Compositing

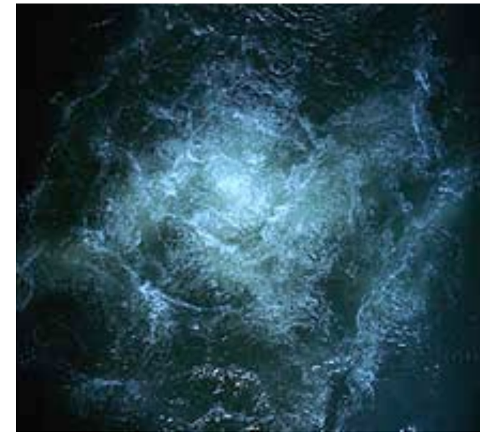
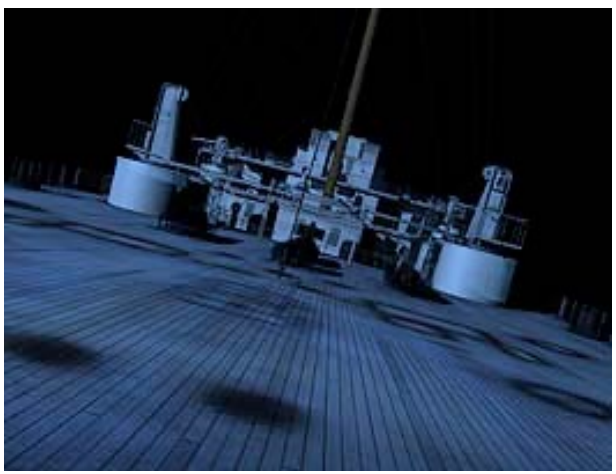
CS4620 Lecture 17

Pixel coverage

- Antialiasing and compositing both deal with questions of pixels that contain unresolved detail
- Antialiasing: how to carefully throw away the detail
- Compositing: how to account for the detail when combining images

Compositing




[*Titanic* ; DigitalDomain; vfxhq.com]

Combining images

- Often useful combine elements of several images
- Trivial example: video crossfade
  - smooth transition from one scene to another

$$r_C = t r_A + (1 - t) r_B$$
$$g_C = t g_A + (1 - t) g_B$$
$$b_C = t b_A + (1 - t) b_B$$

  - note: weights sum to 1.0
    - no unexpected brightening or darkening
    - no out-of-range results
  - this is *linear interpolation*

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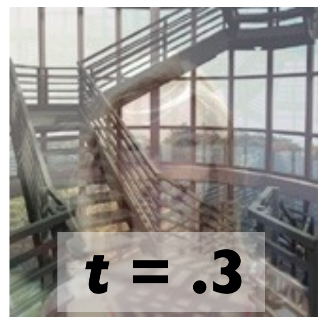

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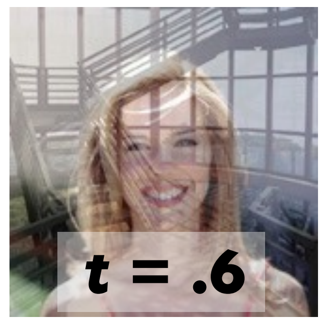

$$
\begin{aligned}
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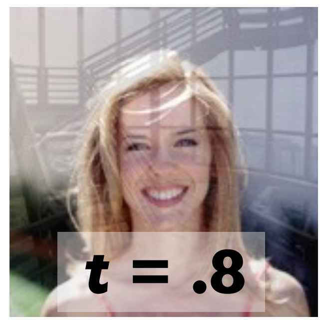

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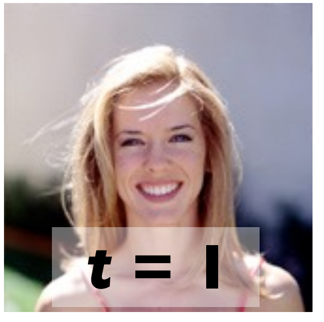

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Foreground and background

- In many cases just adding is not enough
- Example: compositing in film production
  - shoot foreground and background separately
  - also include CG elements
  - this kind of thing has been done in analog for decades
  - how should we do it digitally?

Foreground and background

- How we compute new image varies with position

[Chuang et al. / Corel]

- Therefore, need to store some kind of tag to say what parts of the image are of interest

Binary image mask

- First idea: store one bit per pixel
  - answers question "is this pixel part of the foreground?"

[Chuang et al. / Corel]

  - causes jaggies similar to point-sampled rasterization
  - same problem, same solution: intermediate values

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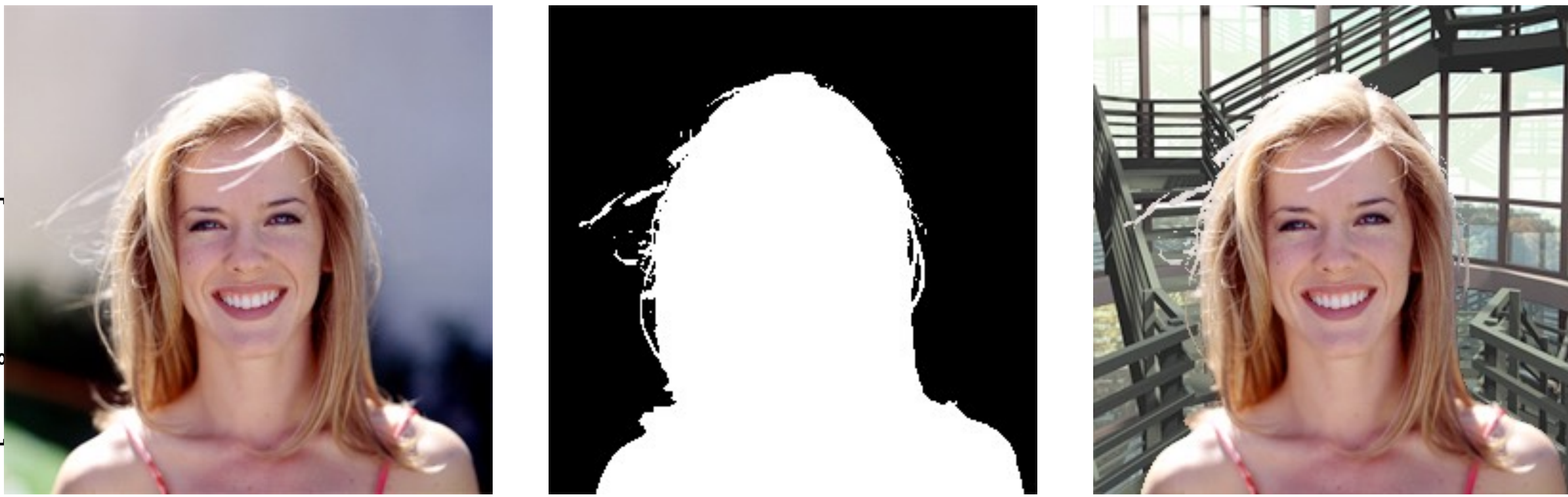

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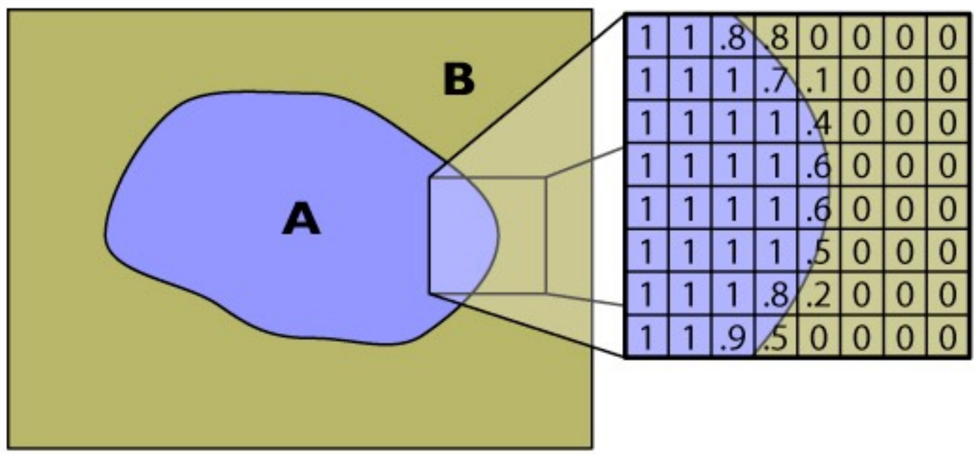
Partial pixel coverage

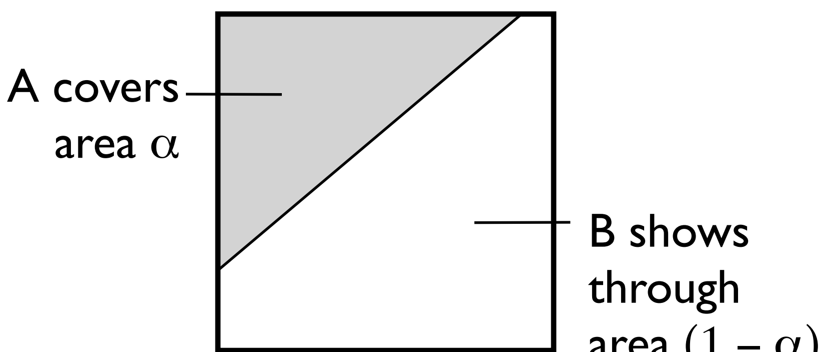
- The problem: pixels near boundary are not strictly foreground or background



  - how to represent this simply?
  - interpolate boundary pixels between the fg. and bg. colors

Alpha compositing

- Formalized in 1984 by Porter & Duff
- Store fraction of pixel covered, called α



$$E = A \textbf{ over } B$$

$$r_E = \alpha_A r_A + (1 - \alpha_A) r_B$$

$$g_E = \alpha_A g_A + (1 - \alpha_A) g_B$$

$$b_E = \alpha_A b_A + (1 - \alpha_A) b_B$$

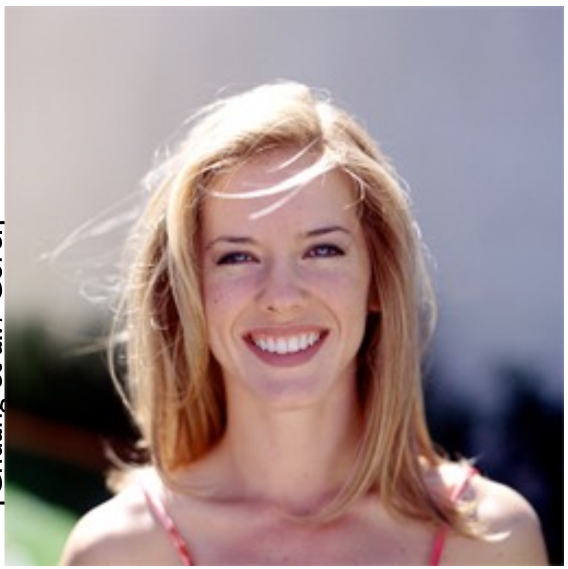
  - this exactly like a spatially varying crossfade
- Convenient implementation
  - 8 more bits makes 32
  - 2 multiplies + 1 add per pixel for compositing

A little notation

- Define $\bar{\alpha} = 1 - \alpha$
- then in $E = A \textbf{ over } B$

$$c_E = \alpha_A c_A + \bar{\alpha}_A c_B$$

Alpha compositing—example



[Chuang et al. / Corel]

Alpha compositing—example

[Chuang et al. / Corel]

Alpha compositing—example

[Chuang et al. / Corel]

Compositing composites

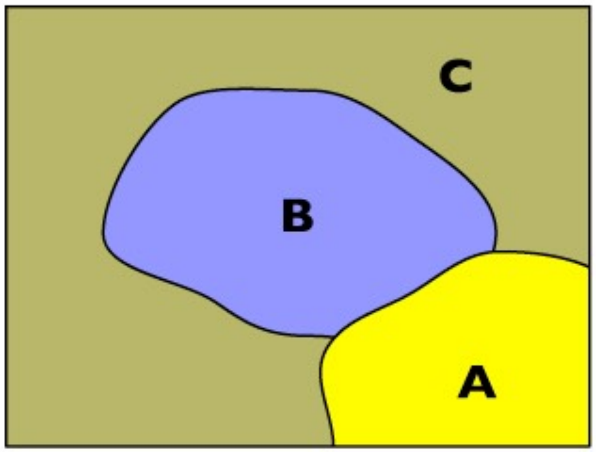
- so far have only considered single fg. over single bg.
- in real applications we have *n* layers
  - *Titanic* example
  - compositing foregrounds to create new foregrounds
    - what to do with α?
    - want (c_A, α_A) $\mathbf{over}$ $(c_B, \alpha_B) = (\alpha_A c_A + \bar{\alpha}_A c_B, ???)$
- desirable property: associativity

$$A \ \mathbf{over}\ (B \ \mathbf{over}\ C) = (A \ \mathbf{over}\ B) \ \mathbf{over}\ C$$

  - to make this work we need to be careful about how α is computed

Compositing composites

- Some pixels are partly covered in more than one layer



  – in D = A **over** (B **over** C) what will be the result?

$$c_D = \alpha_A c_A + \bar{\alpha}_A [\alpha_B c_B + \bar{\alpha}_B c_C]$$
$$= \alpha_A c_A + \bar{\alpha}_A \alpha_B c_B + \bar{\alpha}_A \bar{\alpha}_B c_C$$

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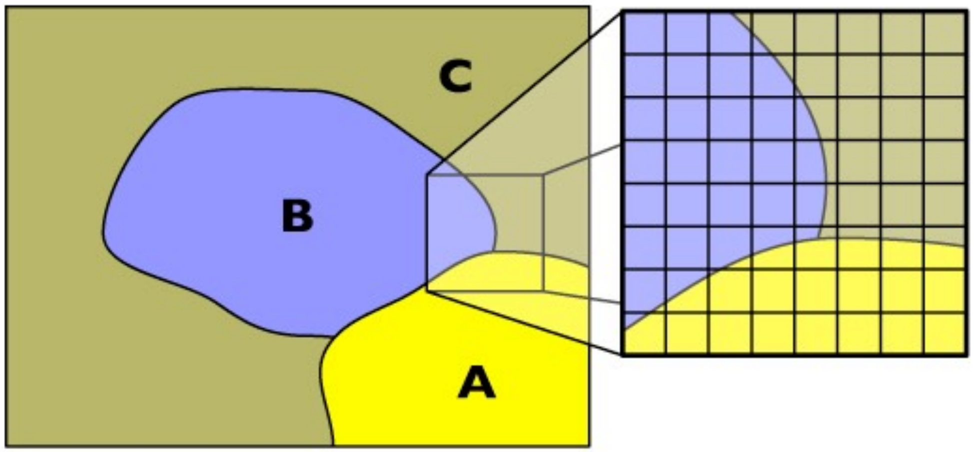

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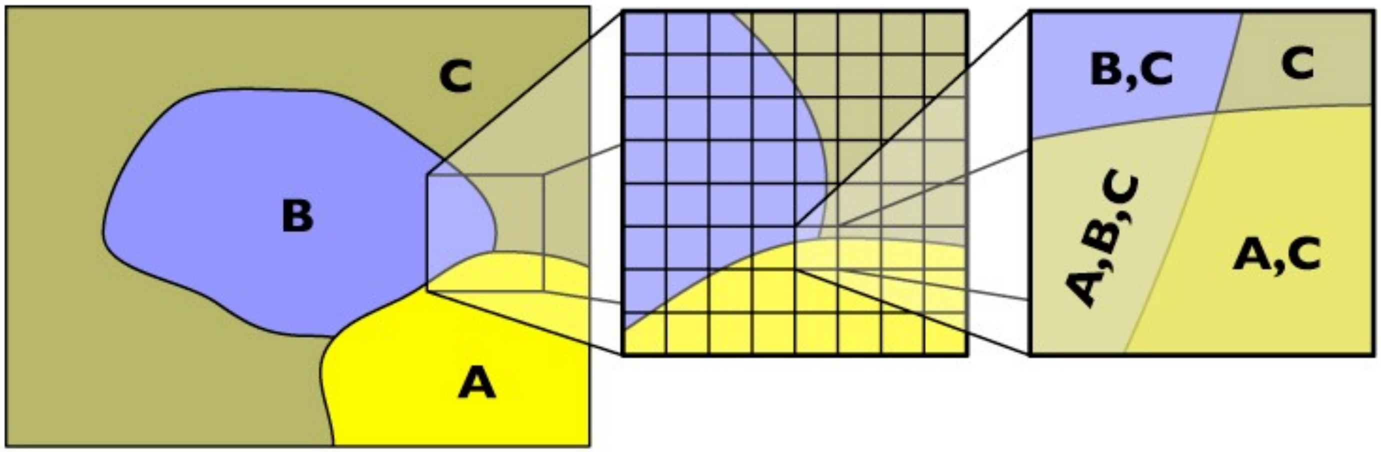

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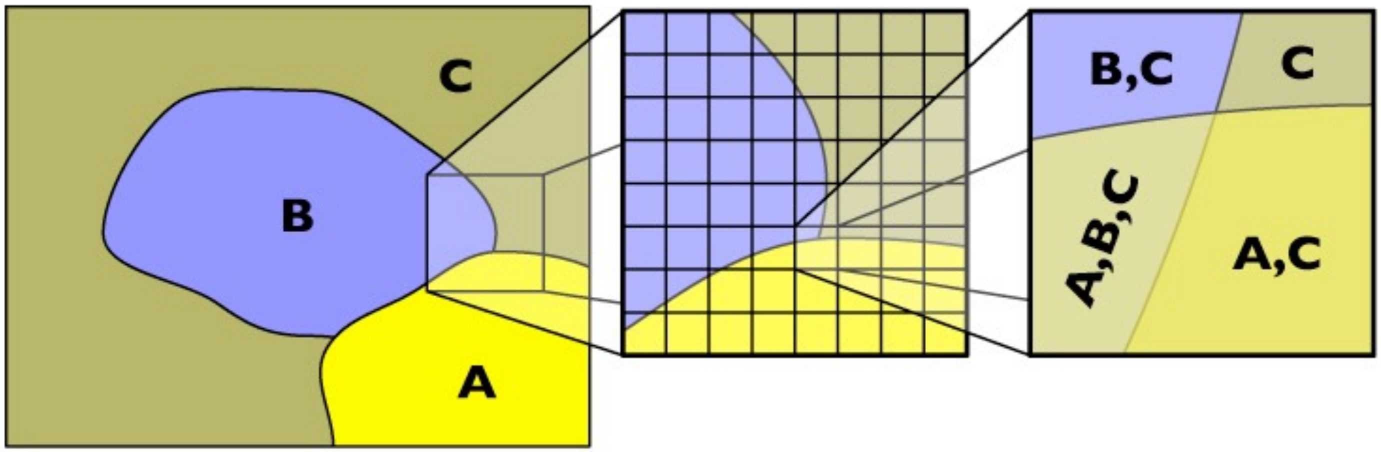


  – in D = A **over** (B **over** C) what will be the result?

$$
\begin{aligned}
c_D &= \alpha_A c_A + \bar{\alpha}_A[\alpha_B c_B + \bar{\alpha}_B c_C] \\
&= \alpha_A c_A + \bar{\alpha}_A \alpha_B c_B + \bar{\alpha}_A \bar{\alpha}_B c_C
\end{aligned}
$$

Fraction covered by neither A nor B

Associativity?

- What does this imply about (A **over** B)?
  - Coverage has to be

$$\begin{aligned}\alpha_{(A\,\mathbf{over}\,B)} &= 1 - \bar{\alpha}_A \bar{\alpha}_B \\ &= \alpha_A + \bar{\alpha}_A \alpha_B = \alpha_B + \alpha_A \bar{\alpha}_B\end{aligned}$$

  - …but the color values then don't come out nicely in D = (A **over** B) **over** C, the color should be the same as in A **over** (B **over** C):

$$\begin{aligned}c_D &= \alpha_{(A\,\mathbf{over}\,B)}(\ldots) + \bar{\alpha}_{(A\,\mathbf{over}\,B)} c_C \\ &= (\alpha_A + \bar{\alpha}_A \alpha_B)(\ldots) + \bar{\alpha}_A \bar{\alpha}_B c_C \\ &= \alpha_A c_A + \bar{\alpha}_A \alpha_B c_B + \bar{\alpha}_A \bar{\alpha}_B c_C\end{aligned}$$

An improvement

- Compositing equation again

$$c_E = \alpha_A c_A + (1 - \alpha_A) c_B$$

- Note c_A appears only in the product $\alpha_A c_A$
  – so why not do the multiplication ahead of time?
- Leads to *premultiplied alpha*:
  – store pixel value (r', g', b', α) where $c' = \alpha c$
  – E = A **over** B becomes

$$c'_E = c'_A + (1 - \alpha_A) c'_B$$

  – this turns out to be more than an optimization…
  – hint: so far the background has been opaque!

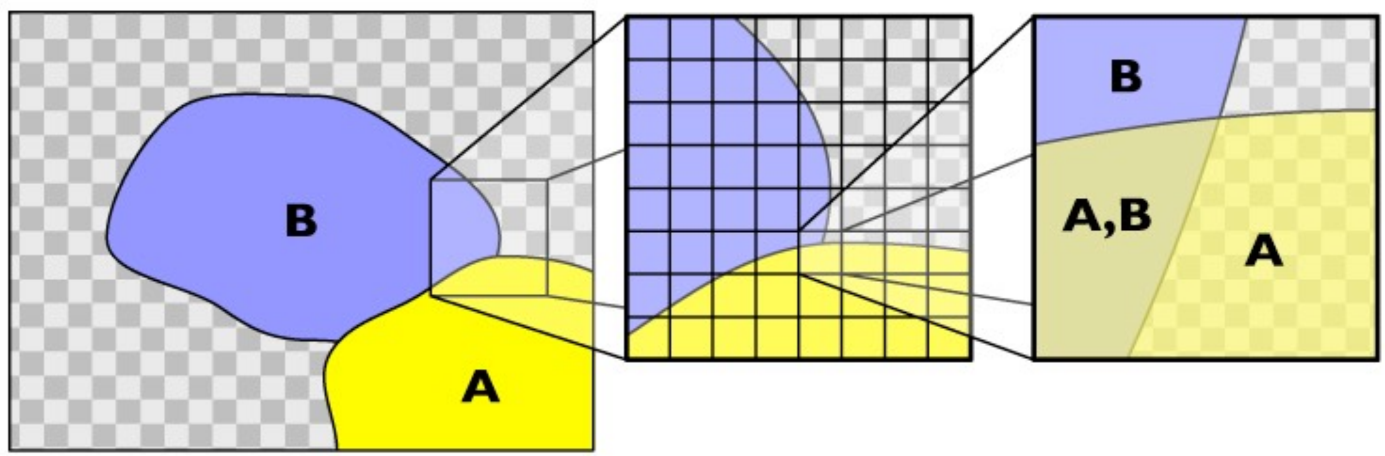
Premultiplied alpha

RGB

α

Compositing composites

- What about just E = A **over** B (with B transparent)?

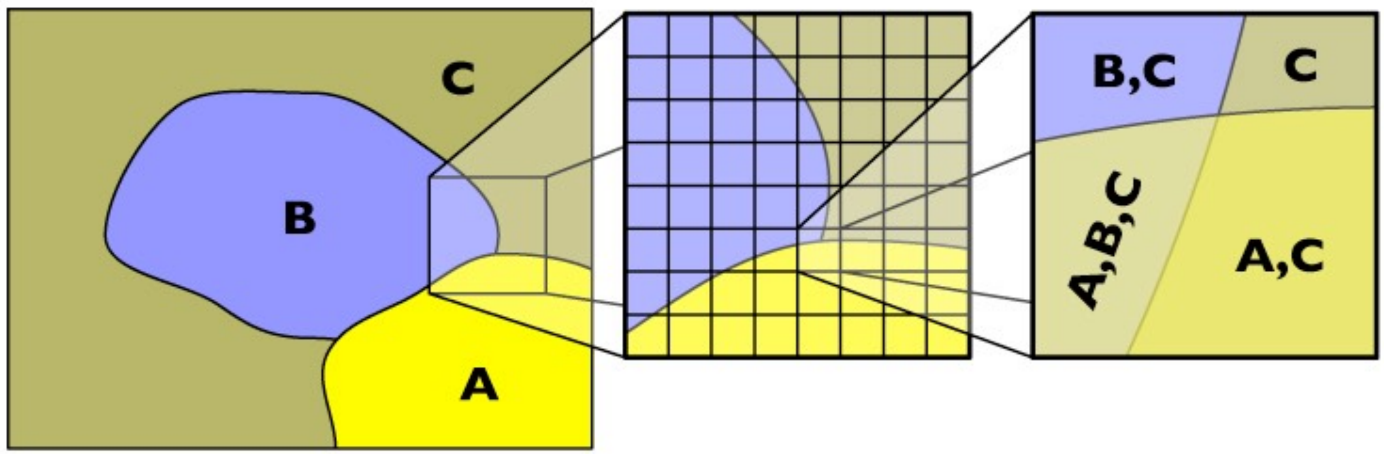


  - in premultiplied alpha, the result

    $$(c'_A, \alpha_A) \textbf{ over } (c'_B, \alpha_B) = (c'_A + \bar{\alpha}_A c_B,\ \alpha'_A + \bar{\alpha}_A \alpha_B)$$

    treats alpha just like colors, and it leads to associativity.

Associativity!



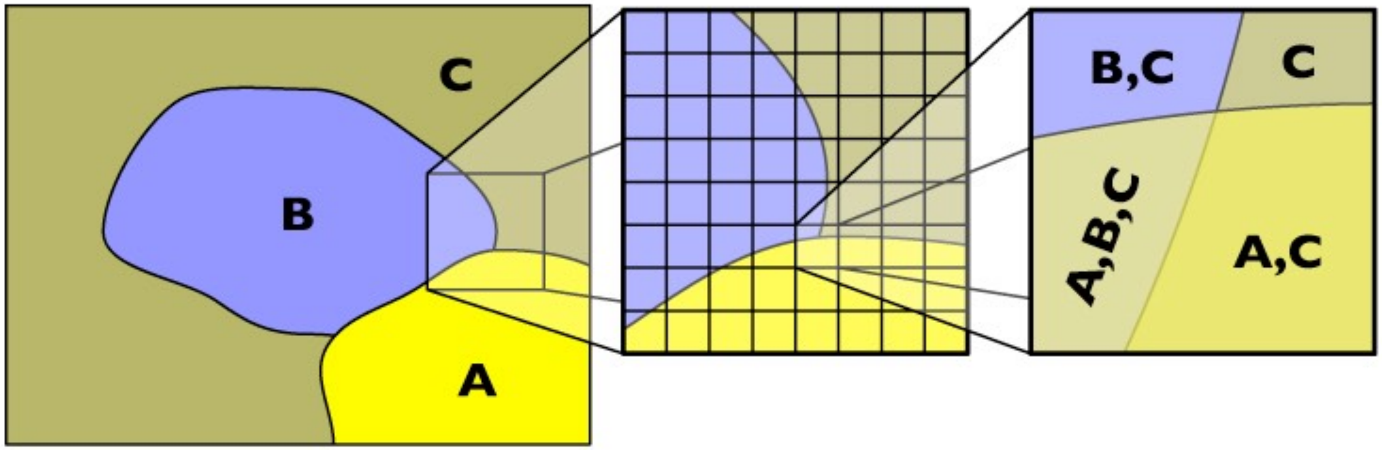
A **over** (B **over** C)

(A **over** B) **over** C

color

alpha

Associativity!



A **over** (B **over** C)

(A **over** B) **over** C

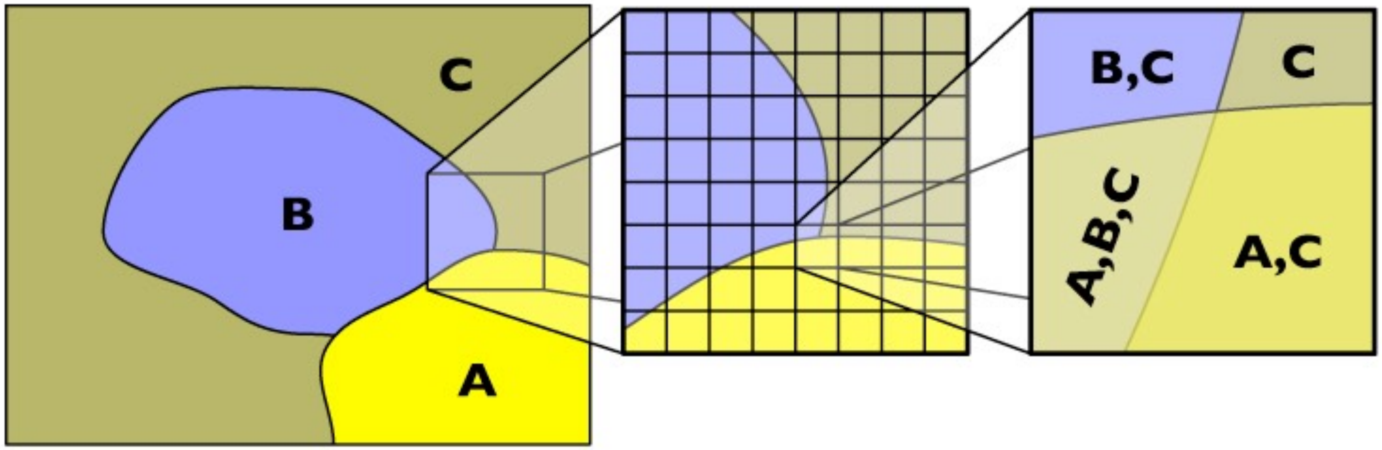
color

$$c'_A + \bar{\alpha}_A[c'_B + \bar{\alpha}_B c'_C]$$

$$c'_A + \bar{\alpha}_A c'_B + \bar{\alpha}_A \bar{\alpha}_B c'_C$$

alpha

Associativity!



A **over** (B **over** C) (A **over** B) **over** C

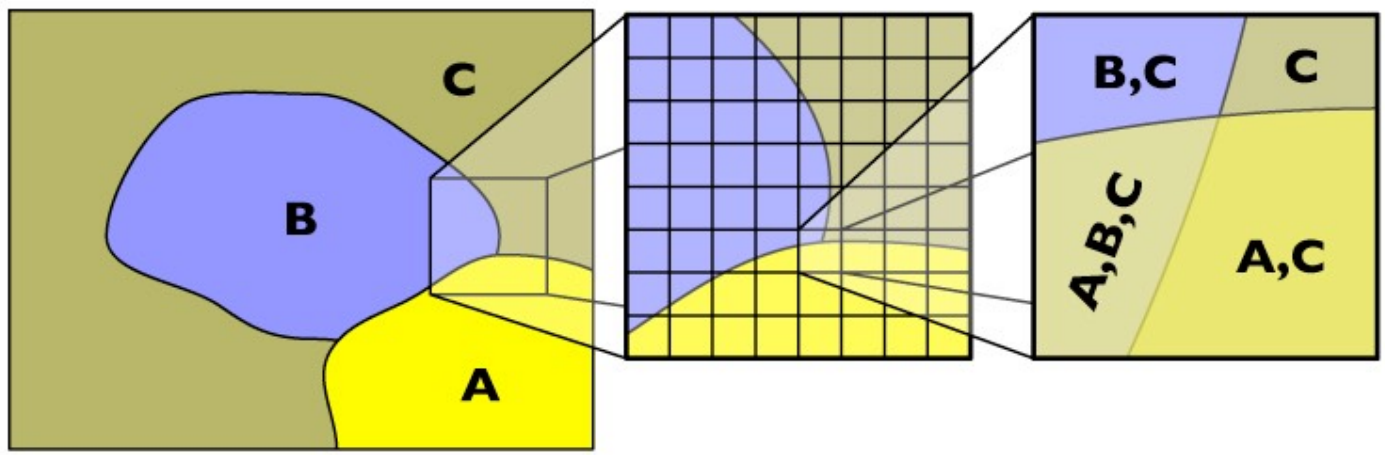
color

$$c'_A + \bar{\alpha}_A[c'_B + \bar{\alpha}_B c'_C]$$
$$c'_A + \bar{\alpha}_A c'_B + \bar{\alpha}_A\bar{\alpha}_B c'_C$$

$$[c'_A + \bar{\alpha}_A c'_B] + \bar{\alpha}_A\bar{\alpha}_B c'_C$$
$$c'_A + \bar{\alpha}_A c'_B + \bar{\alpha}_A\bar{\alpha}_B c'_C$$

alpha

Associativity!



A **over** (B **over** C)

(A **over** B) **over** C

color

$$c'_A + \bar{\alpha}_A[c'_B + \bar{\alpha}_B c'_C]$$

$$c'_A + \bar{\alpha}_A c'_B + \bar{\alpha}_A \bar{\alpha}_B c'_C$$

$$[c'_A + \bar{\alpha}_A c'_B] + \bar{\alpha}_A \bar{\alpha}_B c'_C$$

$$c'_A + \bar{\alpha}_A c'_B + \bar{\alpha}_A \bar{\alpha}_B c'_C$$

alpha

$$\alpha_A + \bar{\alpha}_A[\alpha_B + \bar{\alpha}_B \alpha_C]$$

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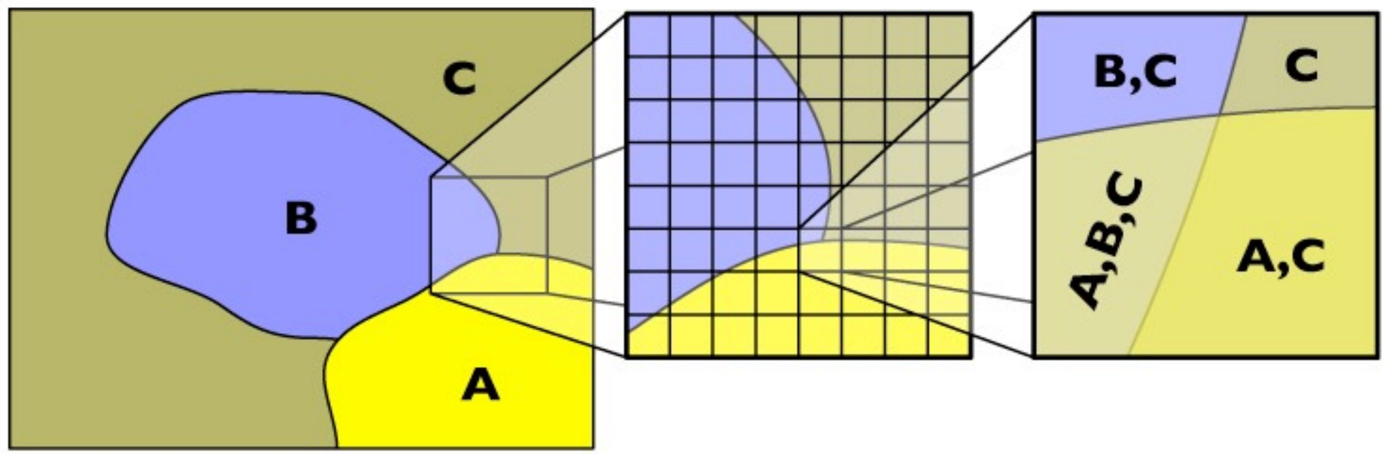
Associativity!



A **over** (B **over** C) | (A **over** B) **over** C

color

$$c'_A + \bar{\alpha}_A[c'_B + \bar{\alpha}_B c'_C] \qquad [c'_A + \bar{\alpha}_A c'_B] + \bar{\alpha}_A \bar{\alpha}_B c'_C$$

$$c'_A + \bar{\alpha}_A c'_B + \bar{\alpha}_A \bar{\alpha}_B c'_C \qquad c'_A + \bar{\alpha}_A c'_B + \bar{\alpha}_A \bar{\alpha}_B c'_C$$

alpha

$$\alpha_A + \bar{\alpha}_A[\alpha_B + \bar{\alpha}_B \alpha_C] \qquad [\alpha_A + \bar{\alpha}_A \alpha_B] + \bar{\alpha}_A \bar{\alpha}_B \alpha_C$$

$$\alpha_A + \bar{\alpha}_A \alpha_B + \bar{\alpha}_A \bar{\alpha}_B \alpha_C \qquad \alpha_A + \bar{\alpha}_A \alpha_B + \bar{\alpha}_A \bar{\alpha}_B \alpha_C$$

Compositing algebras

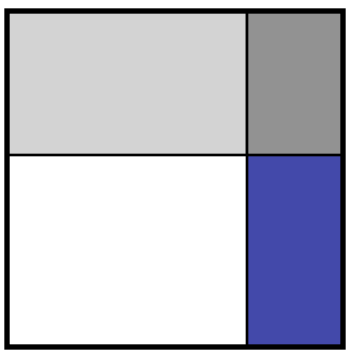
- Compositing is a multiply-and-add operation
  - "take the color you have, scale it by 1-alpha, add some more"
- This is a 1D affine transformation

$$\begin{bmatrix} \bar{\alpha}_A & c'_A \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{\alpha}_B & c'_B \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \bar{\alpha}_A \bar{\alpha}_B & c'_A + \bar{\alpha}_A c'_B \\ 0 & 1 \end{bmatrix}$$

- Once we write it like this, associativity is automatically inherited from matrix multiplication!
  - and we realize we haven't invented any new math after all.

Independent coverage assumption

- Why is it reasonable to blend α like a color?
- Simplifying assumption: covered areas are independent
  - that is, uncorrelated in the statistical sense

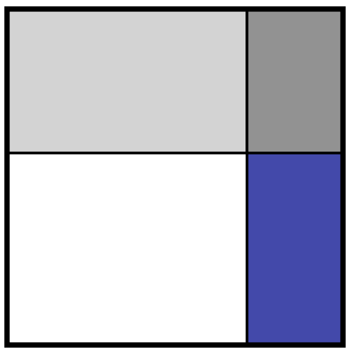


description	*area*
$\bar{A}\cap\bar{B}$	$(1-\alpha_A)(1-\alpha_B)$
$A\cap\bar{B}$	$\alpha_A(1-\alpha_B)$
$\bar{A}\cap B$	$(1-\alpha_A)\alpha_B$
$A\cap B$	$\alpha_A\alpha_B$

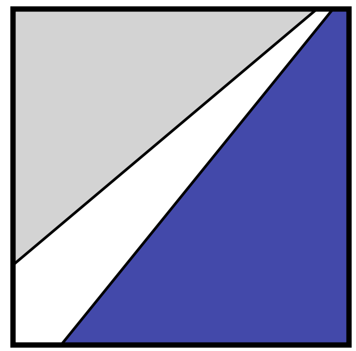
[Porter & Duff 84]

Independent coverage assumption

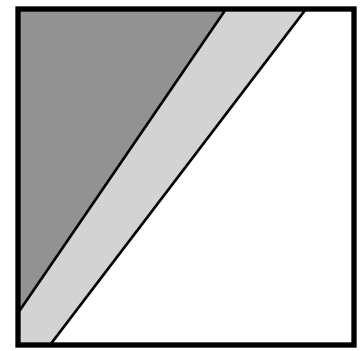
- Holds in most but not all cases



this



not this



or this

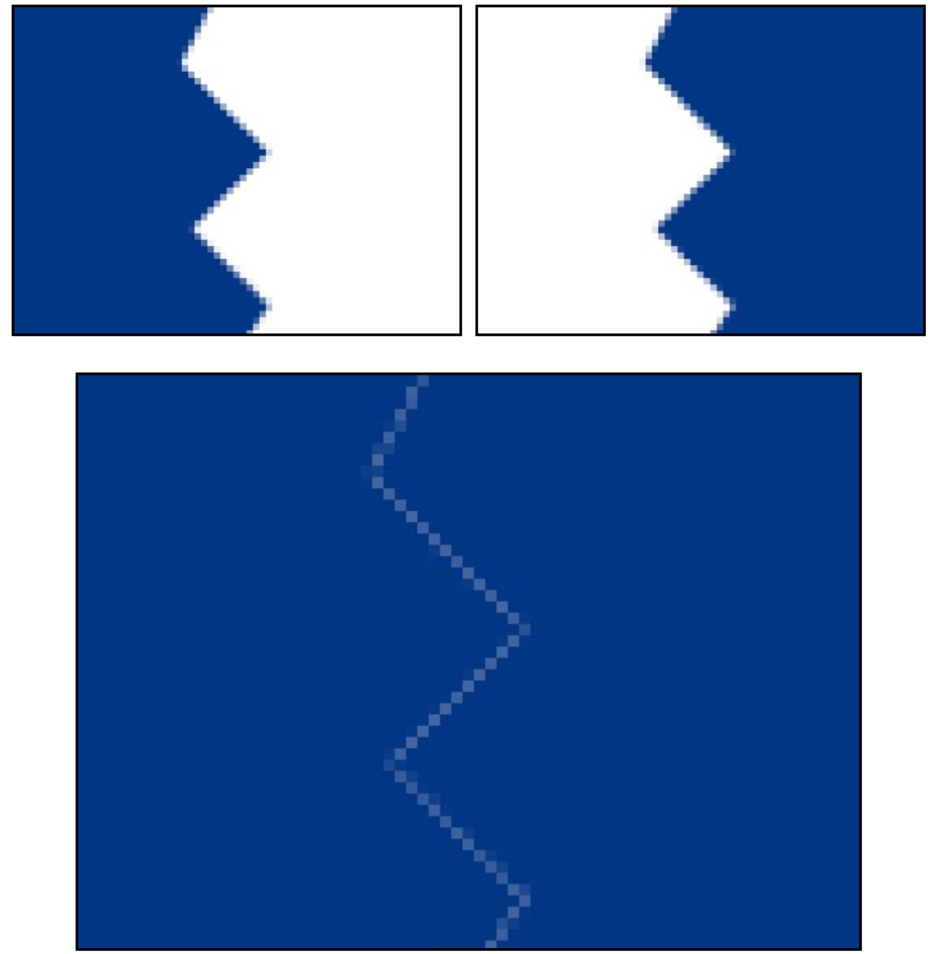
- This will cause artifacts
  - but we'll carry on anyway because it is simple and usually works…

Alpha compositing—failures

[Cornell PCG]

[Chuang et al. / Corel]

positive correlation:
too much foreground

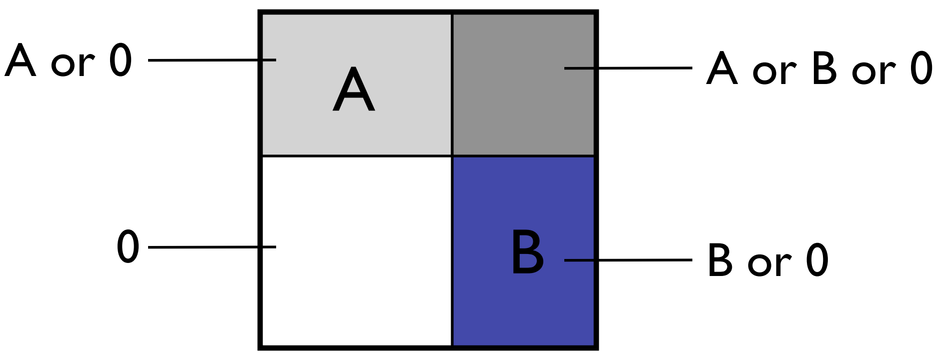


negative correlation:
too little foreground

Other compositing operations

- Generalized form of compositing equation:

$$\alpha_E = A \ \mathbf{op} \ B$$
$$c'_E = F_A c'_A + F_B c'_B$$



1 x 2 x 3 x 2 = 12 reasonable choices

operation	quadruple	diagram	F_A	F_B
clear	(0,0,0,0)		0	0
A	(0,A,0,A)		1	0
B	(0,0,B,B)		0	1
A **over** *B*	(0,A,B,A)		1	$1-\alpha_A$
B **over** *A*	(0,A,B,B)		$1-\alpha_B$	1
A **in** *B*	(0,0,0,A)		α_B	0
B **in** *A*	(0,0,0,B)		0	α_A
A **out** *B*	(0,A,0,0)		$1-\alpha_B$	0
B **out** *A*	(0,0,B,0)		0	$1-\alpha_A$
A **atop** *B*	(0,0,B,A)		α_B	$1-\alpha_A$
B **atop** *A*	(0,A,0,B)		$1-\alpha_B$	α_A
A **xor** *B*	(0,A,B,0)		$1-\alpha_B$	$1-\alpha_A$

[Porter & Duff 84]