CS 465 Homework 5

(revised September 29, 2004)

out: Friday 24 September 2004
due: Friday 8 October 2004

Problem 7: 3D Transformations

In 3D, one often writes a rotation in *axis-angle* form, giving a unit vector $\hat{\mathbf{v}}$ and an angle θ to specify a rotation by θ around an axis throught the origin in the direction $\hat{\mathbf{v}}$.

For this problem let $\hat{\mathbf{v}} = [3\ 4\ 5]^T/\sqrt{50}$ and $\theta = 30°$.

1. Give the matrix for a rotation that maps $\mathbf{e}_1 = [1\ 0\ 0]^T$ to the vector $\hat{\mathbf{v}}$, as a product of two coordinate-axis rotations. *Hint: It may be easier to solve this problem backwards.*

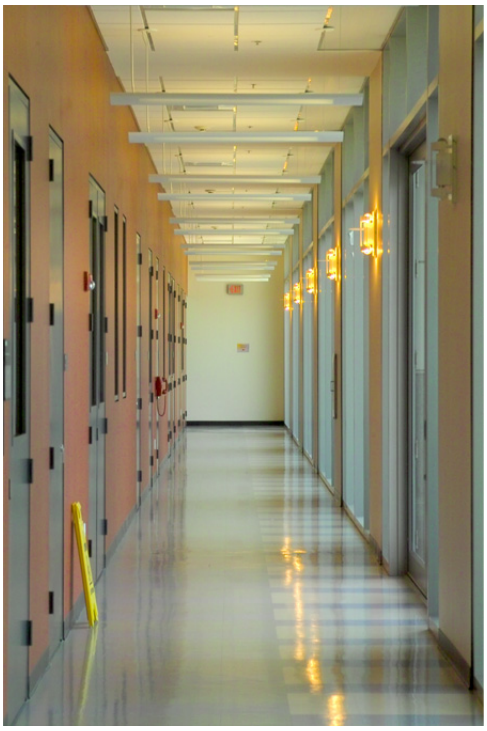
2. Give the matrix for a rotation that maps $\mathbf{e}_1$ to the vector $\hat{\mathbf{v}}$ by constructing its columns as an orthonormal basis.

3. Give the matrix for a rotation by $\theta = 30°$ about the axis (through the origin) in direction $\hat{\mathbf{v}}$, as a product of several transformations.

You might want to check your answer using a computer by ensuring that it is orthonormal and does not move the vector $\hat{\mathbf{v}}$.

Problem 8: Perspective

The following two photos of a hallway in Duffield were taken with the same camera using two different focal lengths.



(a)

(b)

In both images, the hallway's end wall appears the same size.

1. Which image has a larger angular field of view (shorter focal length)?
2. Which camera position is closer to the end wall?

Both images are 1512×2268 pixels. The distance from the exit sign on the end wall to the floor is 413 pixels in both images.

The heights of the center posts of the windows along the right side of the hallway, as measured in the images, are:

(a) 461, 532, 634, 781, and 1015 pixels.
(b) 496, 727, and 1350 pixels.

3. The windows are equally spaced along the hallway. What is the ratio of the focal lengths used for the two images? *Hint: Figure out where the camera is first, by drawing a picture of the projection in side view. You don't need to use all the data.*

4. From the information we have so far, we cannot determine the actual fields of view. Think of two different hallways that, using two different fields of view, would produce identical images that have three posts in view (similar to image (b)). Draw plan and side views of each one showing the camera position, the three window posts, and the projection onto the image plane. *Hint: What transformations can you apply to the whole scene including the camera without changing the picture on the image plane?*

The windows are 21 feet apart and the center of the farthest window is 9 feet from the end wall. The distance from the exit sign to the floor is 9.5 feet.

5. What are the distances of the two camera positions from the end wall? What is the ratio of the distances?

6. What are the horizontal and vertical angular fields of view of the two photographs?

The camera's image sensor measures 20.7mm $\times$ 13.8mm.

7. What are the focal lengths for the two photographs?

For the whole problem, you can assume the camera is pointed parallel to the hallway in both cases, and that the camera's pixels are square.