CS 465 Final Exam

Friday 10 December 2004
2.5 hours

Problem 1: Filtering & Resampling (20 pts)

1. Consider an image consisting of a square of four adjacent white (pixel value = 1) pixels on a black (pixel value = 0) background:

$$\begin{matrix} & & \vdots & & \\ & 0 & 0 & 0 & 0 & \\ \cdots & 0 & 1 & 1 & 0 & \cdots \\ & 0 & 1 & 1 & 0 & \\ & 0 & 0 & 0 & 0 & \\ & & \vdots & & \end{matrix}$$

   What is the result of resampling this image to increase its resolution by a factor of $\frac{3}{2}$ using bilinear interpolation (that is, using a separable tent filter)? Assume that the upper left white pixel aligns exactly with a pixel in the resampled image. Compute your result by the separable filtering method, and give your answer as two arrays of numbers like the one above: one for the intermediate result and one for the final output.

2. Suppose we filter the same 2x2 square twice with the following discrete filter:

$$\frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

   Show how to compute the result easily on paper using a small number of easy multiplication and addition operations (you can do it with less than a dozen individual operations, not counting multiplication by 1 and 0 and addition to 0). Take advantage of separability, associativity, commutativity, and symmetry. Give the final answer as another array of numbers, but use symmetry to reduce writing if you like.

Problem 2: Graphics Pipeline (25 pts)

Consider an eye-space triangle whose vertices have the following positions, normals, and texture coordinates assocated with them:

a: position $(0.4, 0.2, -2)$, normal $(-0.5, -0.3, 1)$, texcoord $(0, 0)$

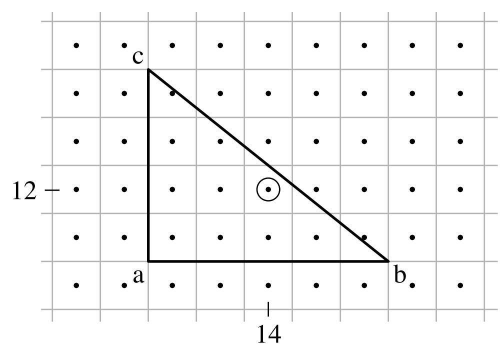
b: position $(1.75, 0.25, -2.5)$, normal $(0.5, -0.3, 1)$, texcoord $(0.8, 0.4)$

c: position $(0.6, 1.5, -3)$, normal $(-0.5, 0.5, 1)$, texcoord $(0.3, 0.9)$

The camera's field of view is 90 degrees horizontally and vertically, and the near and far distances are 1 and 5. The image is 20 by 20 pixels. This means the projection and viewport matrices are:

$$M_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1.5 & -2.5 \\ 0 & 0 & -1 & 0 \end{bmatrix} \qquad M_v = \begin{bmatrix} 10 & 0 & 9.5 \\ 0 & 10 & 9.5 \\ 0 & 0 & 1 \end{bmatrix}$$

(You may not need all this information, but it may be handy in checking answers.) Here is how the triangle looks in screen space:



1. What are the clip space coordinates of the vertices before and after the perspective divide?

2. Assuming the vertex processor hands a screen space position, a normal, and a texture coordinate to the rasterizer, how many quantities will the rasterizer interpolate (internally), and what do they all represent? Assume the rasterizer uses the barycentric coordinates β and γ (leaving α implied) for inside/outside testing and that the fragment processor will finish up perspective correction. Normals are to be interpolated without perspective correction.

3. What are the values at the three vertices from which the rasterizer will interpolate each of these quantities?

The triangle will be shaded using a Lambertian illumination model with no texture mapping (the texture coordinates won't be used). We'll compare two approaches to handling the shading in the pipeline: per-vertex lighting (Gouraud shading) and per-fragment lighting (Phong shading but still Lambertian illumination). The light source is a directional source (infinitely far away) in the direction $(0, 0, 1)$ in eye space, the light intensity is $(1, 1, 1)$, and the diffuse coefficient is a constant $(0.5, 0.5, 1)$. Note that the normals specified by the application are not unit vectors; assume for the purposes of this problem that the vertex or fragment program will normalize the vector just before it's used.

4. In the Phong shading case, what will be the color at the circled fragment? Show how you computed this.

5. In the Gouraud shading case, will the color be darker, the same, or lighter? Why?

Problem 3: Transformations (15 pts)

The matrices

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{ and } \quad \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

are a rotation by $+45°$ about an axis through the origin in the direction $(0, 0, 1)$, a nonuniform scale by 2 about the origin along the direction $(0, 1, 0)$, and a reflection across the plane $x = 0$, respectively. Using the same forms of description, describe what the following products of matrices do:

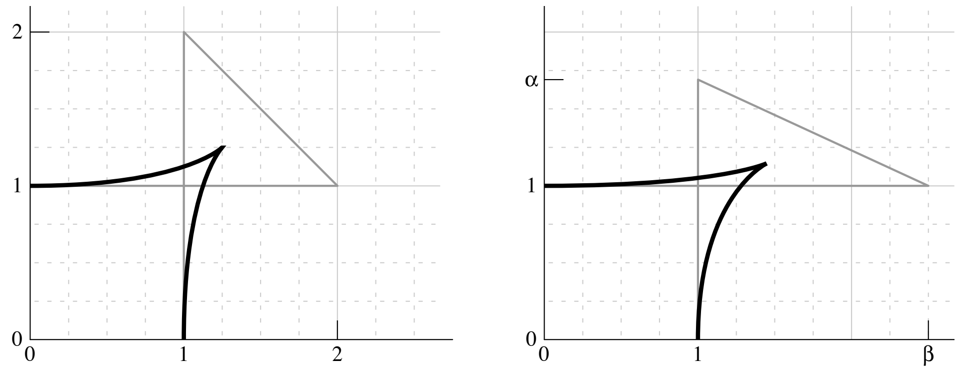
1. $$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -\sqrt{3} & 0 \\ 0 & 2 & 0 & 0 \\ \sqrt{3} & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. $$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hint: That last one is a little tricky.

Problem 4: Splines (25 pts)

A Bézier spline with the control points $(1, 0)$, $(1, 2)$, $(2, 1)$, $(0, 1)$ will form a cusp, as shown here on the left:



1. Sketch plots of the coordinate functions $x(t)$ and $y(t)$ and their derivatives. Explain how you can tell that the cusp happens by loooking at these plots.
2. What are the parametric (C^k) continuity and geometric (G^k) continuity at the cusp?

Let the lengths of the first and last segments of the control polygon be α and β, as shown at right in the figure above. We have seen that a cusp forms when $\alpha = \beta = 2$, but for many other values of α it is also possible to find a β for which a cusp will form.

3. Let t_x and t_y be the values of t at which $x'(t) = 0$ and $y'(t) = 0$ respectively. Find an expression for t_x as a function of β.
4. Use a simple symmetry argument to get t_y as a function of α, and then use your two functions to eliminate t and find β as a function of α.

Hint: Three easy ways to check your answer: (a) make sure it works for $\alpha = 2$; (b) by symmetry, swapping α and β will preserve the cusp; (c) the right-hand diagram is to scale.

Problem 5: Triangle Meshes (15 pts)

Consider the indexed triangle set defined by the vertex list [(–1, –1, –1) (–1, –1, 1) (1, –1, 1) (1, –1, –1) (0, 1, 0)] and the triangle list [0 2 1, 0 2 3, 0 2 4, 4 0 1, 4 1 2, 4 2 3, 4 3 0].

1. What is the shape described?
2. As it stands this is not a manifold mesh. What two changes are required to make it into a manifold mesh?
3. Express the manifold mesh using a single triangle strip.