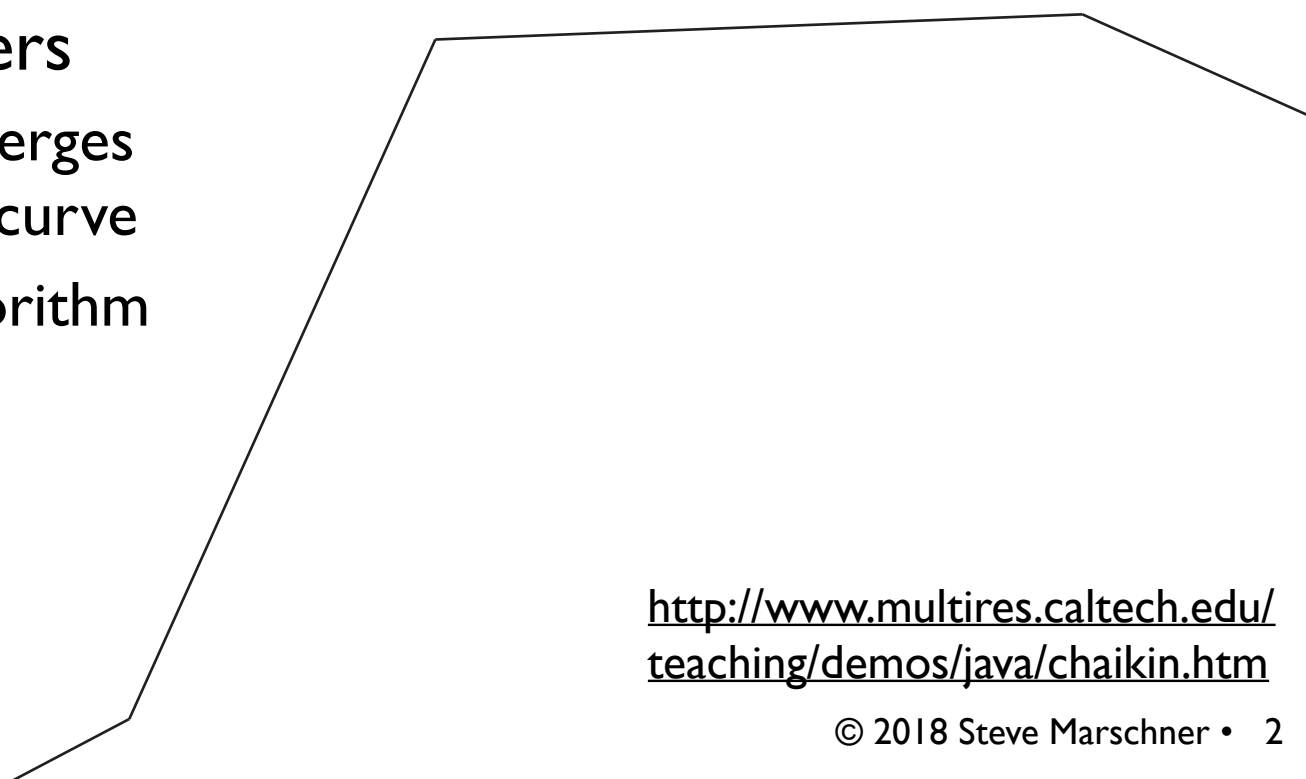# Subdivision overview

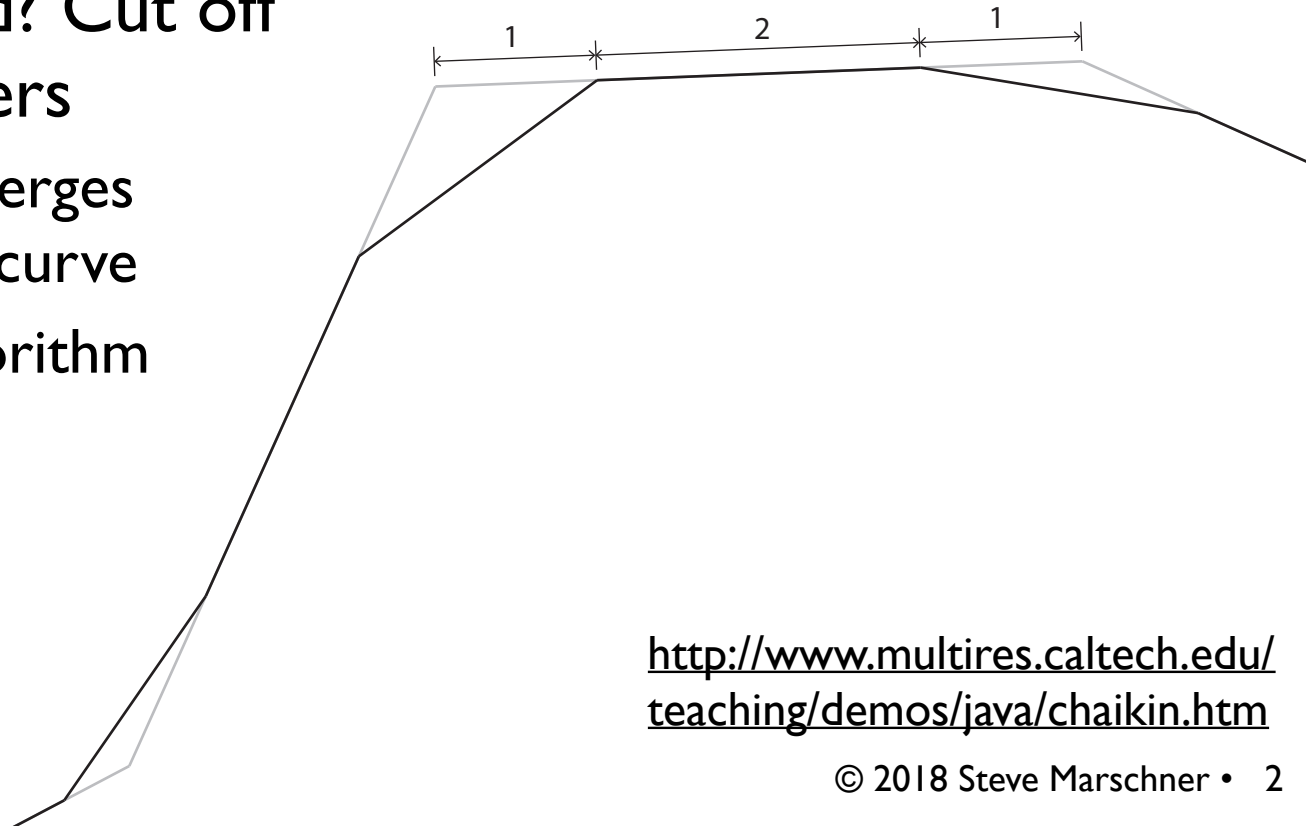## CS4620 Lecture 16

Introduction: corner cutting

- Piecewise linear curve too jagged for you? Lop off the corners!
  - results in a curve with twice as many corners
- Still too jagged? Cut off the new corners
  - process converges to a smooth curve
  - Chaikin's algorithm

http://www.multires.caltech.edu/teaching/demos/java/chaikin.htm
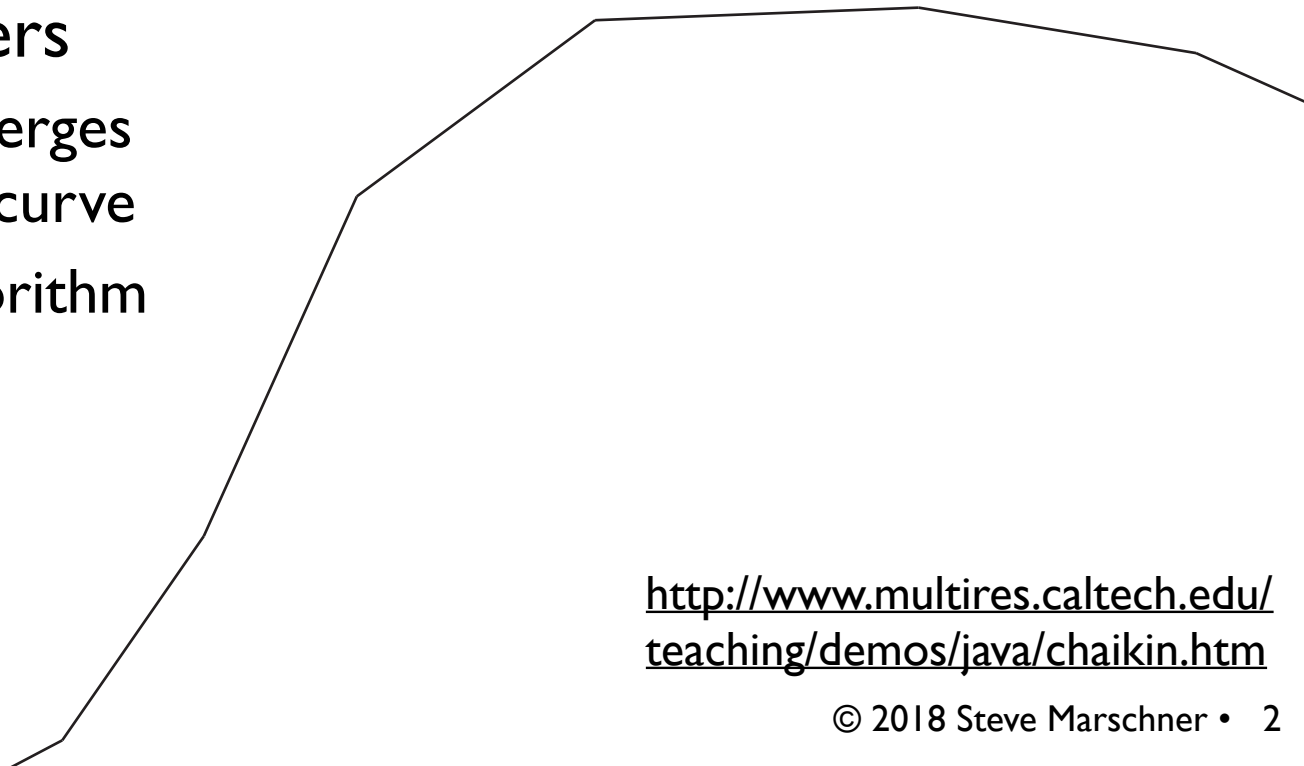
Introduction: corner cutting

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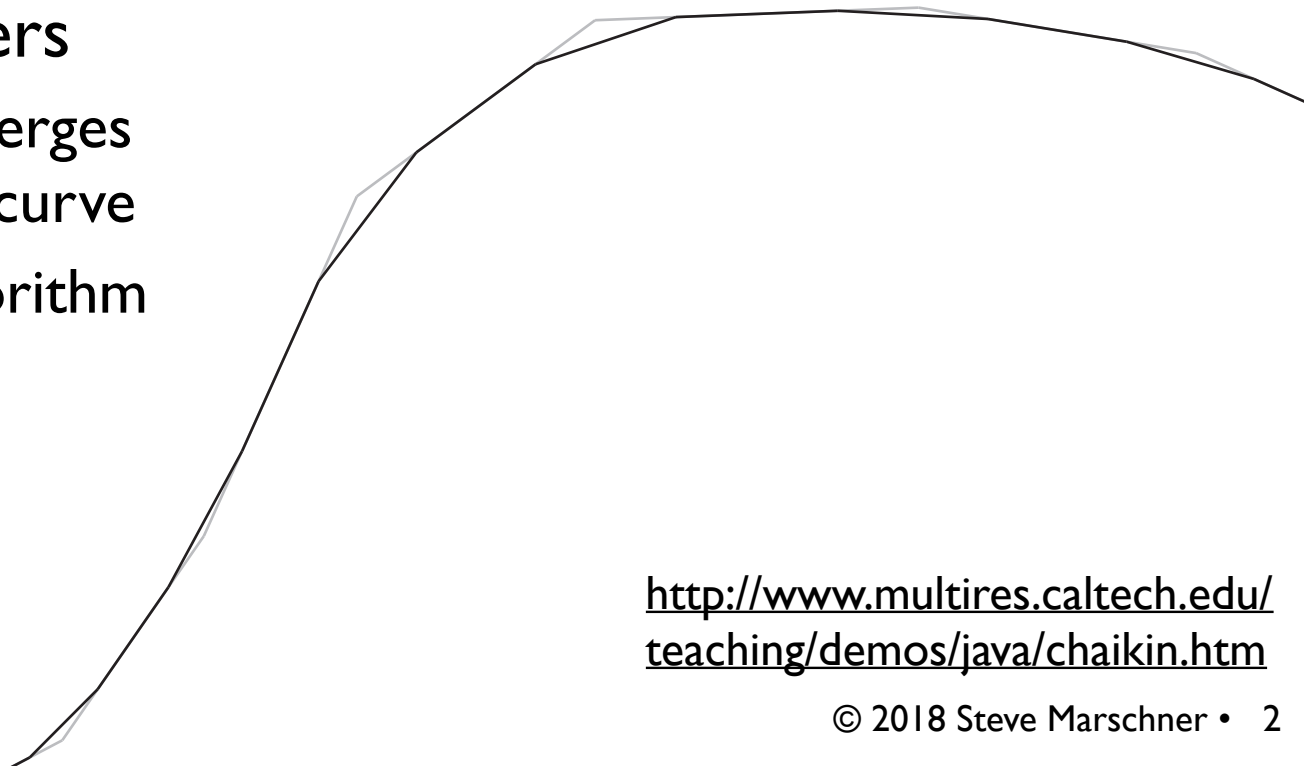# Introduction: corner cutting

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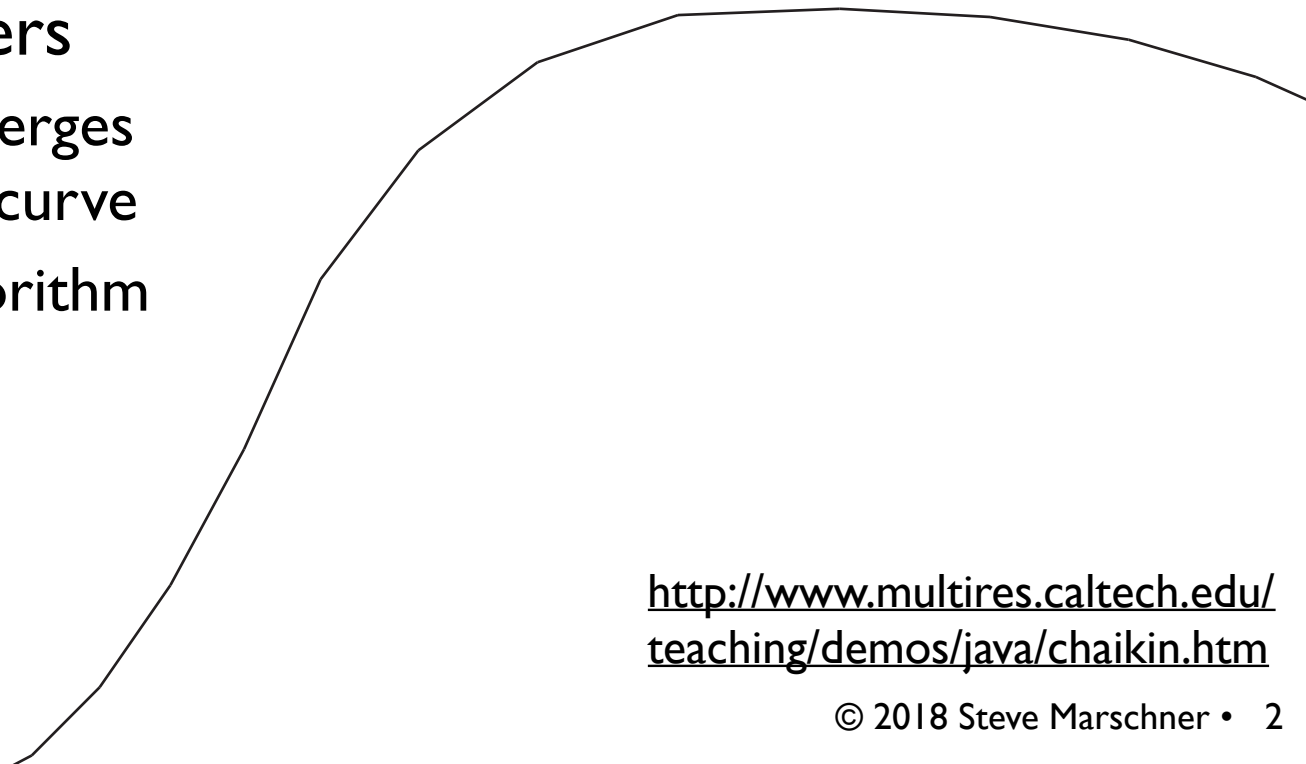
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Corner cutting in equations

- New points are linear combinations of old ones
- Different treatment for odd-numbered and even-numbered points.
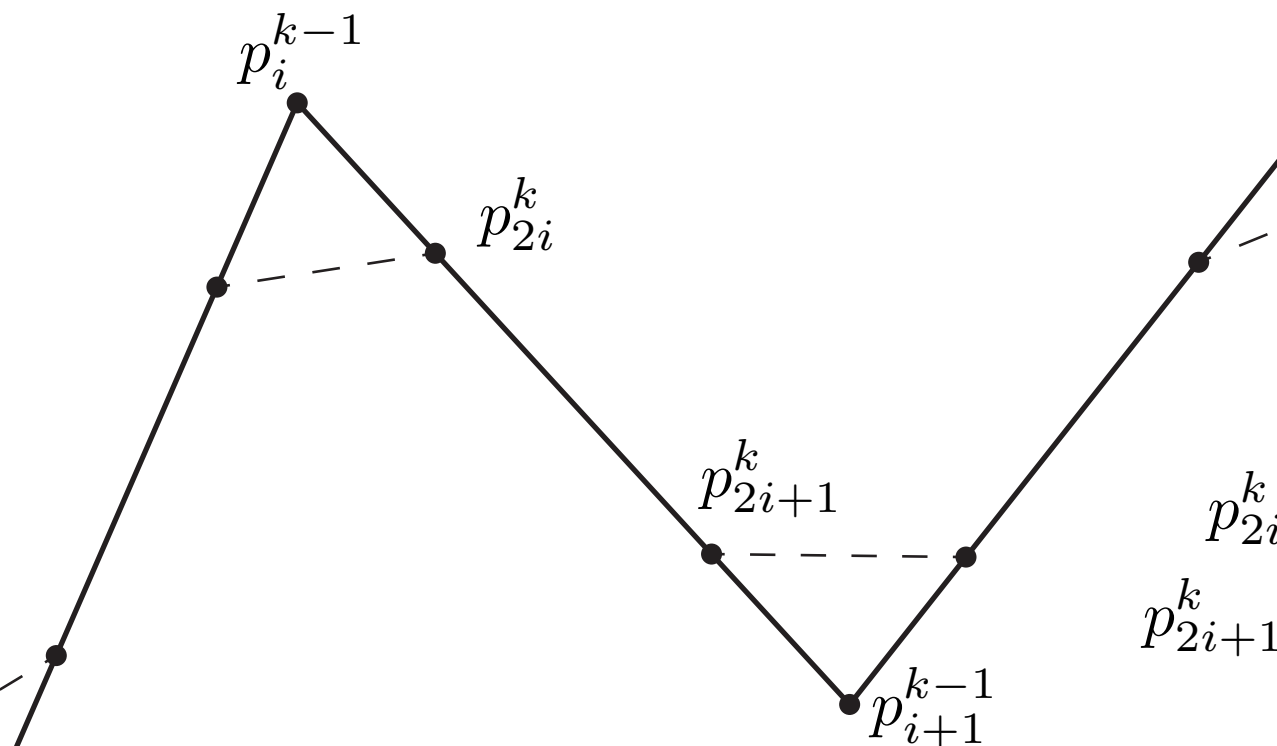
$$p_{2i}^k = (3p_i^{k-1} + p_{i+1}^{k-1})/4$$

$$p_{2i+1}^k = (p_i^{k-1} + 3p_{i+1}^{k-1})/4$$

Spline-splitting math for B-splines

- Can use spline-matrix math from previous lecture to split a B-spline segment in two at $s = t = 0.5$.

- Result is especially nice because the rules for adjacent segments agree (not true for all splines).

$$S_L = \begin{bmatrix} s^3 & & & \\ & s^2 & & \\ & & s & \\ & & & 1 \end{bmatrix}$$

$$P_L = M^{-1} S_L M P$$
$$P_R = M^{-1} S_R M P$$

$$P_L = \begin{bmatrix} 4 & 4 & 0 & 0 \\ 1 & 6 & 1 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 1 & 6 & 1 \end{bmatrix}$$

$$S_R = \begin{bmatrix} s^3 & & & \\ 3s^2(1-s) & s^2 & & \\ 3s(1-s)^2 & 2s(1-s) & s & \\ (1-s)^3 & (1-s)^2 & (1-s) & 1 \end{bmatrix}$$

$$P_R = \begin{bmatrix} 1 & 6 & 1 & 0 \\ 0 & 4 & 4 & 0 \\ 0 & 1 & 6 & 1 \\ 0 & 0 & 4 & 4 \end{bmatrix}$$

Subdivision for B-splines

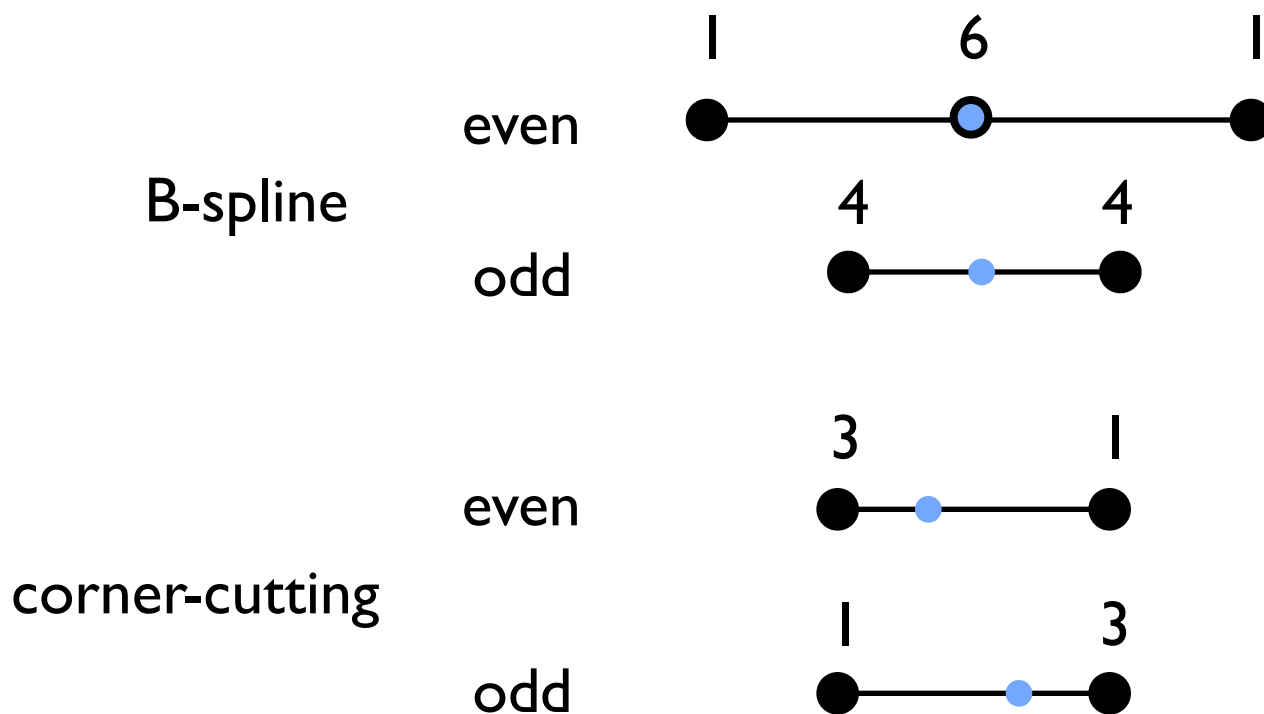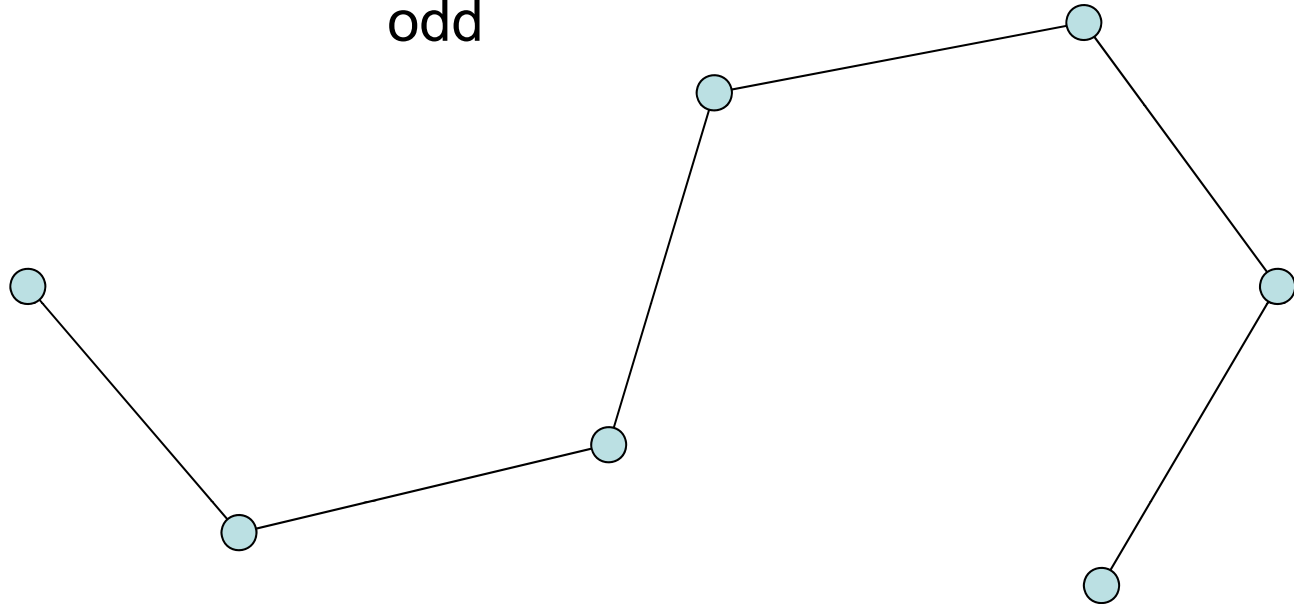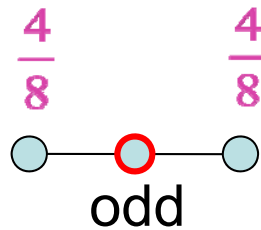- Control vertices of refined spline are linear combinations of the c.v.s of the coarse spline
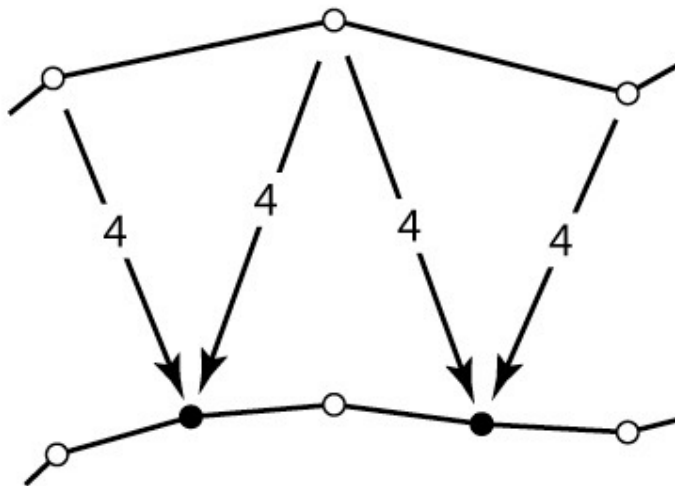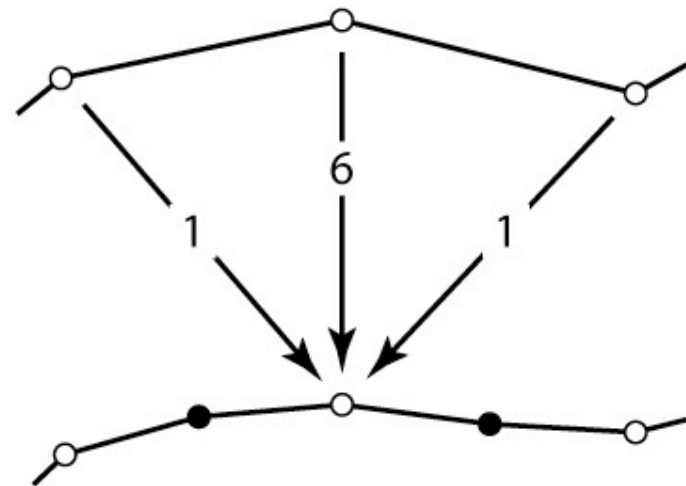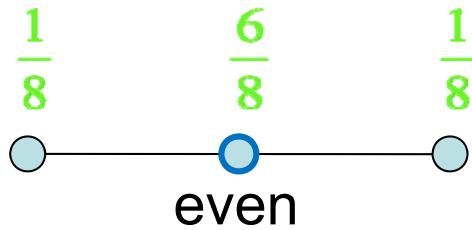

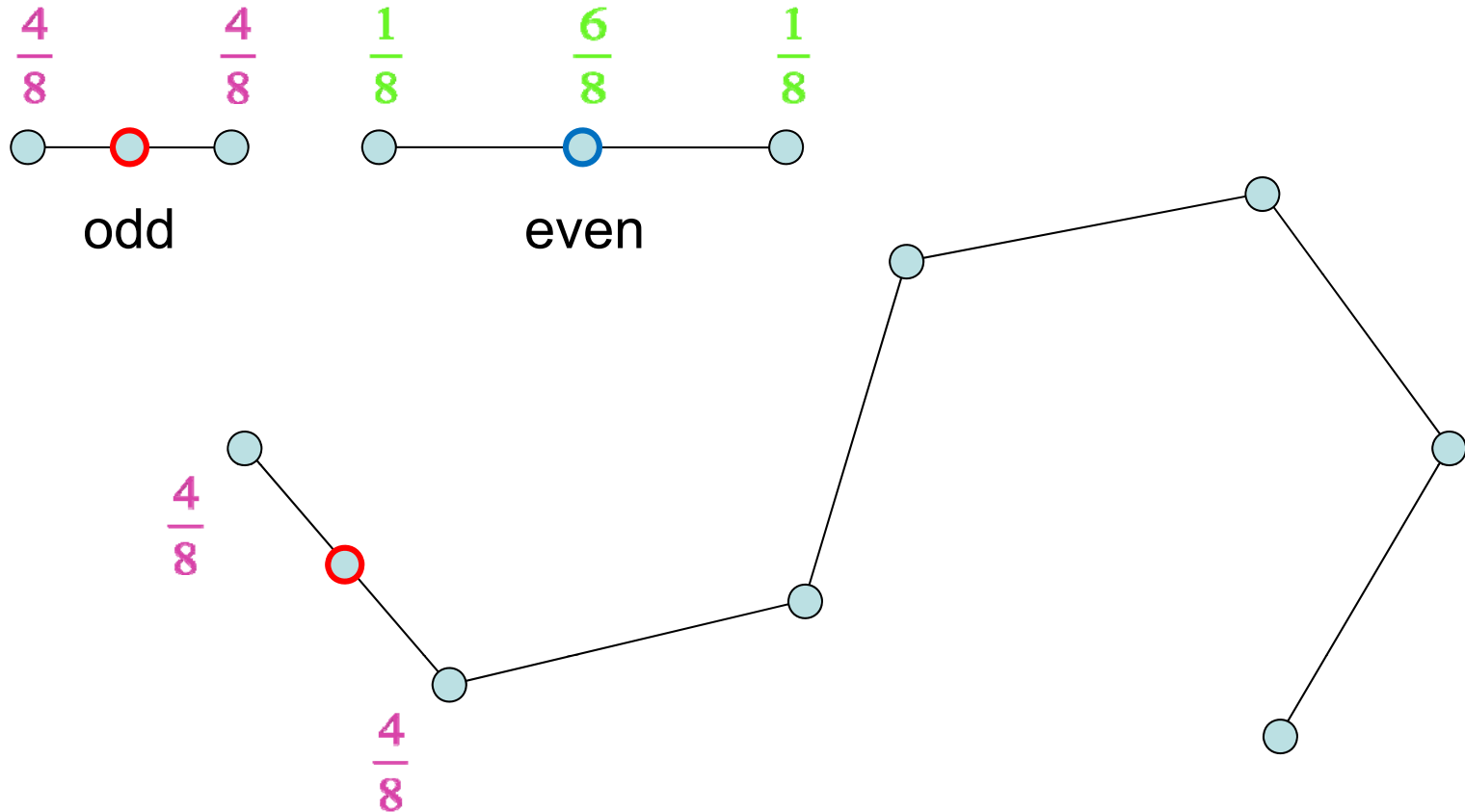**ODD**   **EVEN**

Drawing a picture of the rule

- Conventionally illustrate subdivision rules as a "mask" that you match against the neighborhood
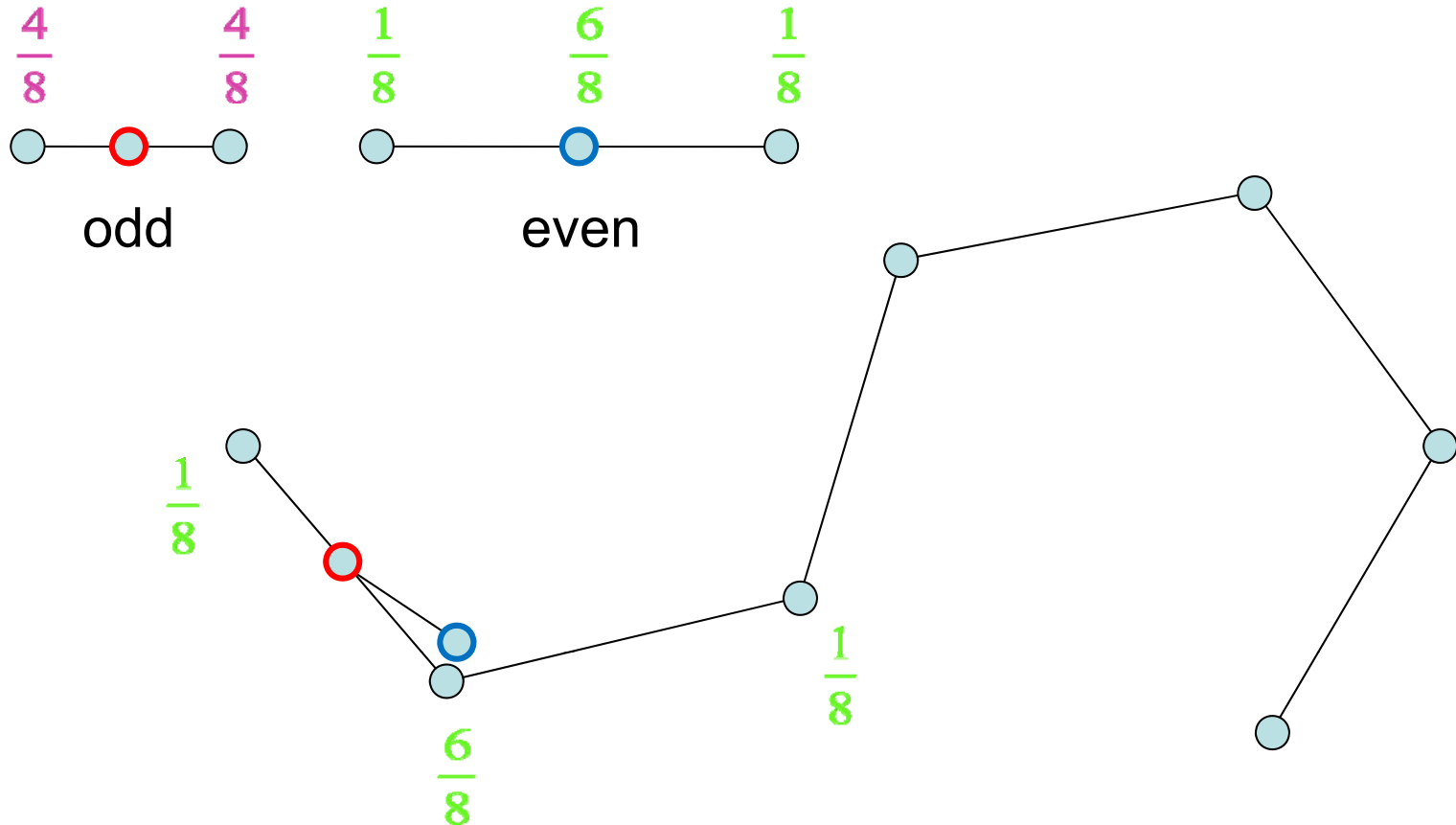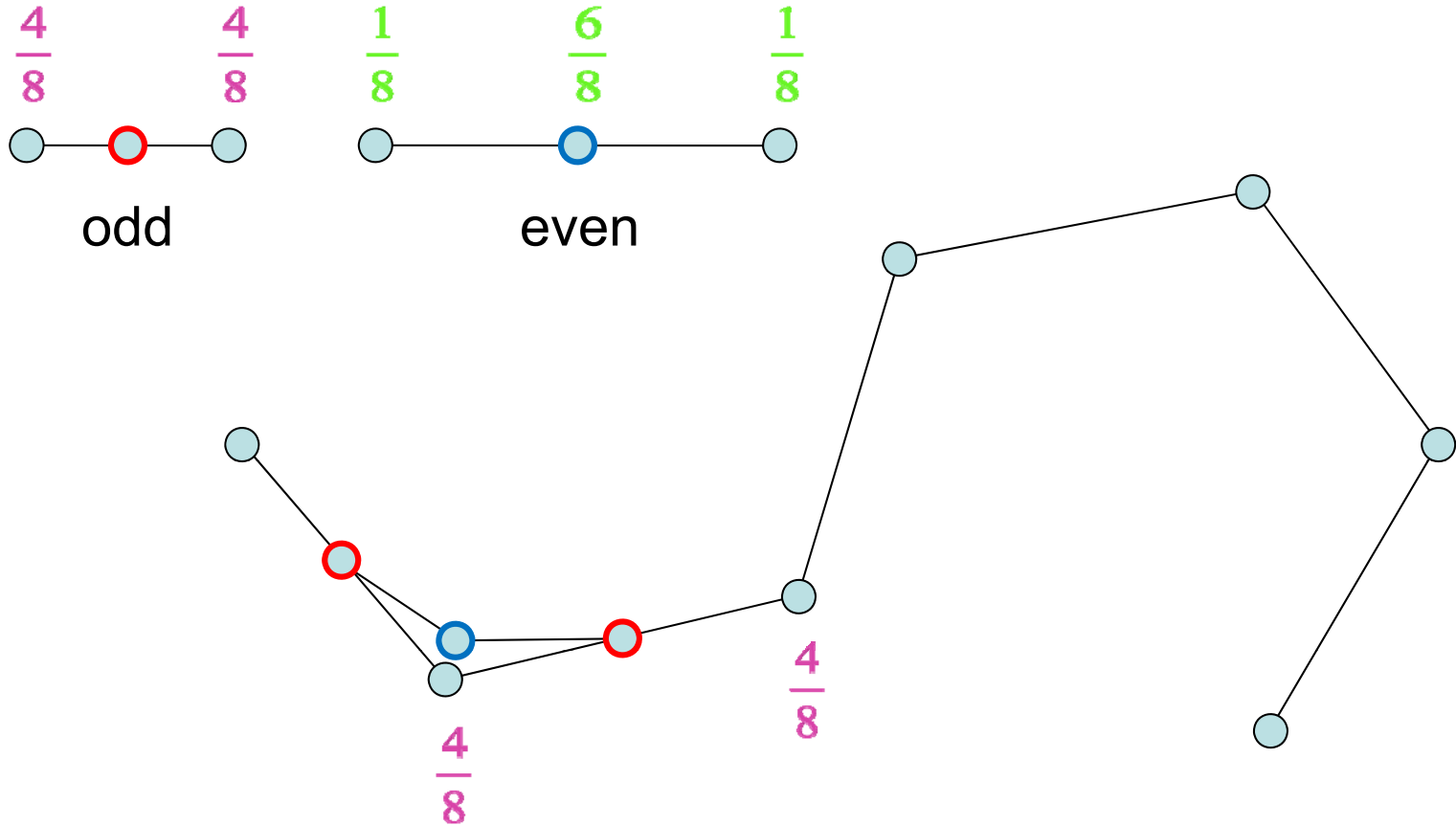 - often implied denominator = sum of weights

[Stanford CS468 Fall 2010 slides]
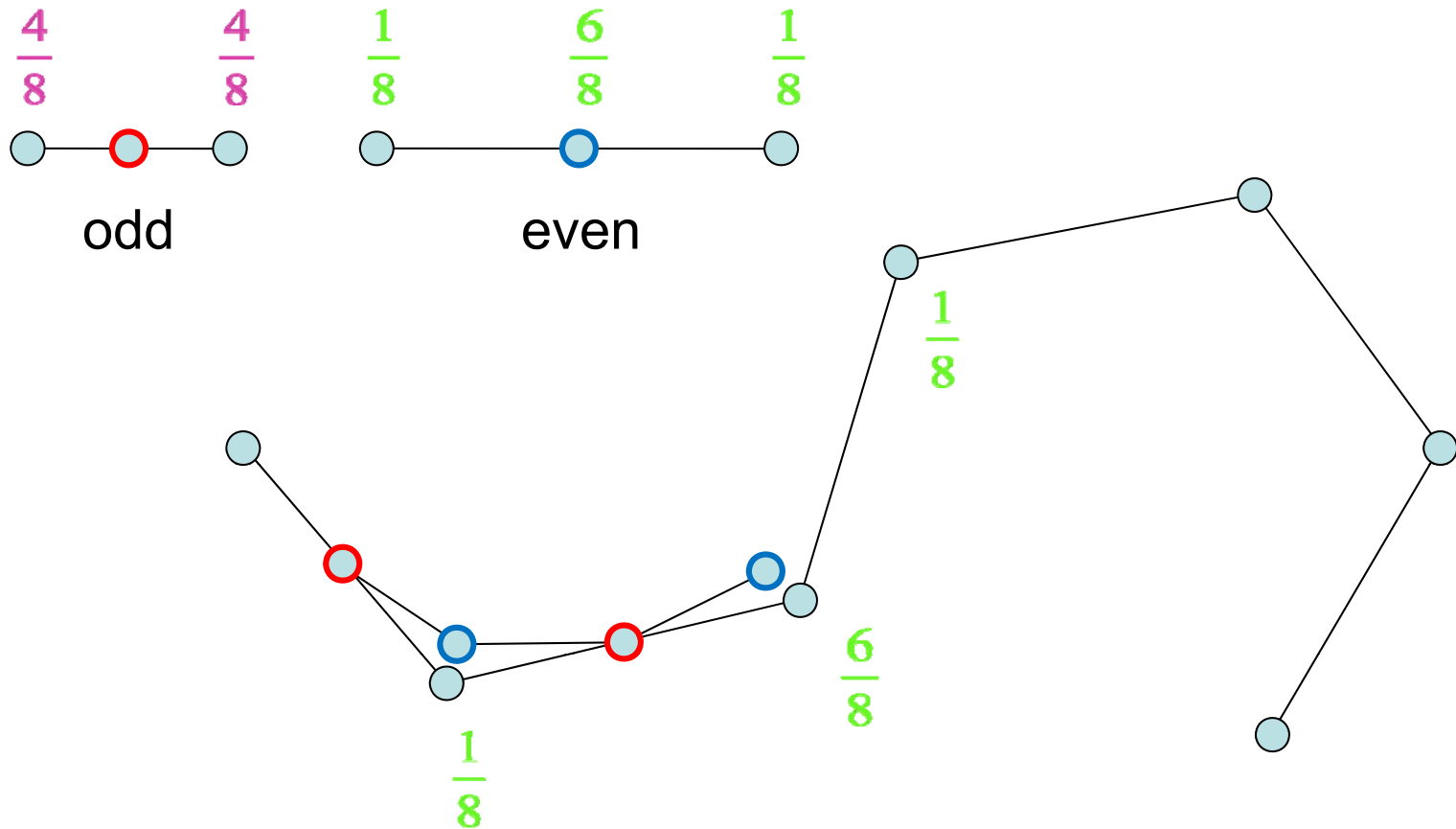
Cubic B-Spline

$$\frac{1}{8} \quad \frac{6}{8} \quad \frac{1}{8} \qquad \frac{4}{8} \quad \frac{4}{8}$$

even

odd

Cubic B-Spline

$$\frac{4}{8} \qquad \frac{4}{8}$$

$$\frac{1}{8} \qquad \frac{6}{8} \qquad \frac{1}{8}$$

odd   even

$$\frac{4}{8}$$

$$\frac{4}{8}$$

Cubic B-Spline

$$\frac{4}{8} \quad \frac{4}{8}$$

$$\frac{1}{8} \quad \frac{6}{8} \quad \frac{1}{8}$$

odd   even

$$\frac{1}{8} \quad \frac{6}{8} \quad \frac{1}{8}$$

Cubic B-Spline

$\frac{4}{8}$ $\frac{4}{8}$ $\frac{1}{8}$ $\frac{6}{8}$ $\frac{1}{8}$

odd   even

$\frac{4}{8}$

$\frac{4}{8}$

$\frac{4}{8}$

Cubic B-Spline

$$\frac{4}{8} \qquad \frac{4}{8}$$

odd

$$\frac{1}{8} \qquad \frac{6}{8} \qquad \frac{1}{8}$$

even

$$\frac{1}{8}$$

$$\frac{6}{8}$$

$$\frac{1}{8}$$

Cubic B-Spline

$$\frac{4}{8} \qquad \frac{4}{8}$$

$$\frac{1}{8} \qquad \frac{6}{8} \qquad \frac{1}{8}$$

odd   even

$$\frac{4}{8}$$

$$\frac{4}{8}$$

Cubic B-Spline

$$\frac{4}{8} \qquad \frac{4}{8}$$

odd

$$\frac{1}{8} \qquad \frac{6}{8} \qquad \frac{1}{8}$$

even

$$\frac{6}{8}$$

$$\frac{1}{8}$$

$$\frac{1}{8}$$

Cubic B-Spline

$$\frac{4}{8} \qquad \frac{4}{8}$$

$$\frac{1}{8} \qquad \frac{6}{8} \qquad \frac{1}{8}$$

odd   even

$$\frac{4}{8}$$

$$\frac{4}{8}$$

Cubic B-Spline

$$\frac{4}{8} \qquad \frac{4}{8}$$

$$\frac{1}{8} \qquad \frac{6}{8} \qquad \frac{1}{8}$$

odd

even

$$\frac{1}{8} \qquad \frac{6}{8}$$

$$\frac{1}{8}$$

Cubic B-Spline

$$\frac{4}{8} \qquad \frac{4}{8}$$

$$\frac{1}{8} \qquad \frac{6}{8} \qquad \frac{1}{8}$$

odd   even

$$\frac{4}{8}$$

$$\frac{4}{8}$$

Cubic B-Spline

$$\frac{4}{8} \qquad \frac{4}{8}$$

$$\frac{1}{8} \qquad \frac{6}{8} \qquad \frac{1}{8}$$

odd   even

$$\frac{1}{8}$$

$$\frac{6}{8}$$

$$\frac{1}{8}$$

Cubic B-Spline

$$\frac{4}{8} \quad \frac{4}{8}$$

$$\frac{1}{8} \quad \frac{6}{8} \quad \frac{1}{8}$$

odd   even

$$\frac{4}{8}$$

$$\frac{4}{8}$$

Cubic B-Spline

$$\frac{4}{8} \qquad \frac{4}{8}$$

$$\frac{1}{8} \qquad \frac{6}{8} \qquad \frac{1}{8}$$

odd   even
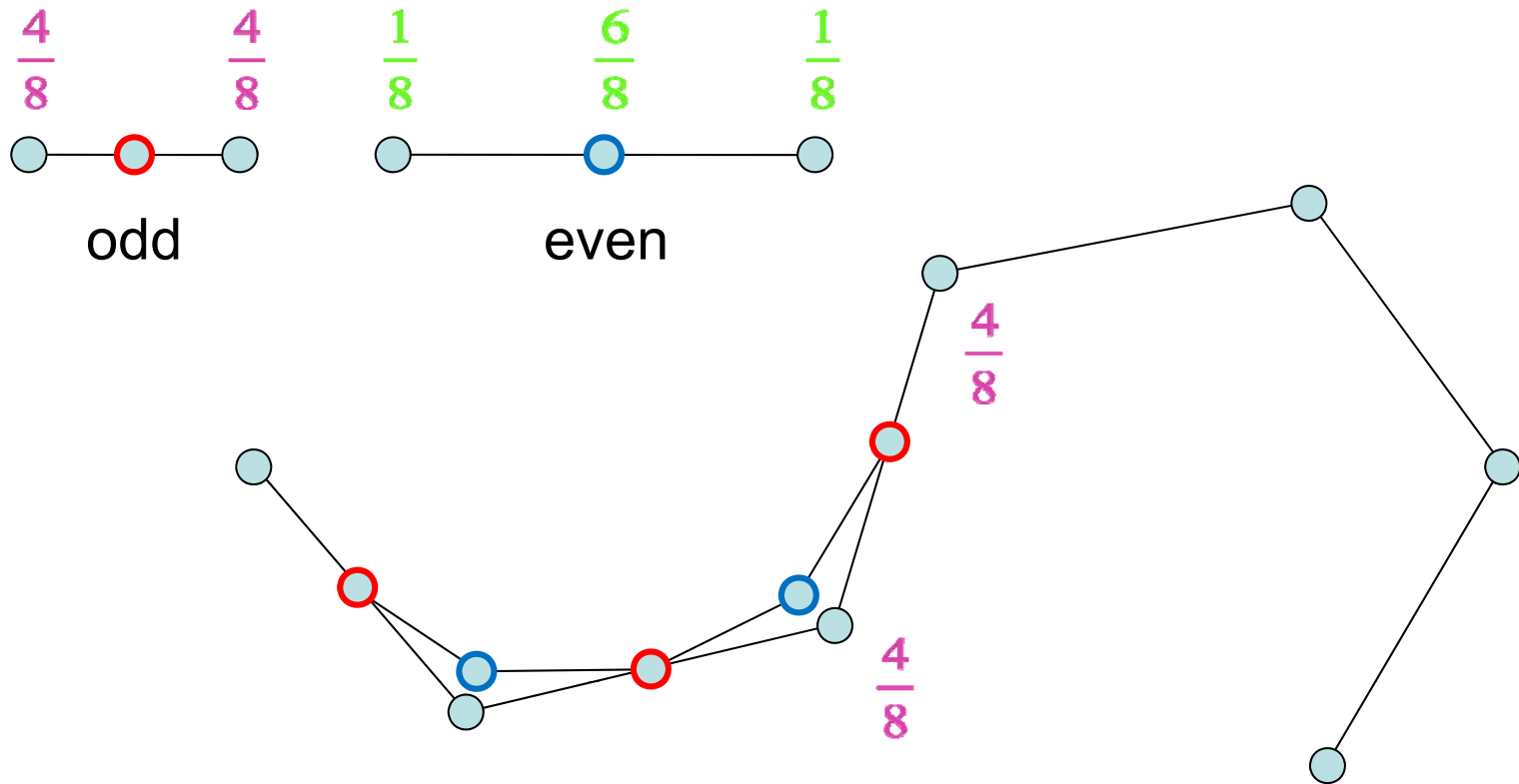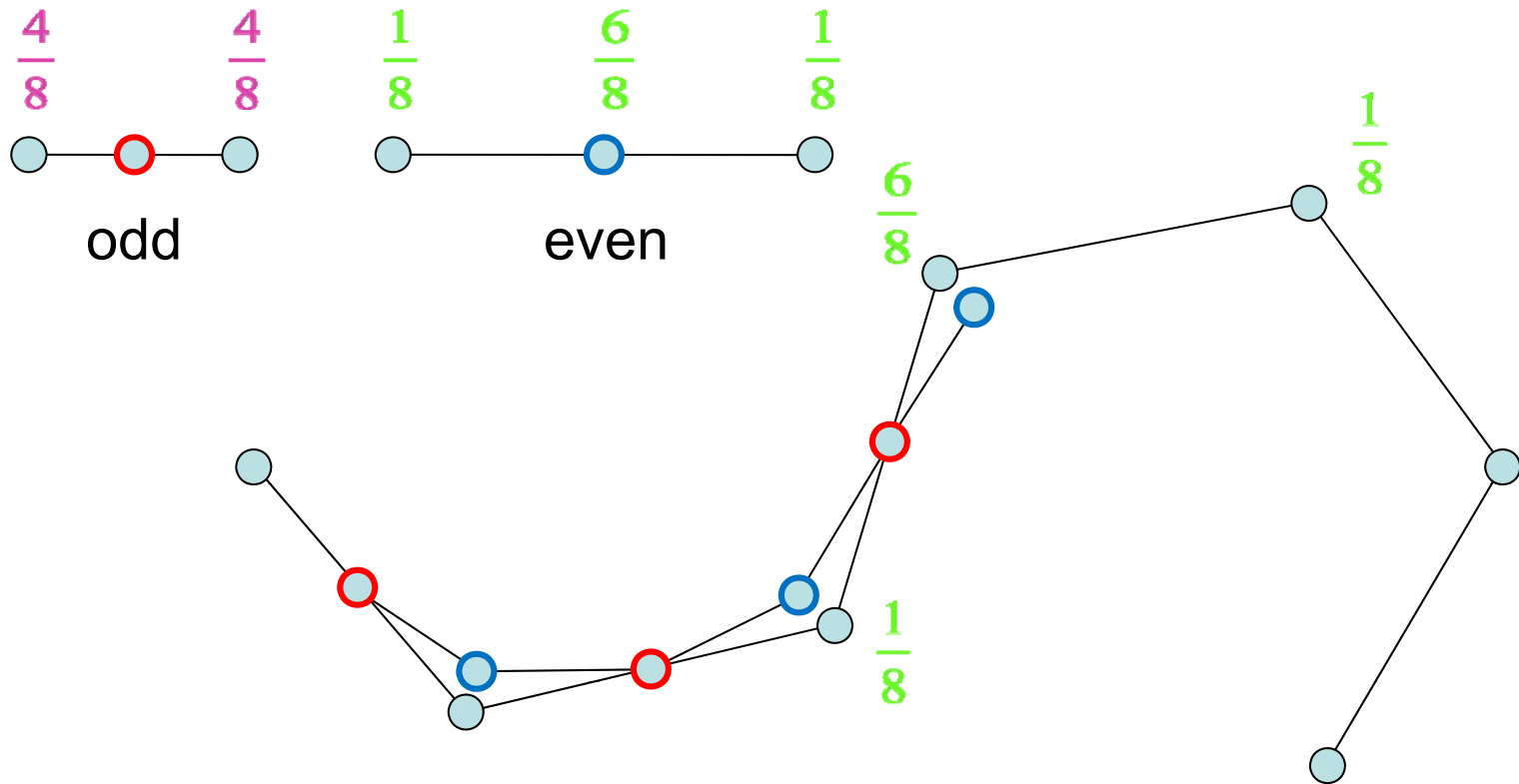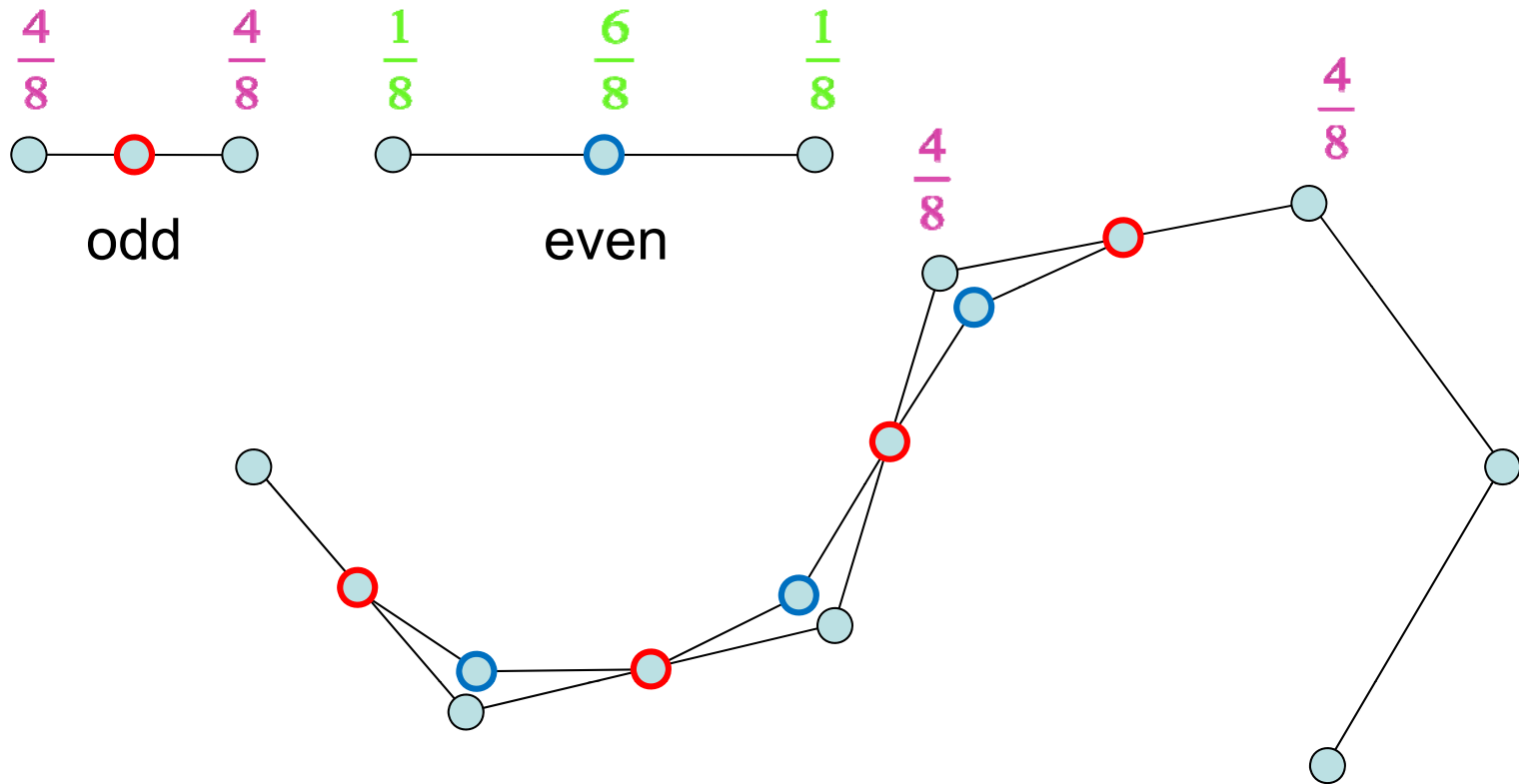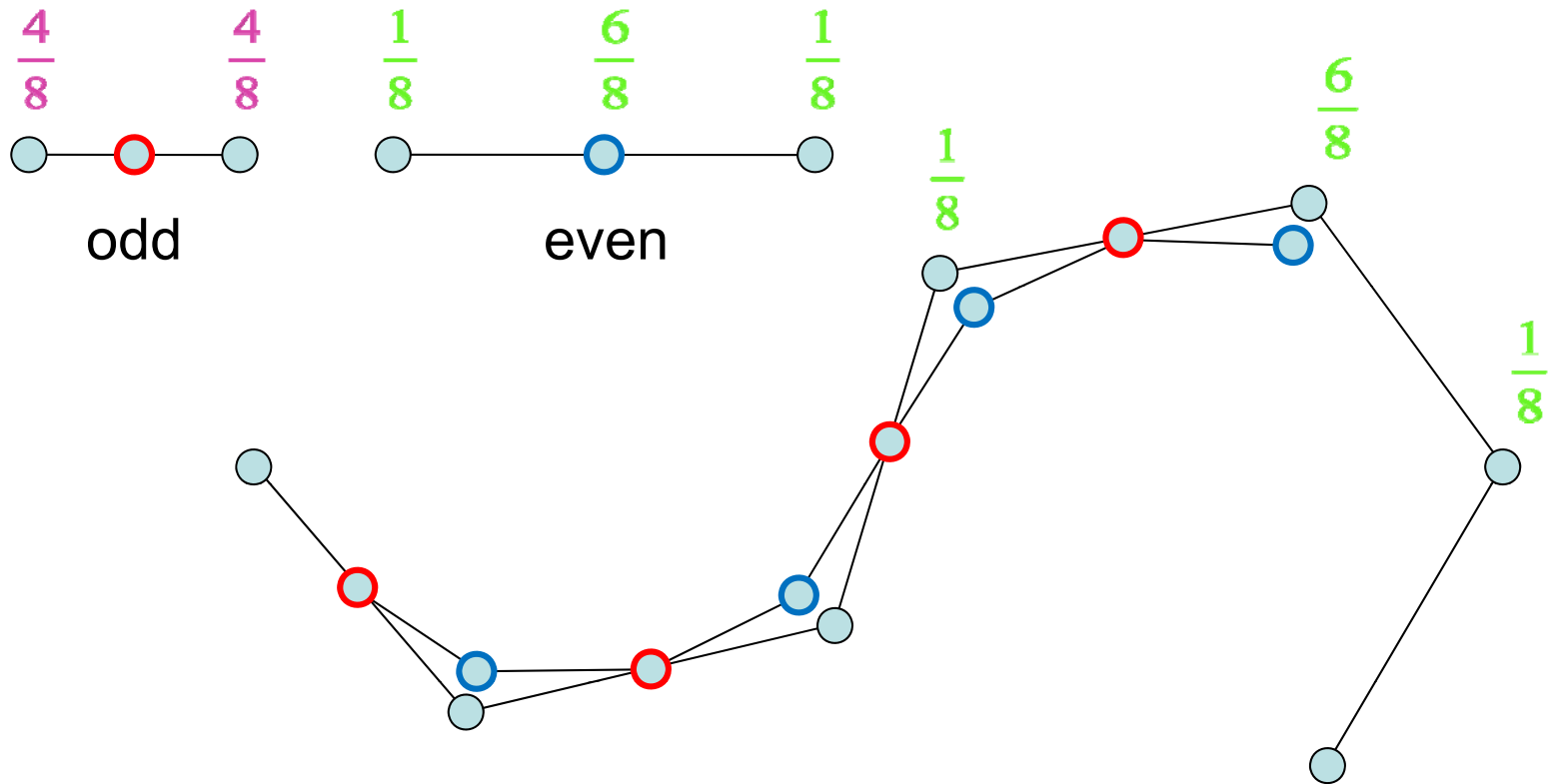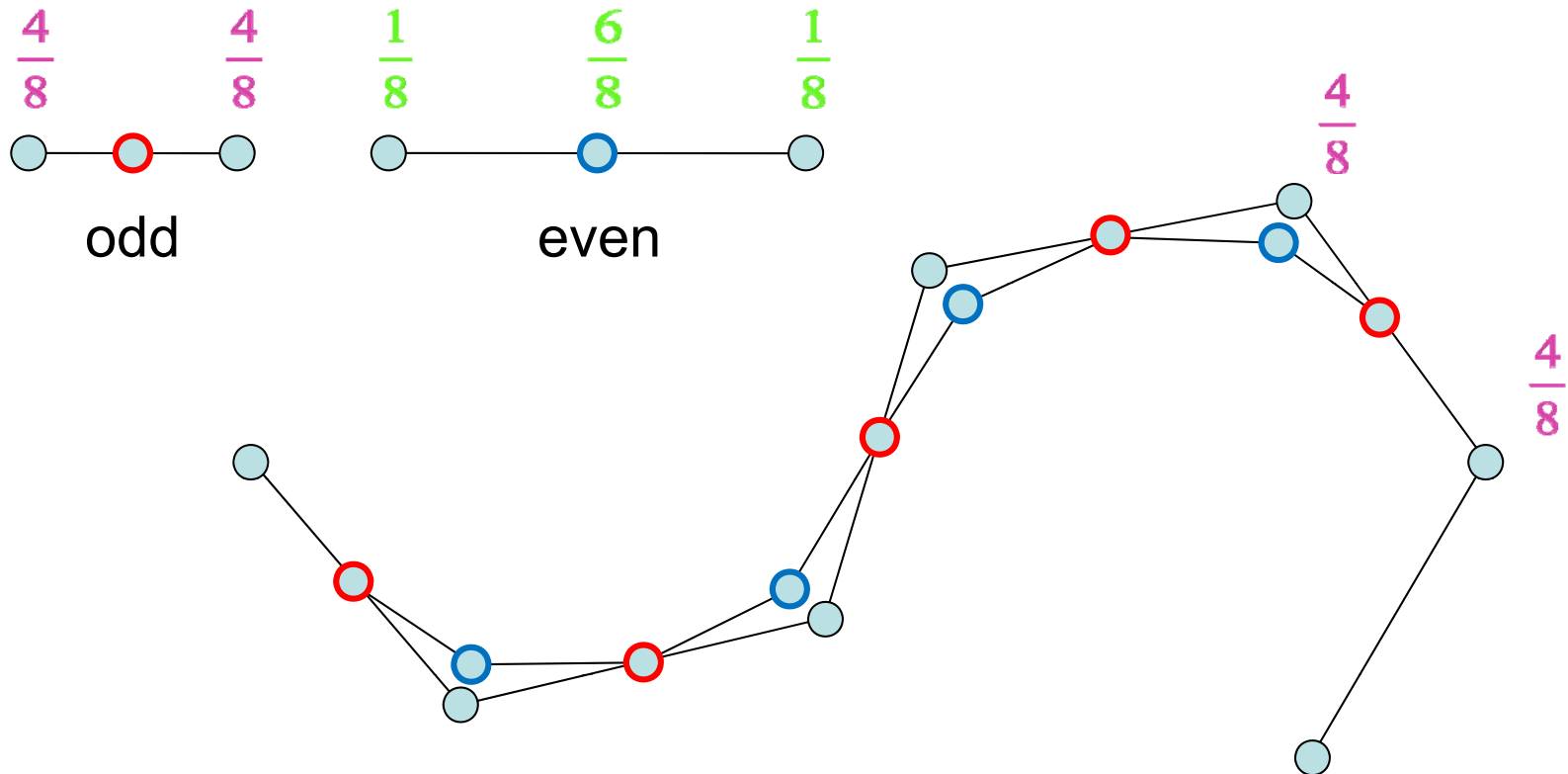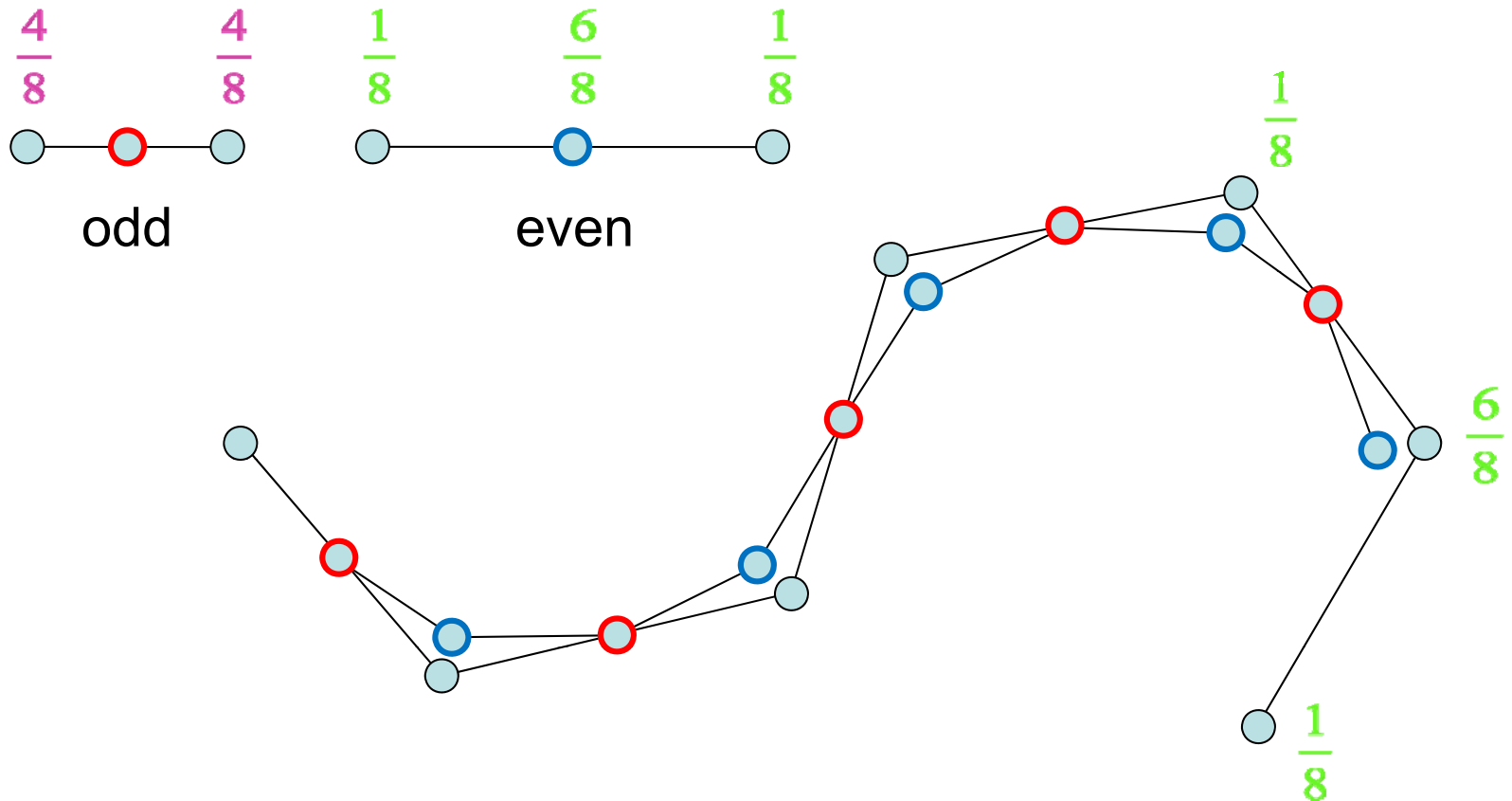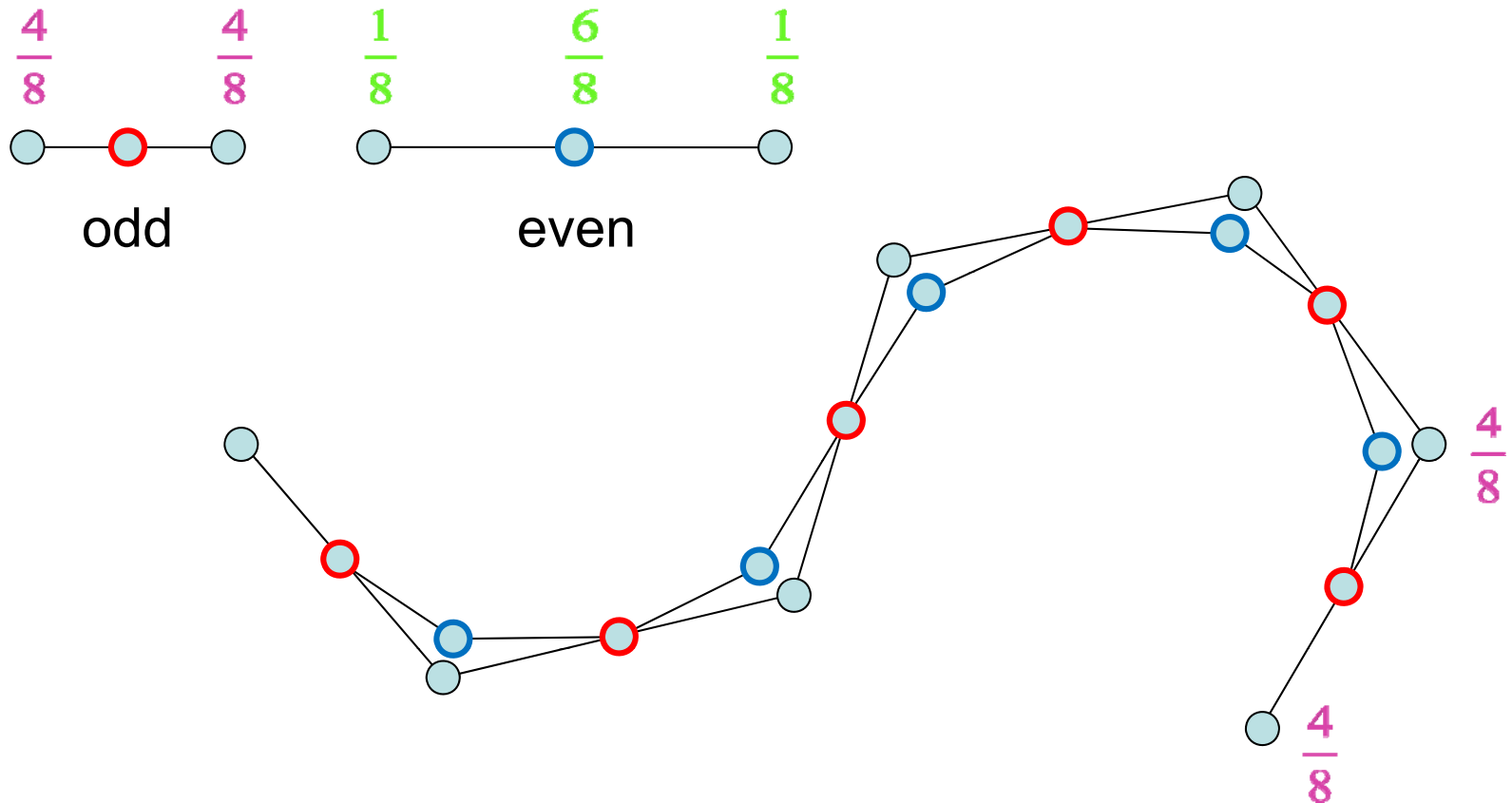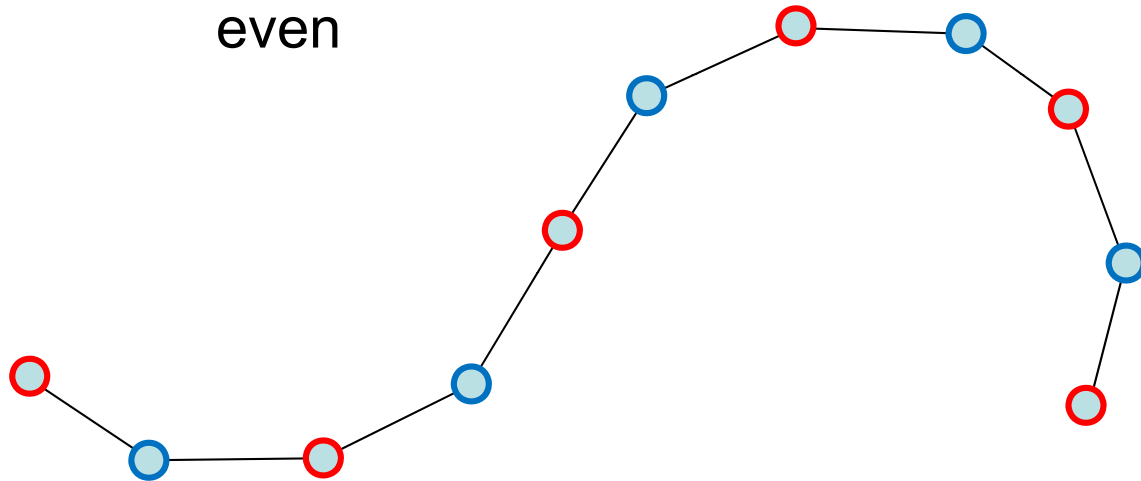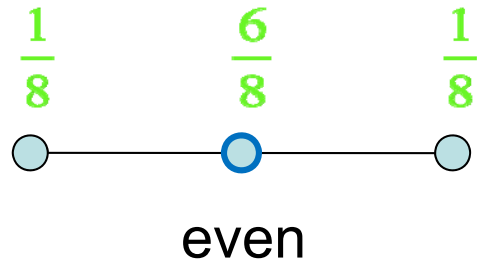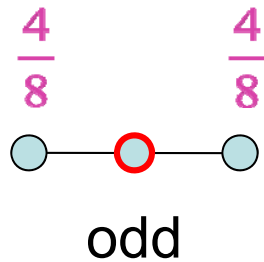
Subdivision curves

- Key idea: let go of the polynomials as the definition of the curve, and let the refinement rule define the curve

- Curve is defined as the *limit of a refinement process*
  - properties of curve depend on the rules
  - some rules make polynomial curves, some don't
  - complexity shifts from implementations to proofs
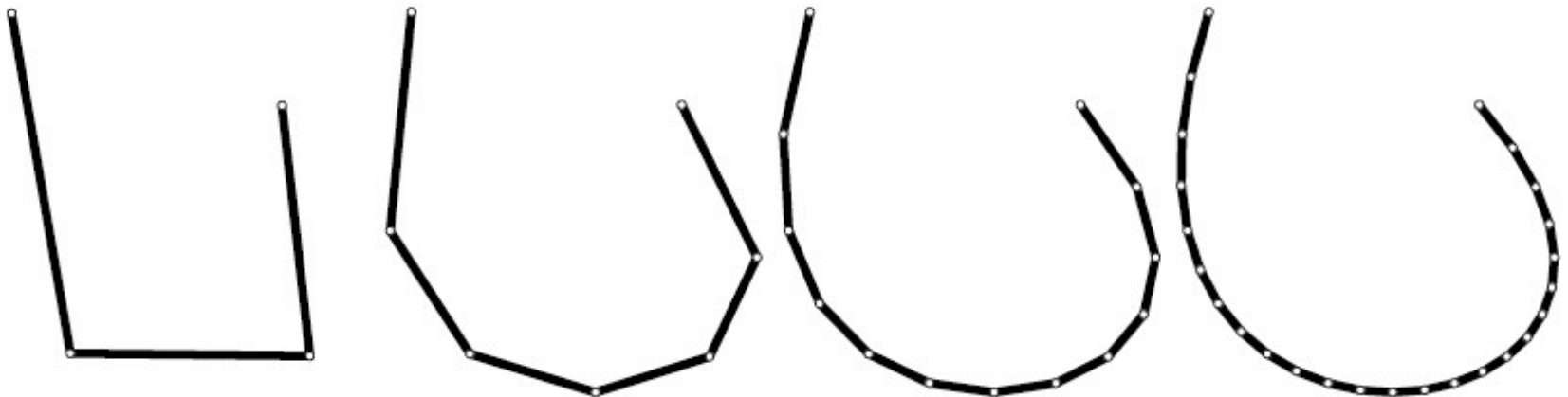
Playing with the rules

- Once a curve is *defined* using subdivision we can customize its behavior by making exceptions to the rules.

- Example: handle endpoints by simply using the mask [1] at that point.

- Resulting curve *is* a uniform B-spline in the middle, but near the exceptional points it is something different.
    - it might not be a polynomial
    - but it is still linear, still has basis functions
    - the three coordinates of a surface point are still separate

ubdivision defines a smooth curve or surface as
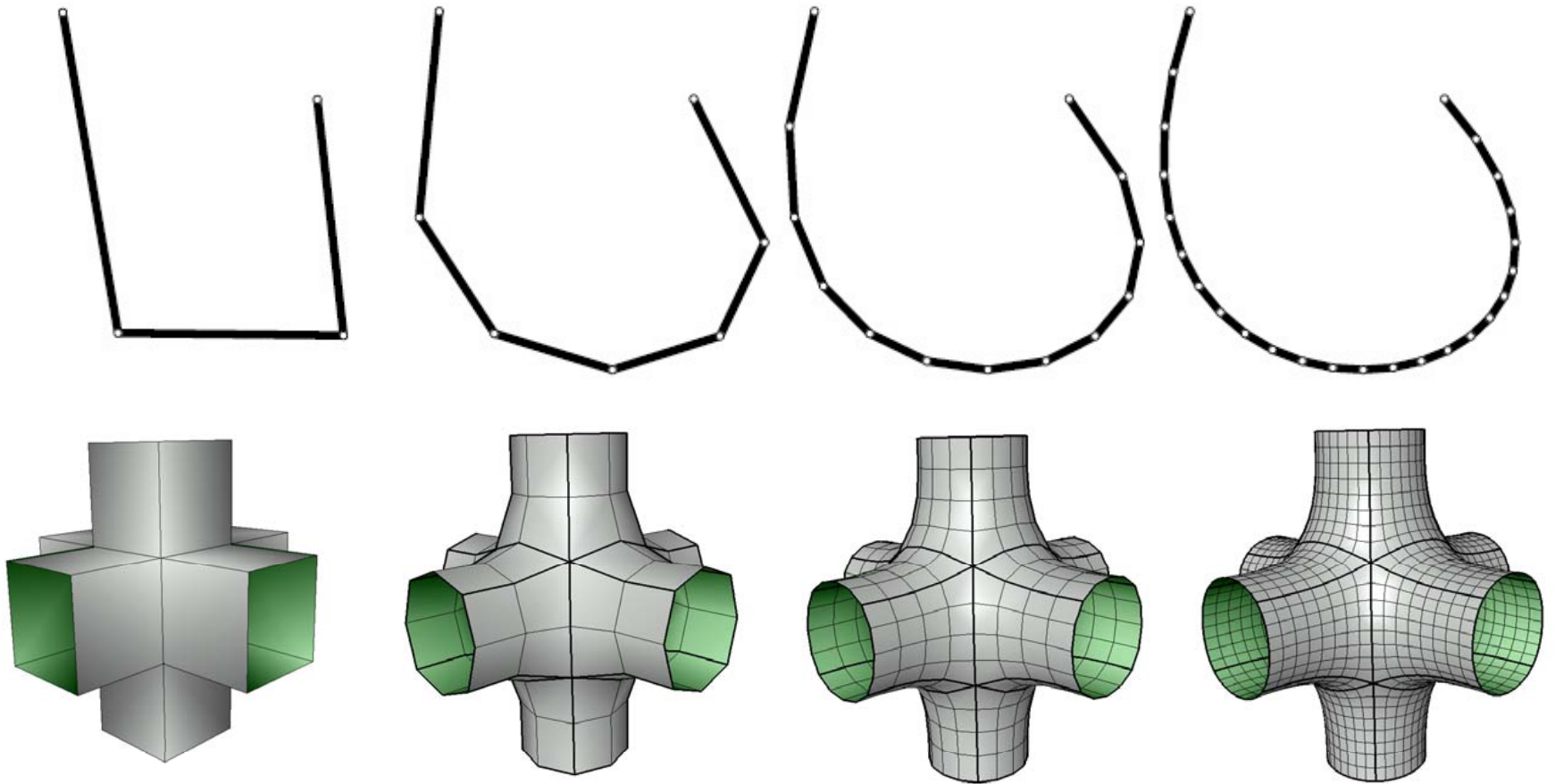
efinements

Subdivision surfaces
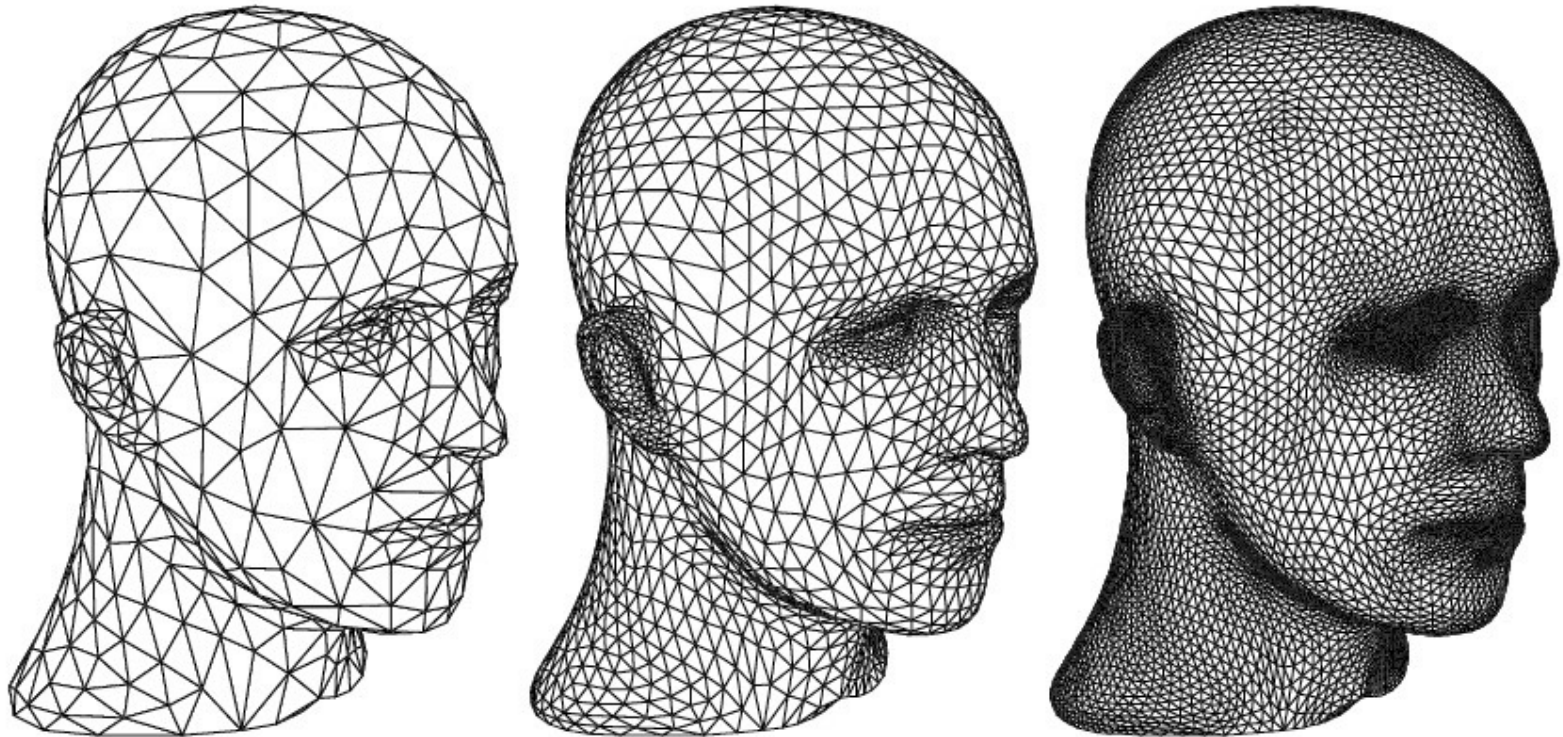
Figure 2.2: *Example of subdivision for a surface, showing 3 successive levels of refinement. On the left an initial triangular mesh approximating the surface. Each triangle is split into 4 according to a particular subdivision rule (middle). On the right the mesh is subdivided in this fashion once again.*

Generalizing from curves to surfaces

- Two parts to subdivision process
- Subdividing the mesh (computing new topology)
  - For curves: replace every segment with two segments
  - For surfaces: replace every face with some new faces
- Positioning the vertices (computing new geometry)
  - For curves: two rules (one for *odd* vertices, one for *even*)
    - New vertex's position is a weighted average of positions of old vertices that are nearby along the sequence
  - For surfaces: two kinds of rules (still called odd and even)
    - New vertex's position is a weighted average of positions of old vertices that are nearby in the mesh

Subdivision of meshes

- ## Quadrilaterals
  - ### Catmull-Clark 1978
- ## Triangles
  - ### Loop 1987
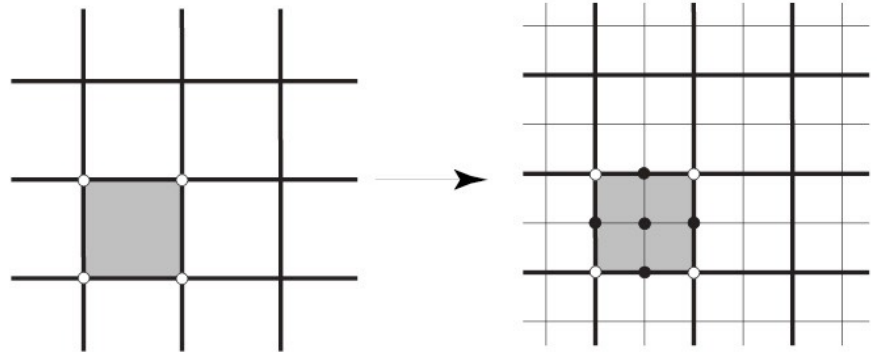
Face split for quads
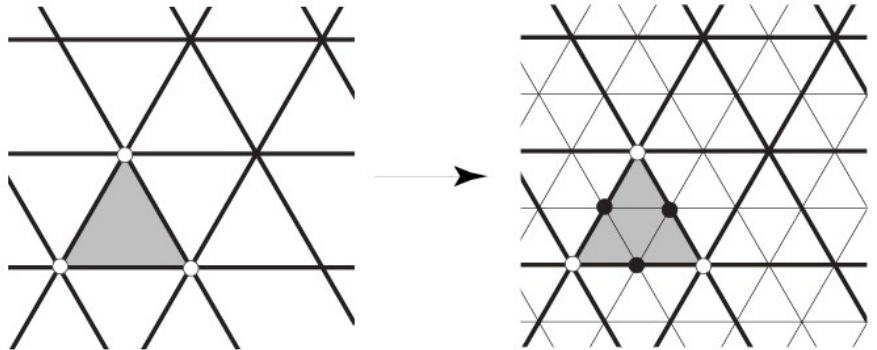
Face split for triangles

Loop regular rules
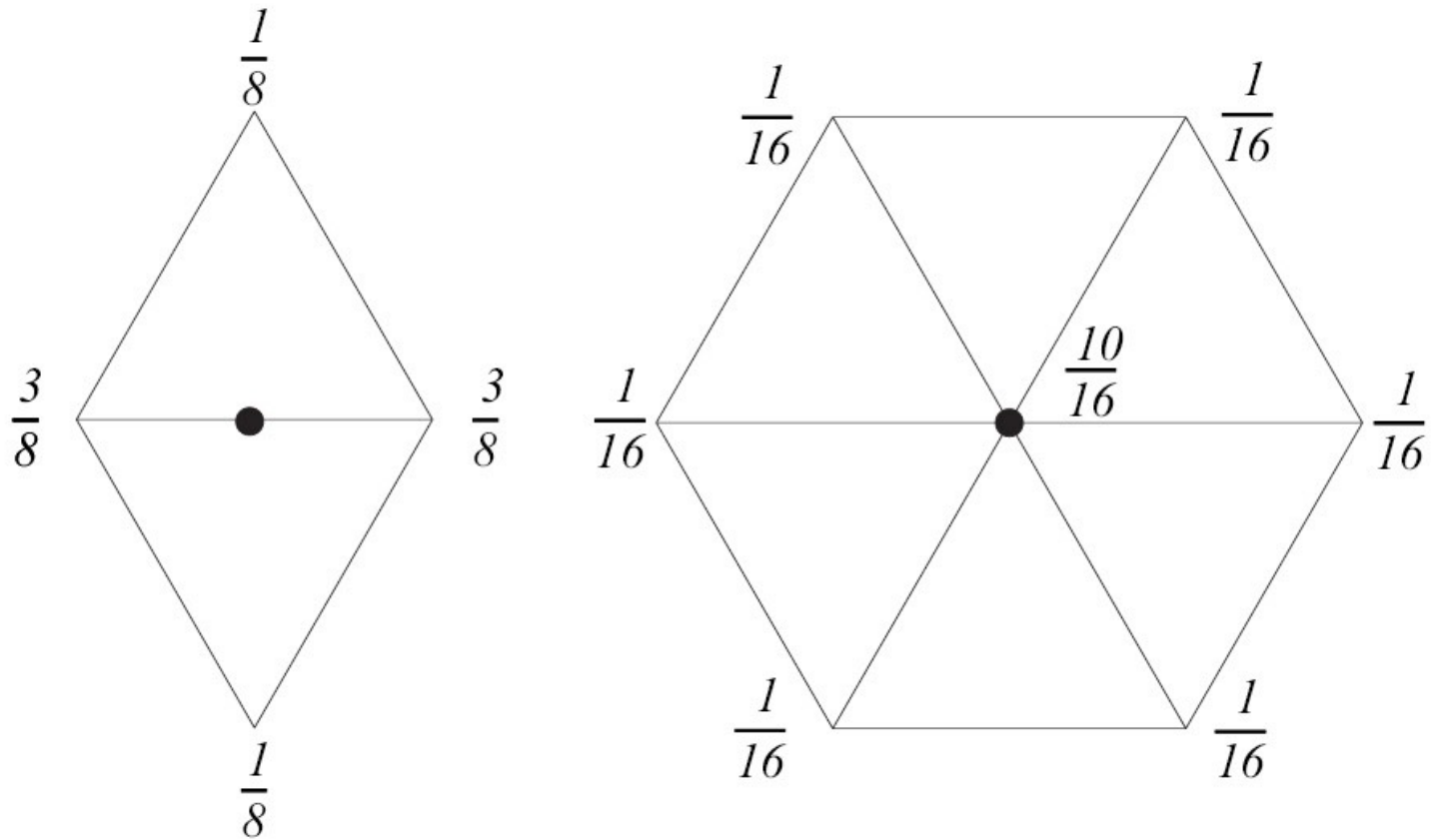
Catmull-Clark regular rules
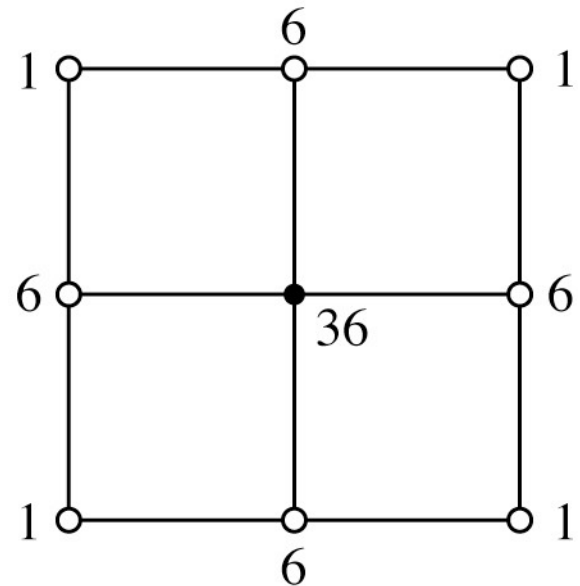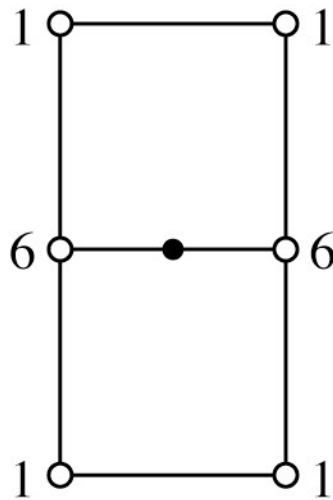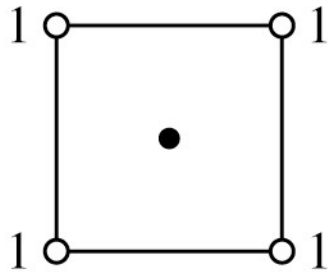
Creases

- With splines, make creases by turning off continuity constraints
- With subdivision surfaces, make creases by marking edges "sharp"
  - use different rules for vertices with sharp edges
  - these rules produce B-splines that depend only on vertices along crease
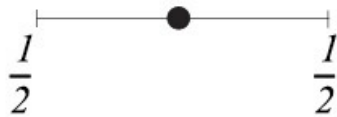
a. Masks for odd vertices
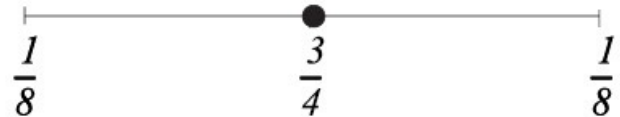
b. Masks for even vertices

Boundaries

- At boundaries the masks do not work
  - mesh is not manifold; edges do not have two triangles
- Solution: same as crease
  - shape of boundary is controlled only by vertices along boundary

Crease and boundary

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{8}$ $\frac{3}{4}$ $\frac{1}{8}$
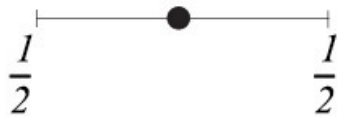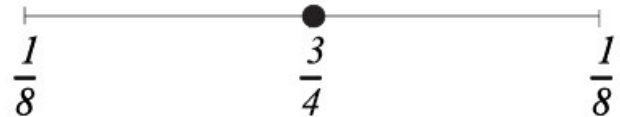
a. Masks for odd vertices   *b. Masks for even vertices*

Extraordinary vertices

- Vertices that don't have the "standard" valence
- Unavoidable for most topologies
- Difference from splines
  - treatment of extraordinary vertices is really the only way subdivision surfaces are different from spline patches
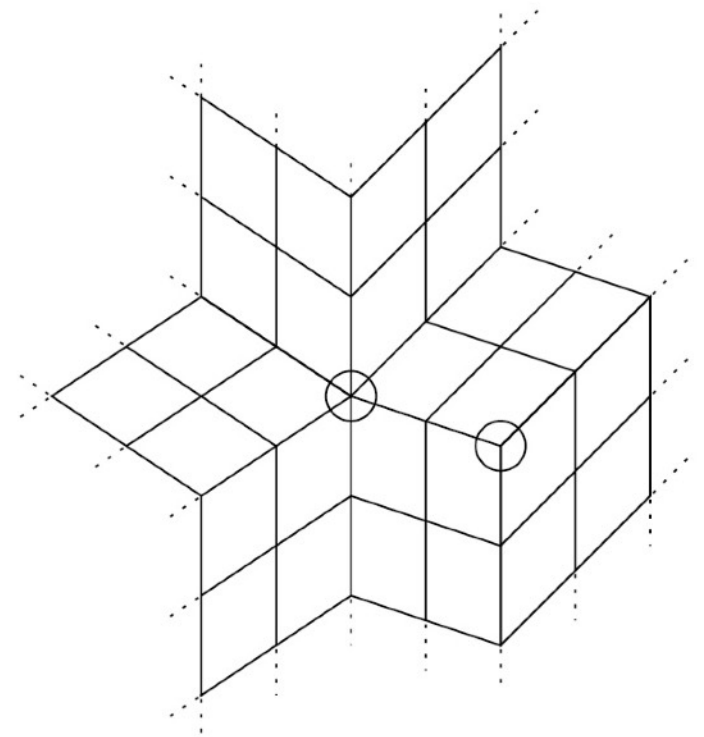
[Schröder & Zorin SIGGRAPH 2000 course 23]

Full Loop rules (triangle mesh)

$$\beta = \frac{5}{8} - \frac{(3 + 2\cos(2\pi/n))^2}{64}$$
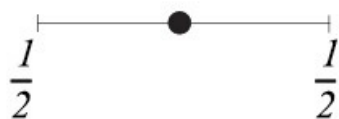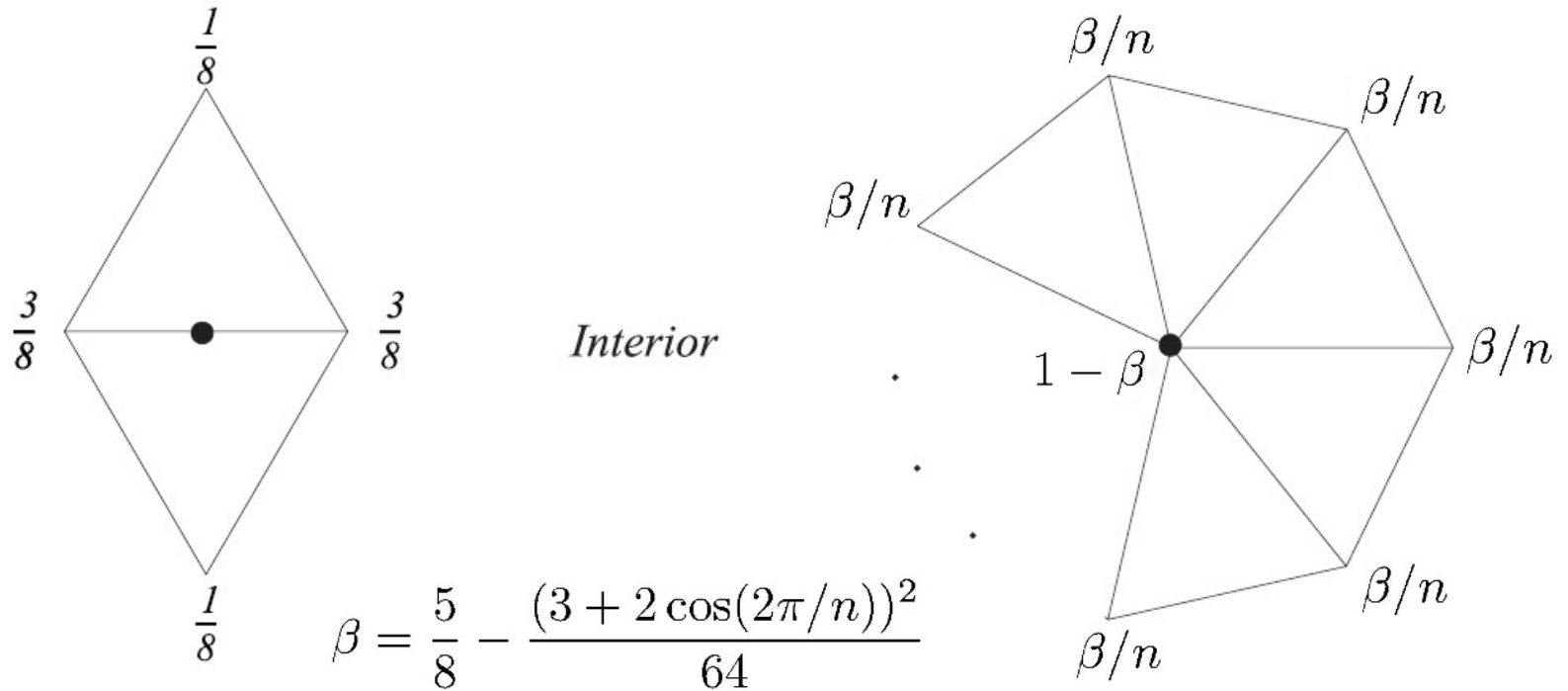
Interior

Crease and boundary

a. *Masks for odd vertices*
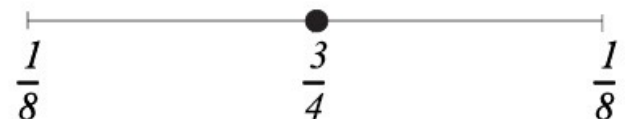
b. *Masks for even vertices*

Full Catmull-Clark rules (quad mesh)
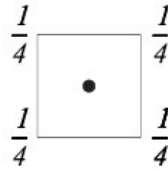
Mask for a face vertex
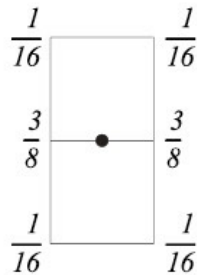
Mask for an edge vertex

Mask for a boundary odd vertex

Interior
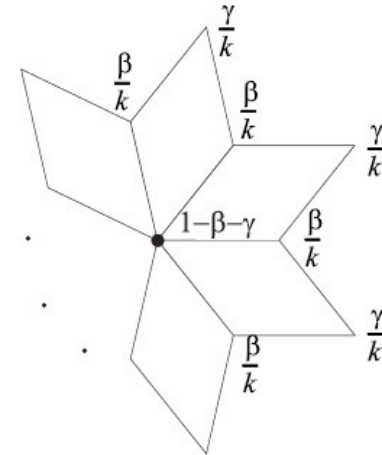
$$\beta = 3/2k; \gamma = 1/4k$$

Crease and boundary

a. Masks for odd vertices
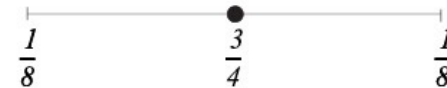
b. Mask for even vertices

Loop Subdivision Example

control polyhedron
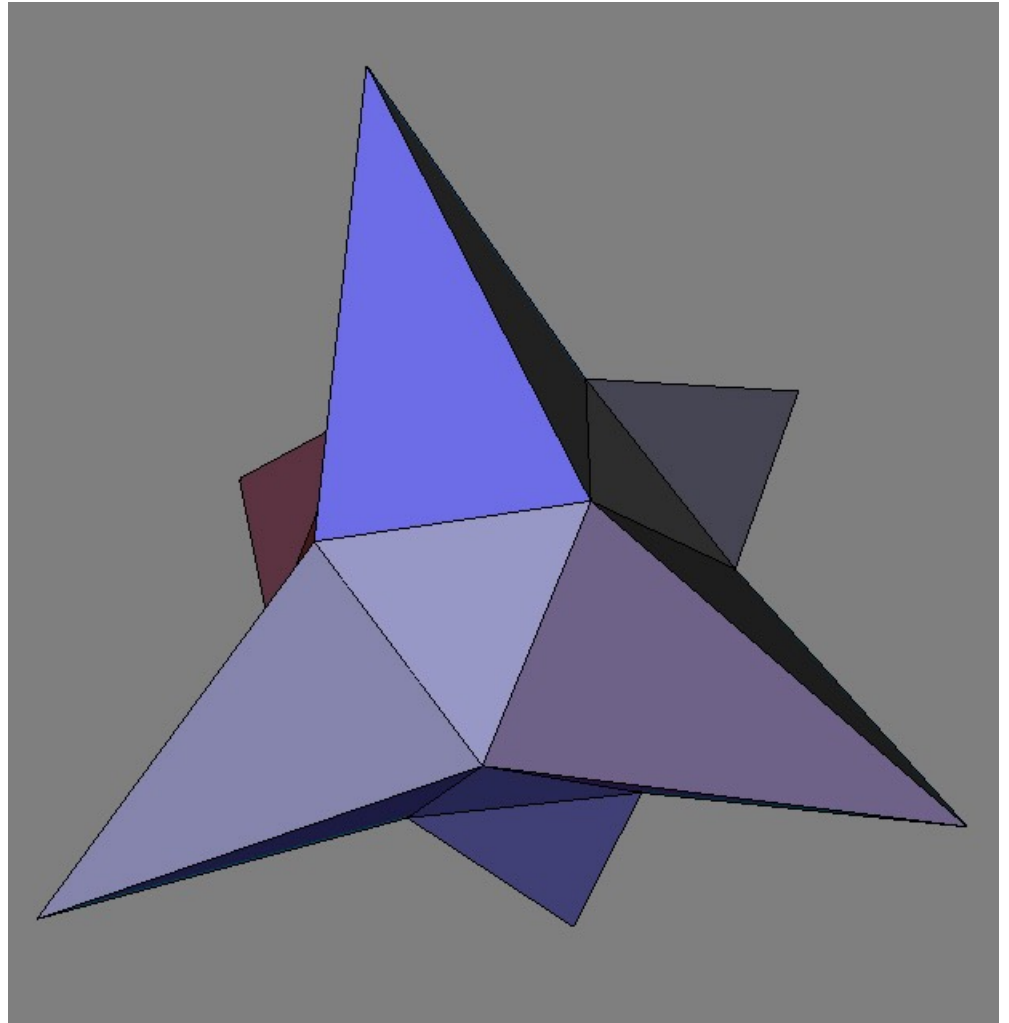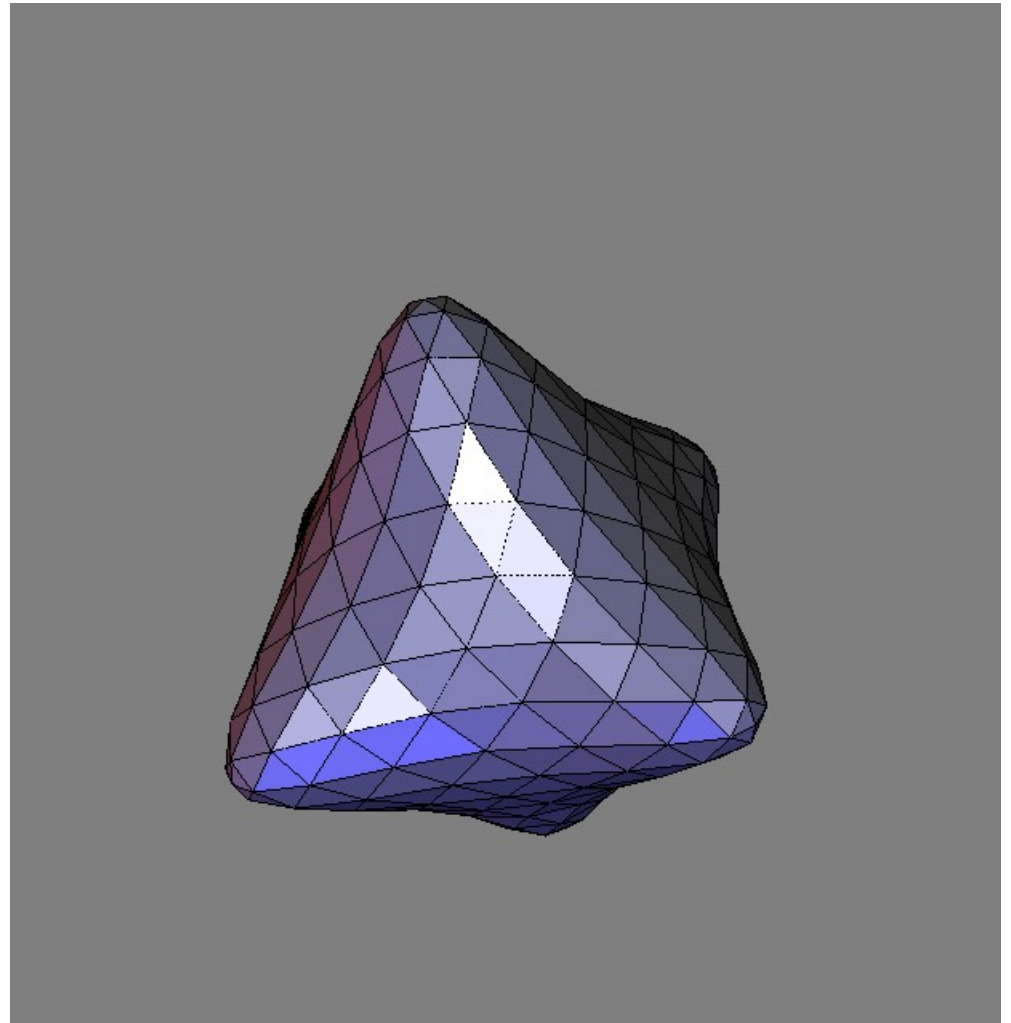
Loop Subdivision Example

refined
control polyhedron
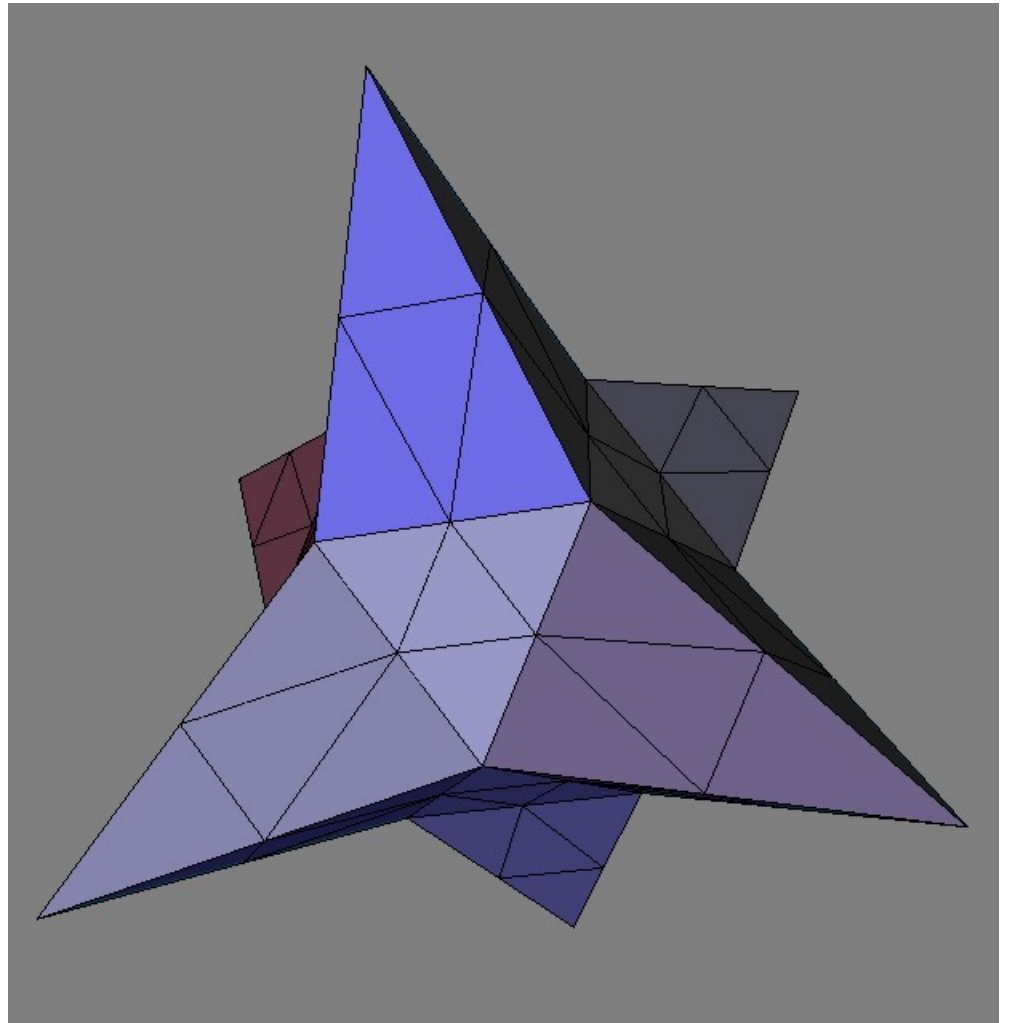
Loop Subdivision Example

odd
subdivision mask
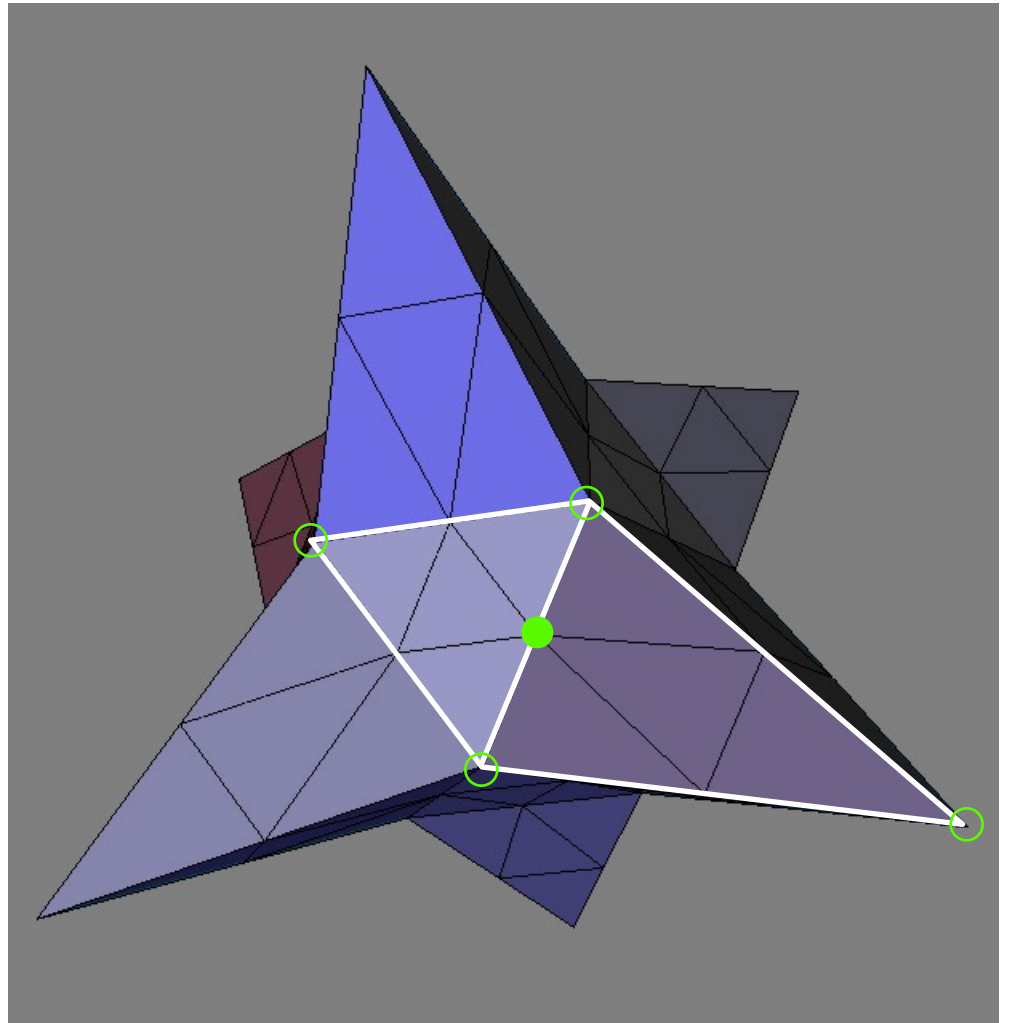
Loop Subdivision Example
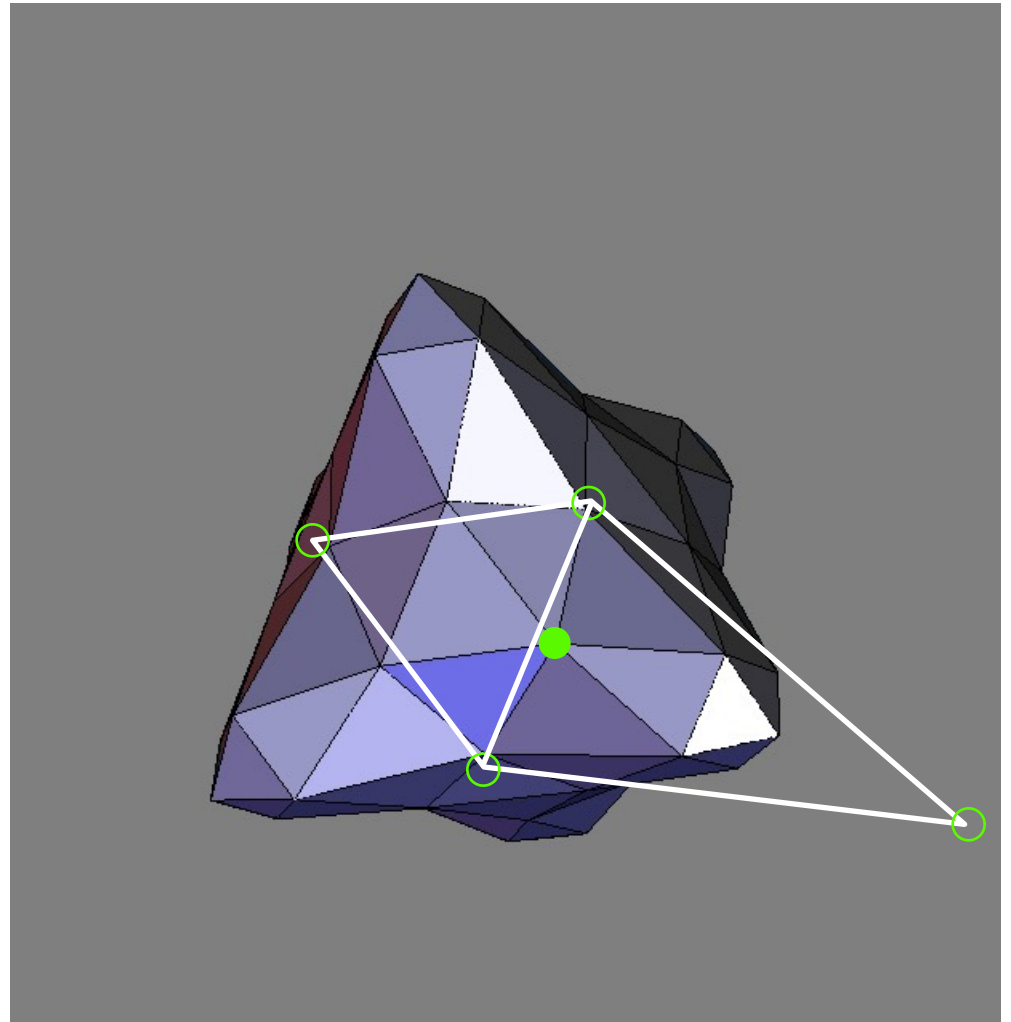
subdivision level 1

Loop Subdivision Example

even
subdivision mask
(ordinary vertex)
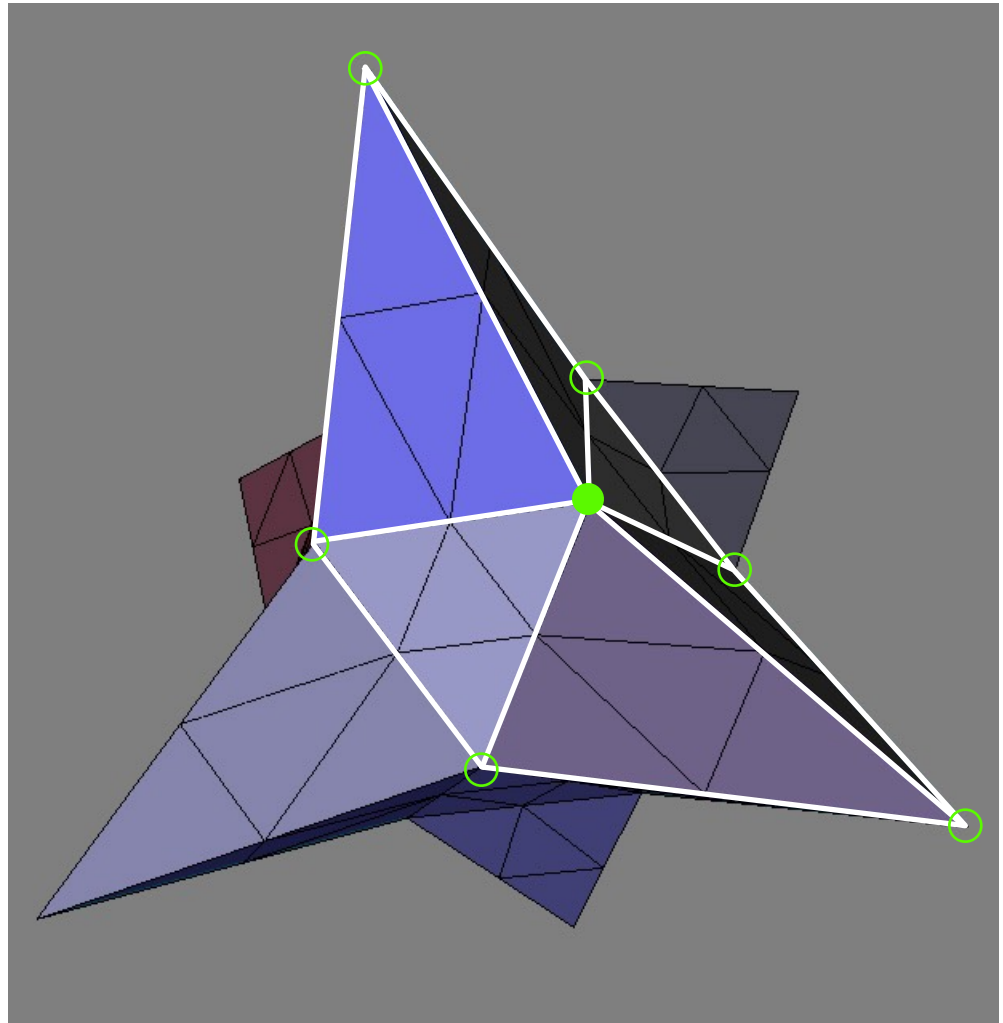
Loop Subdivision Example
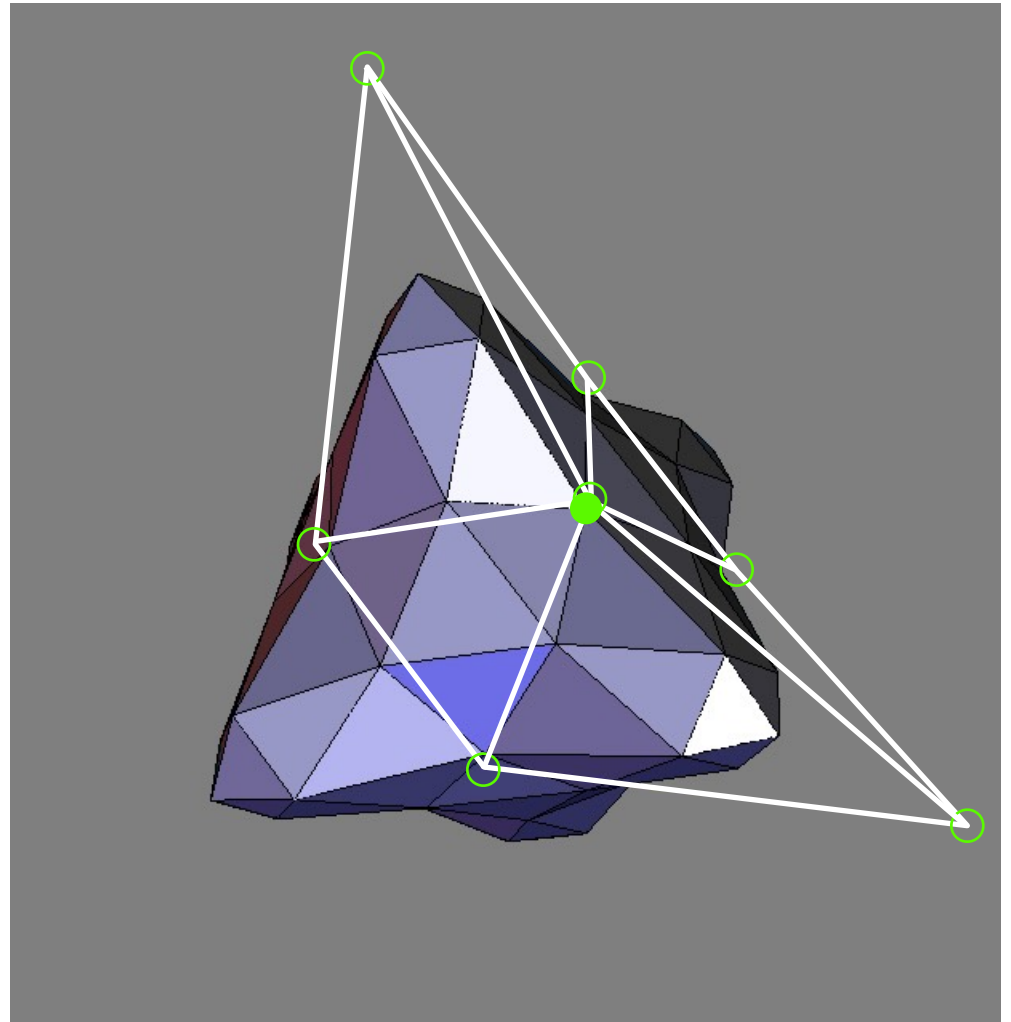
subdivision level 1

Loop Subdivision Example

even
subdivision mask
(extraordinary
vertex)
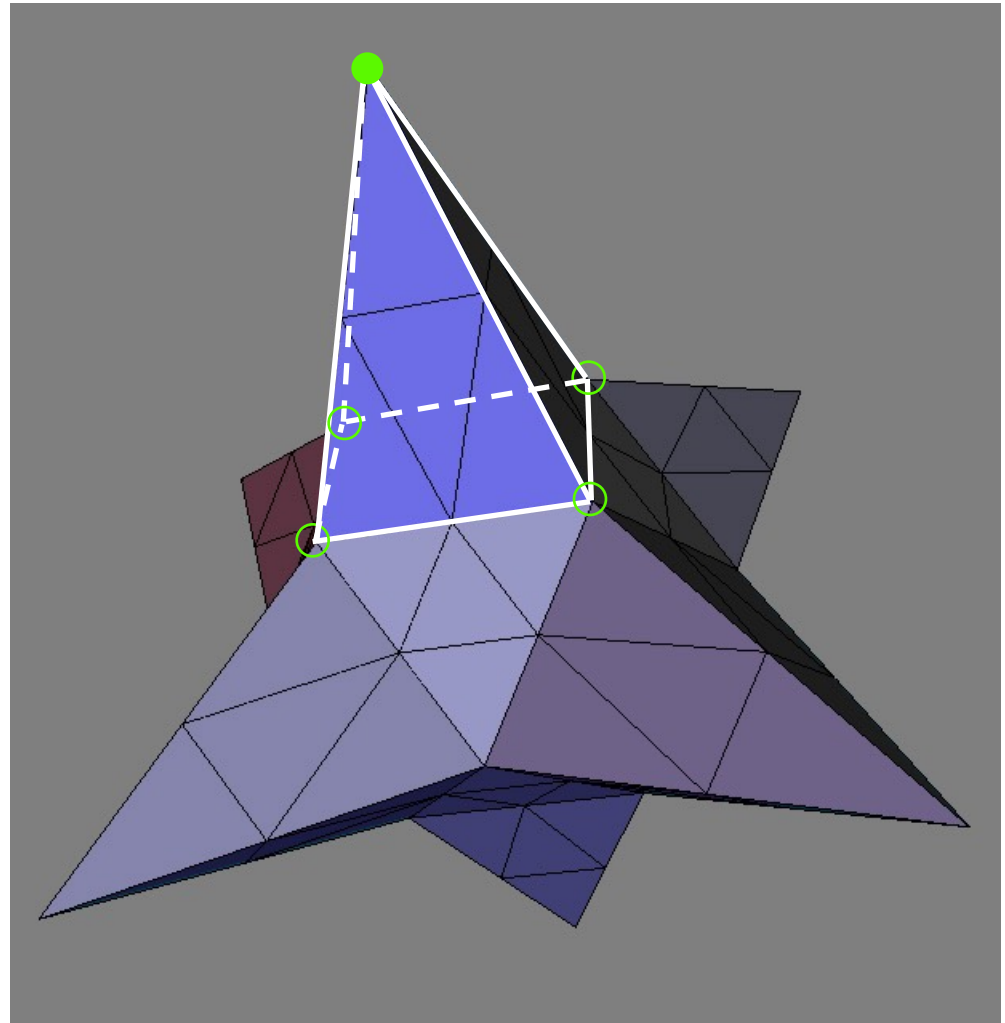
Loop Subdivision Example
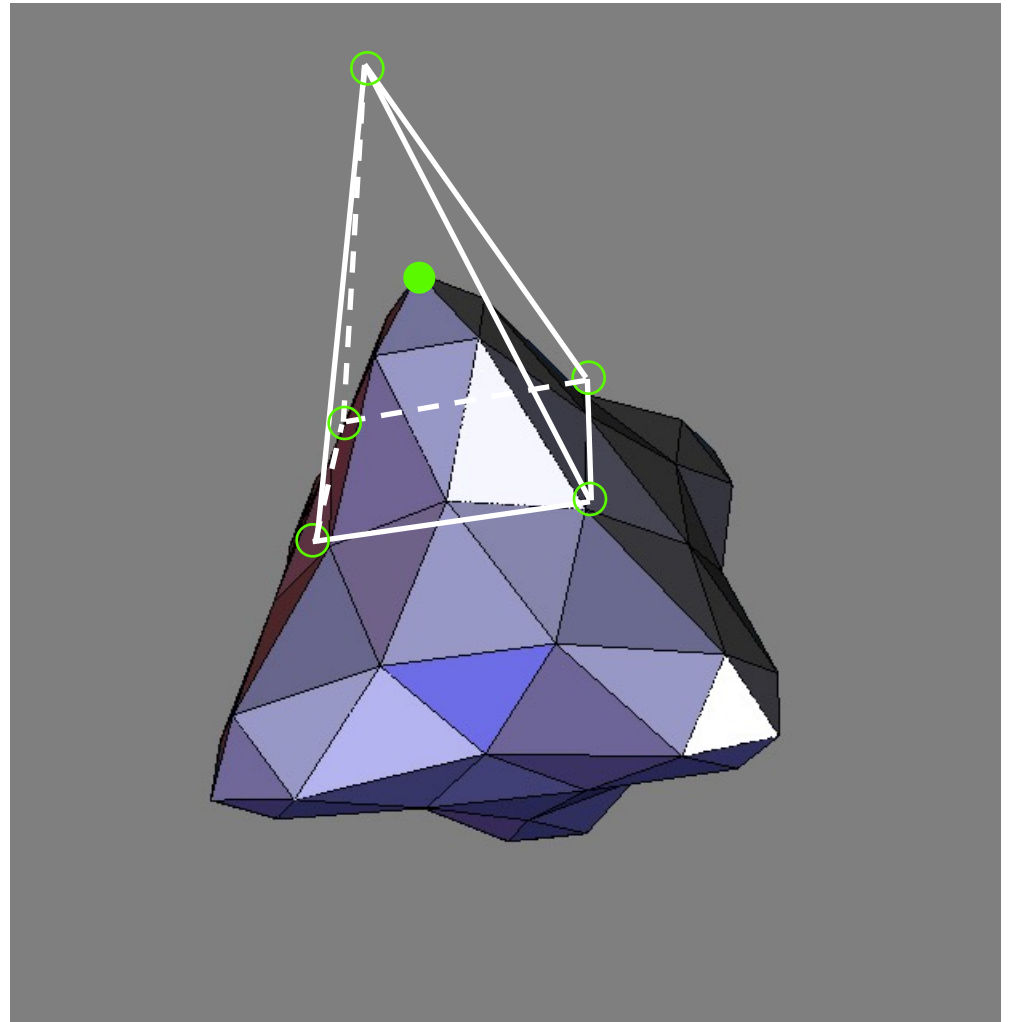
subdivision level 1

Loop Subdivision Example
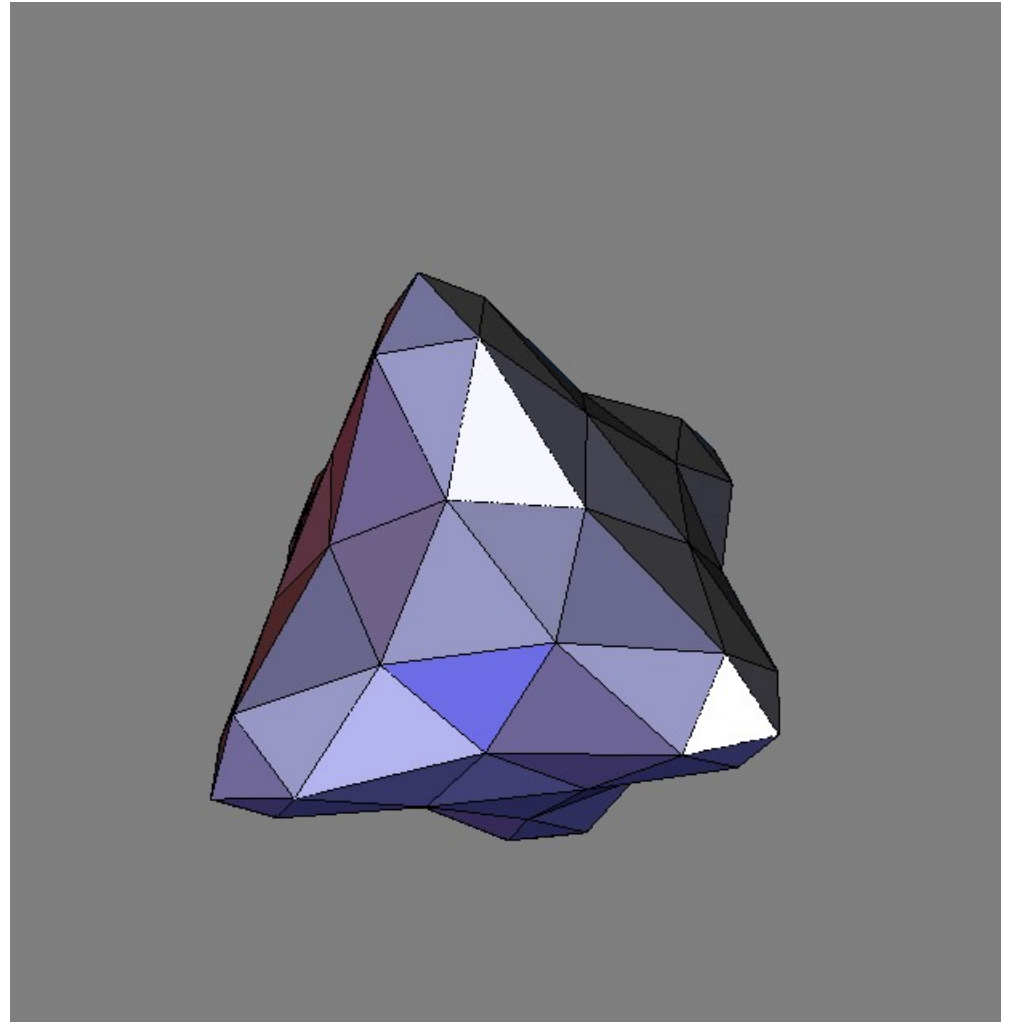
subdivision level 1

Loop Subdivision Example
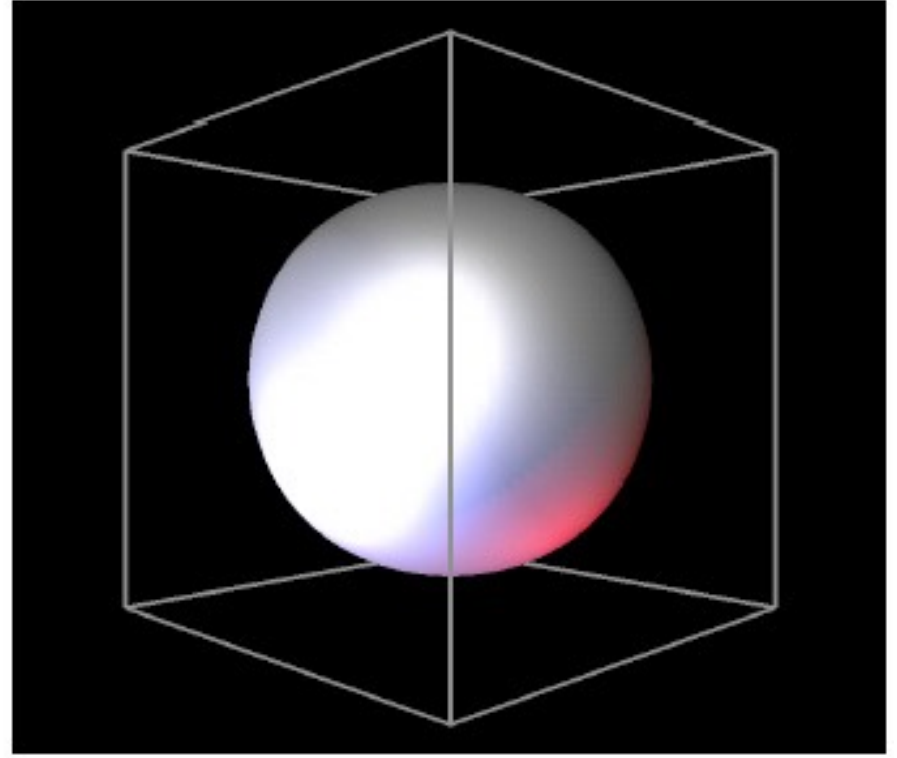

subdivision level 2

Loop Subdivision Example
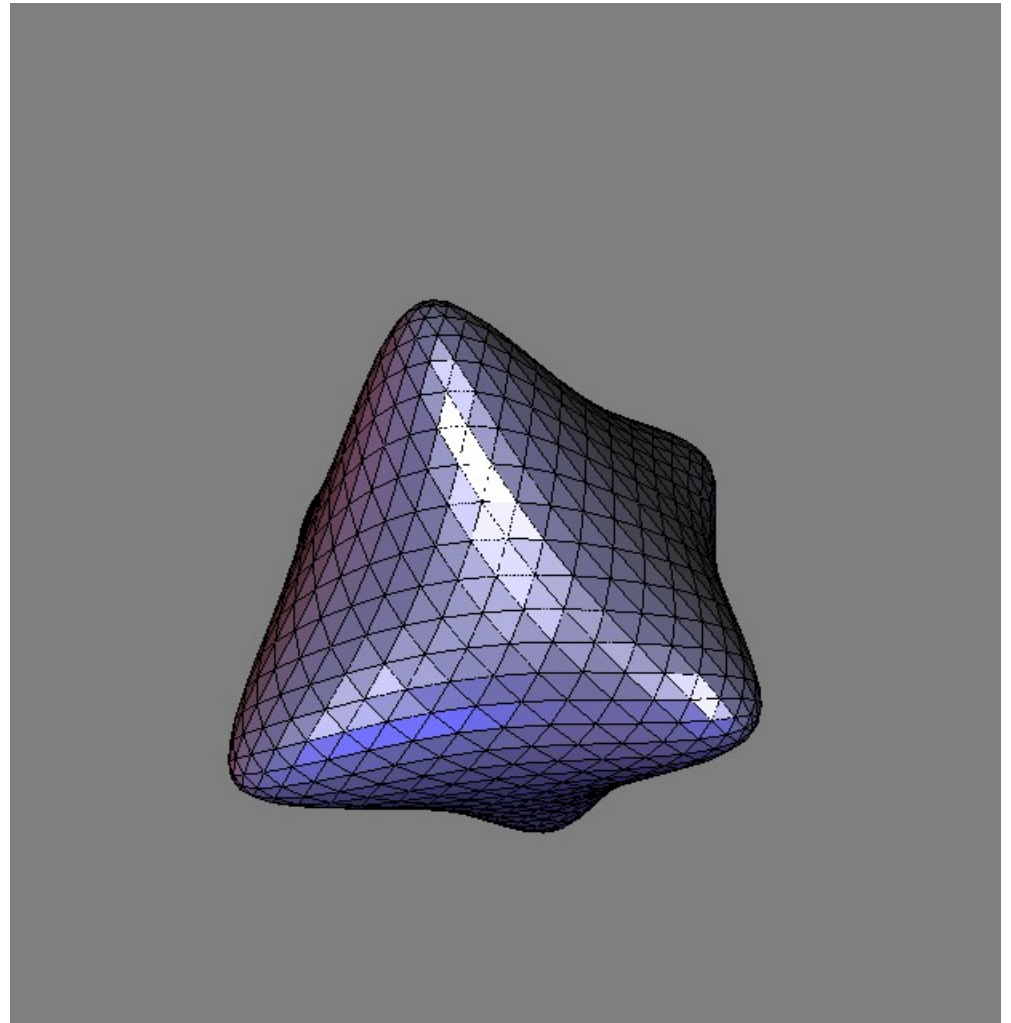
subdivision level 3

Loop Subdivision Example
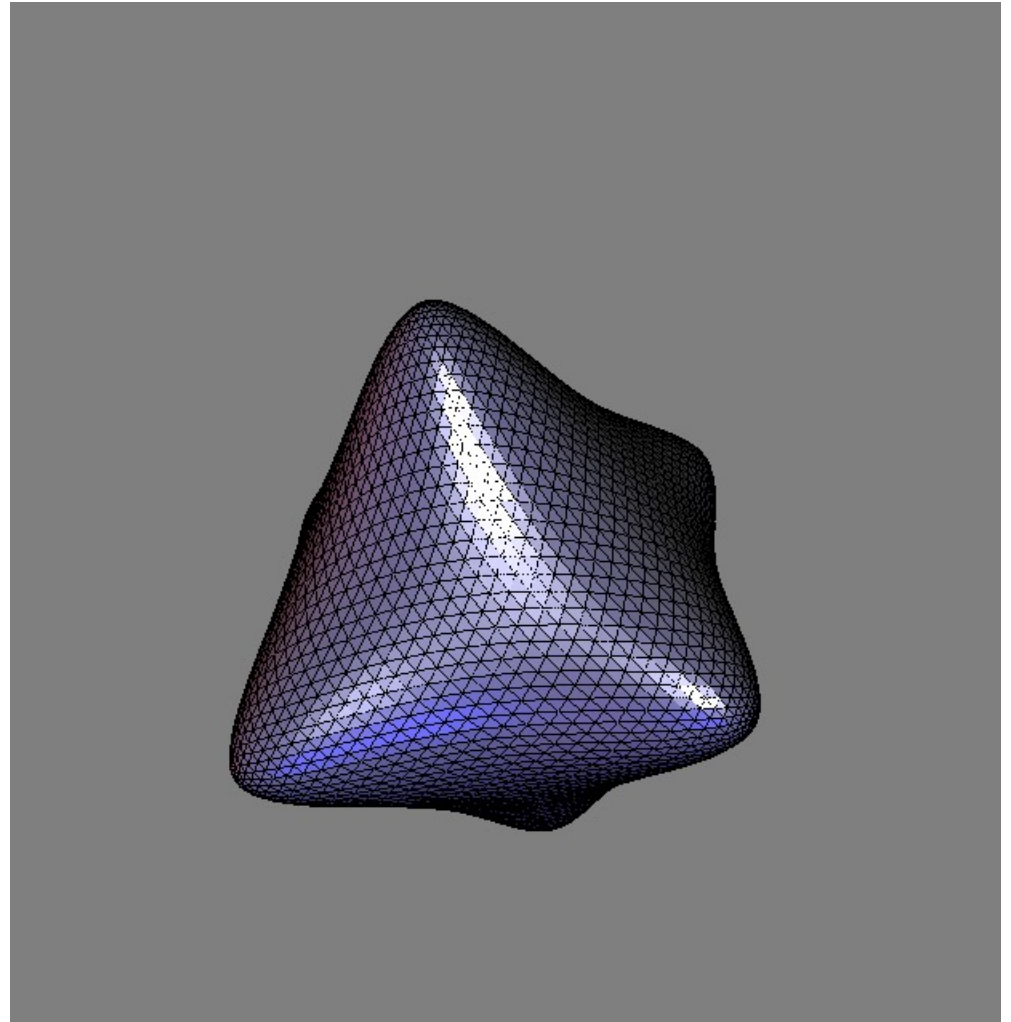
subdivision level 4

Loop Subdivision Example
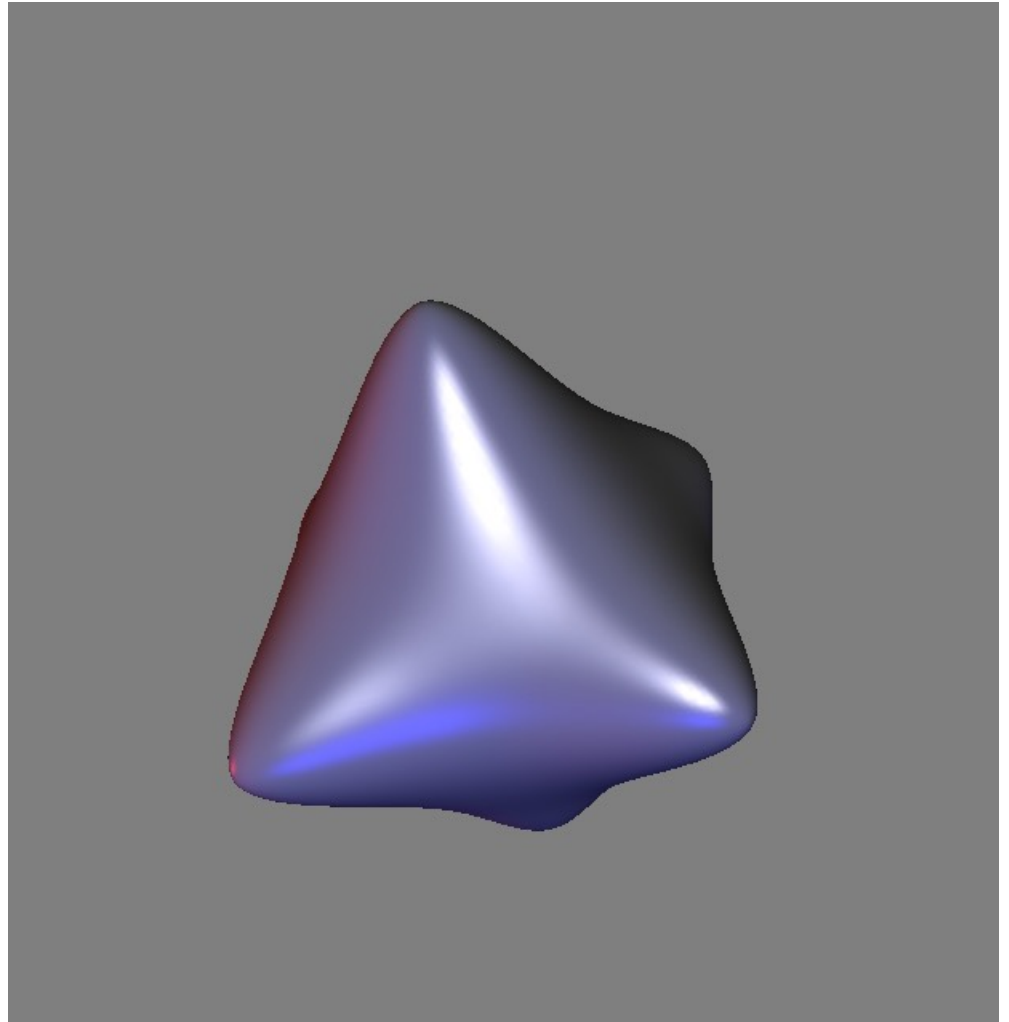
limit surface

Relationship to splines

- In regular regions, behavior is identical

- At extraordinary vertices, achieve C^1
  - near extraordinary, different from splines

- Linear everywhere
  - mapping from parameter space to 3D is a linear combination of the control points
  - "emergent" basis functions per control point
    - match the splines in regular regions
    - "custom" basis functions around extraordinary vertices

Loop vs. Catmull-Clark

Loop

Catmull-Clark

Loop vs. Catmull-Clark

Loop

Catmull-Clark

Loop vs. Catmull-Clark

Loop
(after splitting faces)

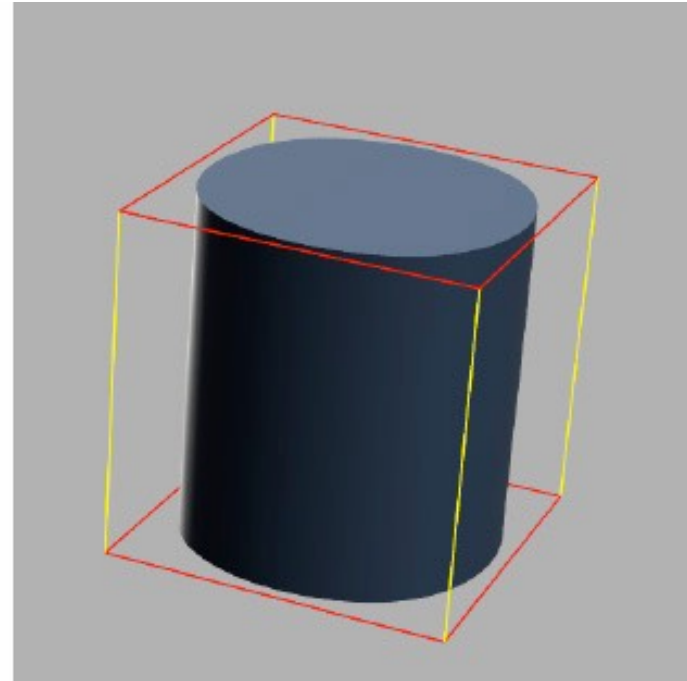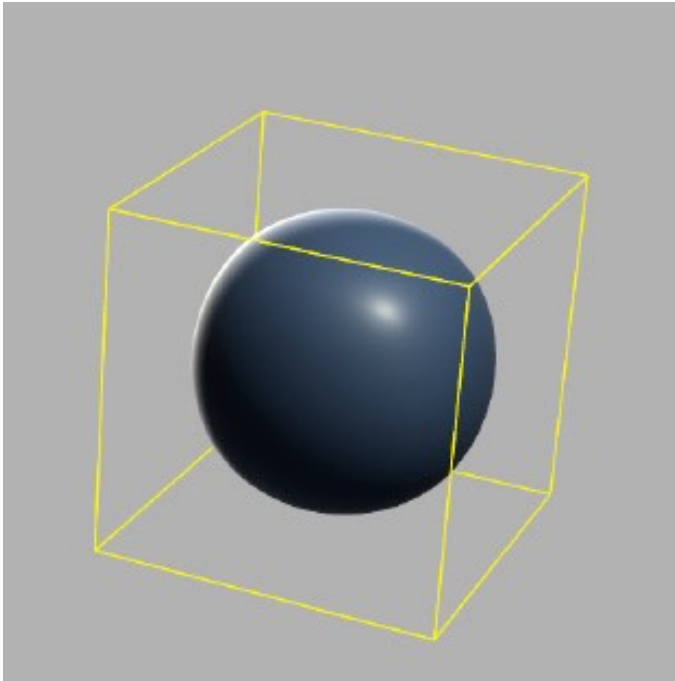Catmull-Clark

Loop with creases

(a-d) Loop's subdivision scheme: control mesh, meshes after 1 and 2 subdivision steps, and smooth limit surface
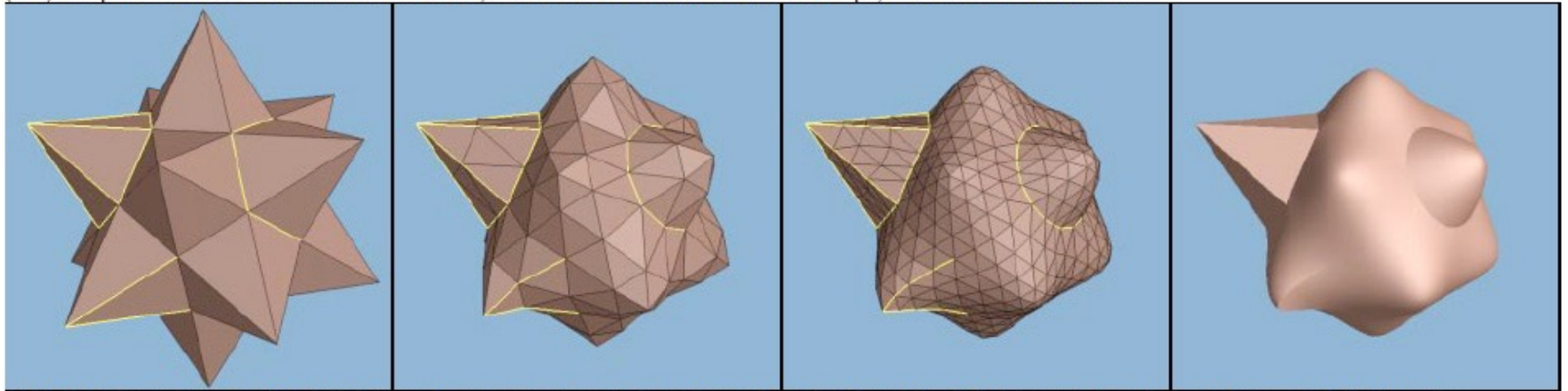
(e-h) Our piecewise smooth subdivision scheme: tagged control mesh, meshes after 1 and 2 subdivision steps, and piecewise smooth limit surface
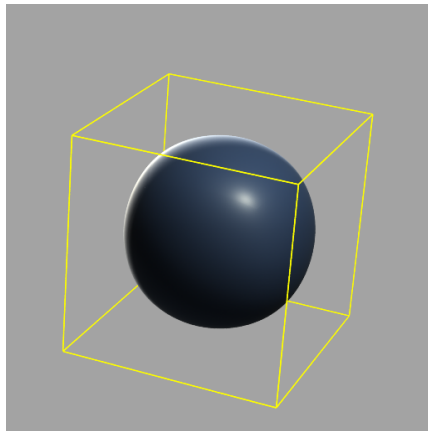
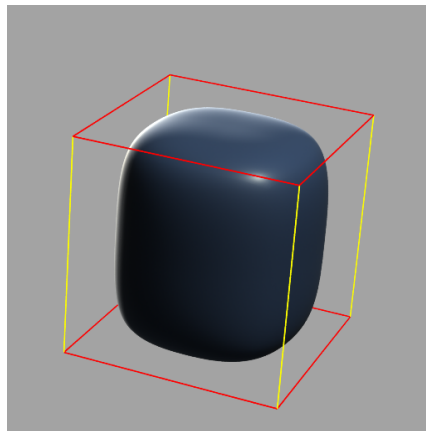[Hugues Hoppe]

Catmull-Clark with creases
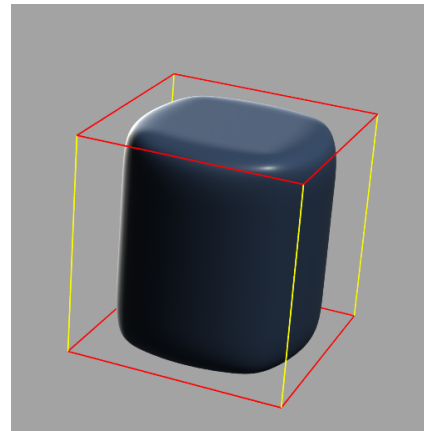
Variable sharpness creases

- Idea: subdivide for a few levels using the crease rules, then proceed with the normal smooth rules.

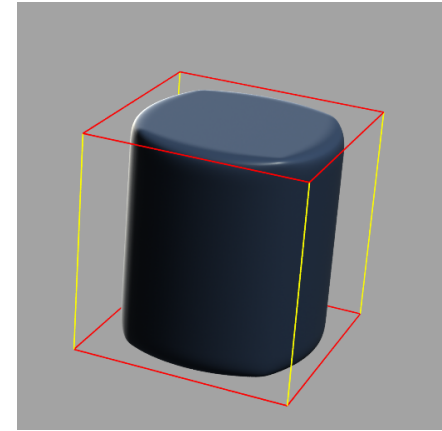- Result: a soft crease that gets sharper as we increase the number of levels of sharp subdivision steps

sharpness 0   sharpness 1   sharpness 2   sharpness 3

Geri's Game

- Pixar short film to test subdivision in production
  - Catmull-Clark (quad mesh) surfaces
  - complex geometry
  - extensive use of creases
  - subdivision surfaces to support cloth dynamics

[DeRose et al. SIGGRAPH 1998]