CS 465 Homework 4a

out: Friday 17 October 2003
due: Wednesday 29 October 2003

Problem 1: 2D transformations

Each of the following matrices defines a 2D affine transformation. For each transformation,

1. Find the images of the four corners of the unit square, $(0, 0)$, $(1, 0)$, $(0, 1)$, and $(1, 1)$.
2. Express the transformation as a sequence of one or more of the following elementary transformations from the lecture slides: translation (by a vector); uniform scaling (about the origin); nonuniform scaling (about the origin, along the coordinate axes); rotation (about the origin); reflection (across the coordinate axes); and shearing (along the coordinate axes).
3. Suppose we generalize the elementary transformations to be centered about arbitrary points, rather than just the origin. How can we write the transformation as a sequence of fewer elementary transformations?

For example, the matrix

$$\begin{bmatrix} 2 & 0 & 1 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

takes the four given points to $(1, 0)$, $(3, 0)$, $(1, 2)$, and $(3, 2)$. It can be expressed as a uniform scale by 2 followed by a translation of $(1, 0)$, or when scales can be around arbitrary points, it is just a scale by 2 around the point $(-1, 0)$.

The matrices are:

(a) $\begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{3}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{3}{2} & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Problem 2: 3D axis-angle rotations

In 3D, one often writes a rotation in *axis-angle* form, giving a point $\mathbf{p}$, a unit vector $\hat{\mathbf{v}}$, and an angle θ to specify a rotation by θ around the line through $\mathbf{p}$ in the direction $\hat{\mathbf{v}}$.

1. Let $\mathbf{p} = [2\ 0\ 3]^T$ and $\hat{\mathbf{v}} = [3\ 4\ 0]^T/\sqrt{5}$.
   (a) Give the matrix for a rotation that maps $\mathbf{e}_1 = [1\ 0\ 0]^T$ to the vector $\hat{\mathbf{v}}$.
   (b) Give the matrix for a rigid motion that maps the line segment from $\mathbf{0}$ to $\mathbf{0} + \mathbf{e}_1$ to the line segment from $\mathbf{p}$ to $\mathbf{p} + \hat{\mathbf{v}}$.
   (c) Give the matrix for a rotation by $\theta = 30°$ about $\mathbf{p}$ and $\hat{\mathbf{v}}$. You are welcome (encouraged, even) to express the answer as a product of simpler transformations.

   You might want to check your answer using a computer by ensuring that it does not move the point $\mathbf{p}$ or the point $\mathbf{p} + \hat{\mathbf{v}}$.
2. For an arbitrary point $\mathbf{p}$ and direction $\hat{\mathbf{v}} = [v_1\ v_2\ v_3]^T$, write the axis-angle rotation as a product of matrices $MR_\theta M^T$:
   (a) where M is the product of a translation and rotations about two coordinate axes.
   (b) where M is a coordinate frame matrix constructed using cross products.

Problem 3: Viewing

Consider a cubical box (a cube with one side missing) of size 2.0 by 2.0 by 2.0, centered at the origin. There is an ordinary perspective camera looking into the box from a distance of 5.0 units from the origin. The camera's aspect ratio is 1.0, and its field of view is such that the box exactly fills the image. For this problem use image coordinates that run from $(-1, -1)$ to $(1, 1)$.

1. What is the camera's field of view?
2. Sketch the image produced by this camera.
3. Give the viewing and projection matrices of the camera.
4. To what image coordinates will the 8 corners of the box project? How does the answer change if the camera distance is increased to 10.0 (with the box still filling the image)?
5. To what image coordinates will the midpoints of the 12 edges of the box project? (Symmetry can save you some writing here.) For each midpoint, at what parameter along the edge does it appear? (For example, if the 3D midpoint maps to the 2D midpoint the answer is 0.5.)
6. How does the answer to the previous part change if the camera distance is increased to 10.0? Are they closer to or farther from being in the centers of the box's sides in the image?