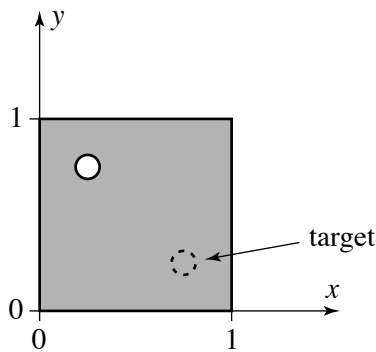# CS 465 Homework 4

out: Friday 17 September 2004
due: Friday 24 September 2004

Problem 6: 2D Transformations

Suppose you have a square metal plate with an off-center hole in it:

and you are in charge of a robot that is supposed to bolt the plate to a surface that has a similar hole. To do this, the robot needs to align the two holes. However, the robot only speaks affine transformations, so you need to give it a transformation matrix to tell it how to move the plate.

The plate is currently positioned as shown in the illustration. The hole in the plate is at $(\frac{1}{4}, \frac{3}{4})$, and the target hole is at $(\frac{3}{4}, \frac{1}{4})$. For each of the following types of rigid motions, what transformation do we need to apply to align the holes?

1. a rotation pivoting about the plate's lower left corner

2. a rotation pivoting about the plate's upper right corner

Now suppose that, rather than a boring material like metal, the plate is made out of an amazing high-tech material that can be deformed only by linear transformations (of course it can still undergo rigid motions too). Now answer the question for the following kinds of non-rigid affine motions:

3. a uniform scale about the plate's upper left corner

4. a nonuniform scale about the plate's lower left corner

5. a shear along an axis through the plate's lower left corner

For each part, give (a) a verbal description of the transformation (e. g. "a rotation about $(3, 4)$ by 52 degrees"); (b) the 3 by 3 homogeneous matrix for the transformation; and (c) a drawing that shows the final position of the plate overlaid on the starting position. For example:

0. a translation

   (a) A translation by $\frac{1}{2}$ in x and $-\frac{1}{2}$ in y.

   (b) $T = \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$

   (c)