CS 465 Homework 7

out: Friday 22 October 2004
due: Friday 29 October 2004

Problem 10: Super-Hermite splines

Hermite splines are cubic splines defined by point and tangent (that is, value and derivative) constraints at the two endpoints of each segment. More specifically, the following constraints define a Hermite segment:

$$\begin{aligned}\mathbf{p}(0) &= \mathbf{p}_0 \\ \mathbf{p}'(0) &= \mathbf{v}_0 \\ \mathbf{p}(1) &= \mathbf{p}_1 \\ \mathbf{p}'(1) &= \mathbf{v}_1\end{aligned}$$

The resulting spline is defined as follows:

$$\mathbf{p}(t) = \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}^T \begin{bmatrix} 2 & 1 & -2 & 1 \\ -3 & -2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{v}_0 \\ \mathbf{p}_1 \\ \mathbf{v}_1 \end{bmatrix}$$

For added control we decide to add the ability to control the second derivative (the curvature) of the spline at the two endpoints as well as the first derivative:

$$\begin{aligned}\mathbf{p}''(0) &= \mathbf{w}_0 \\ \mathbf{p}''(1) &= \mathbf{w}_1\end{aligned}$$

1. What degree polynomial is required to match these two additional constraints?
2. Give the matrix equation that defines this new spline. Please put the controls in the order $\mathbf{p}_0$, $\mathbf{v}_0$, $\mathbf{w}_0$, $\mathbf{p}_1$, $\mathbf{v}_1$, $\mathbf{w}_1$.
3. Plot the basis functions for the new spline over the interval $[0, 1]$.
4. Suppose the spline is set up as a vertical line segment with $\mathbf{p}_0$ at the bottom and $\mathbf{p}_1$ at the top. What happens to the middle of the curve if we move $\mathbf{w}_0$ to point rightwards?

Problem 11: Spline conversions

A B-spline curve segment is defined by the four control points

$$\mathbf{p}_0 = (-1, -1)$$
$$\mathbf{p}_1 = (1, -1)$$
$$\mathbf{p}_2 = (1, 1)$$
$$\mathbf{p}_3 = (-1, 1)$$

1. Give the Bézier control points that will produce the same curve segment.
2. Plot the Bézier control polygon with the B-spline control polygon and a sketch of the curve.

We now use the same points to define a close B-spline curve. In a closed spline curve, the control point indices wrap around, so that the first point comes after the last one. In this way n control points define an n-segment B-spline (rather than $n - 3$ segments as with an open spline). So our 4-point curve will have a segment for $(\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$, one for $(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_0)$, and so on.

3. Plot the B-spline control points with a sketch of the closed curve, and plot the Bézier control points that are required to produce the closed curve.