputs:
  - eye direction
  - light direction
    (for each of many lights)
  - surface normal
  - surface parameters
    (color, shininess, …)
- More on this in the
  next lecture

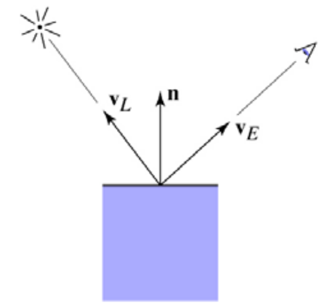
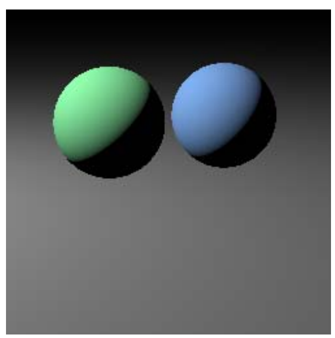


Image so far

```
trace(Ray ray, tMin, tMax) {
   surface, t = hit(ray, tMin, tMax);
   if (surface != null) {
      point = ray.evaluate(t);
      normal = surface.getNormal(point);
      return surface.shade(ray, point,
         normal, light);
   }
   else return black;
}

…

shade(ray, point, normal, light) {
   v_E = –normalize(ray.direction);
   v_L = normalize(light.pos - point);
   // compute shading
}
```

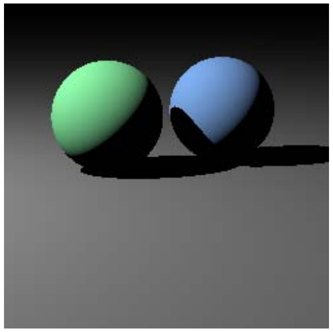


Shadows

- Surface is only illuminated if nothing blocks its view of the light.
- With ray tracing it's easy to check
  - just intersect a ray with the scene!

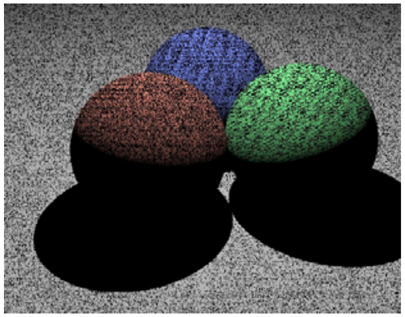
Image so far

```
shade(ray, point, normal, light) {
    shadRay = (point, light.pos - point);
    if (shadRay not blocked) {
        v_E = –normalize(ray.direction);
        v_L = normalize(light.pos - point);
        // compute shading
    }
    return black;
}
```



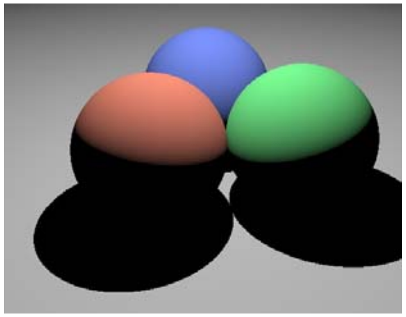
Shadow rounding errors

- Don't fall victim to one of the classic blunders:



- What's going on?
  - hint: at what *t* does the shadow ray intersect the surface you're shading?

- Solution: shadow rays start a tiny distance from the surface



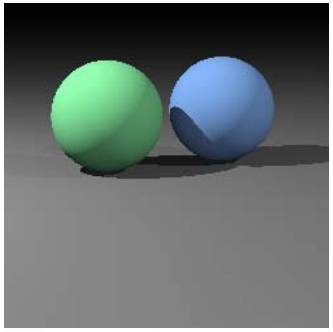
- Do this by moving the start point, or by limiting the *t* range

Multiple lights

- Important to fill in black shadows
- Just loop over lights, add contributions
- Ambient shading
  - black shadows are not really right
  - one solution: dim light at camera
  - alternative: all surface receive a bit more light
    - just add a constant "ambient" color to the shading…

Image so far

```
shade(ray, point, normal, lights) {
    result = ambient;
    for light in lights {
        if (shadow ray not blocked) {
            result += shading contribution;
        }
    }
    return result;
}
```



Ray tracer architecture 101

- You want a class called Ray
  - point and direction; evaluate(t)
  - possible: tMin, tMax
- Some things can be intersected with rays
  - individual surfaces
  - the whole scene
  - often need to be able to limit the range (*e.g.* shadow rays)
- Once you have the visible intersection, compute the color
  - this is an object that's associated with the object you hit
  - its job is to compute the color

Architectural practicalities

- Return values
  - surface intersection tends to want to return multiple values
    - *t*, surface or shader, normal vector, maybe surface point
  - in many programming languages (*e.g*. Java) this is a pain
  - typical solution: an *intersection record*
    - a class with fields for all these things
    - keep track of the intersection record for the closest intersection
    - be careful of accidental aliasing (which is very easy if you're new to Java)
- Efficiency
  - in Java the (or, a) key to being fast is to minimize creation of objects
  - what objects are created for every ray? try to find a place for them where you can reuse them.
  - Shadow rays can be cheaper (any intersection will do, don't need closest)
  - but: "Get it Right, Then Make it Fast"