Rasterization

CS4620 Lecture 10

The graphics pipeline

- **The standard approach to object-order graphics**
- **Many versions exist**
  - software, e.g. Pixar's REYES architecture
    - many options for quality and flexibility
  - hardware, e.g. graphics cards in PCs
    - amazing performance: millions of triangles per frame
- **We'll focus on an abstract version of hardware pipeline**
- **"Pipeline" because of the many stages**
  - very parallelizable
  - leads to remarkable performance of graphics cards (many times the flops of the CPU at ~1/5 the clock speed)

Pipeline

you are here → APPLICATION

COMMAND STREAM

3D transformations; shading → VERTEX PROCESSING

TRANSFORMED GEOMETRY

conversion of primitives to pixels → RASTERIZATION

FRAGMENTS

blending, compositing, shading → FRAGMENT PROCESSING

FRAMEBUFFER IMAGE

user sees this → DISPLAY

Primitives

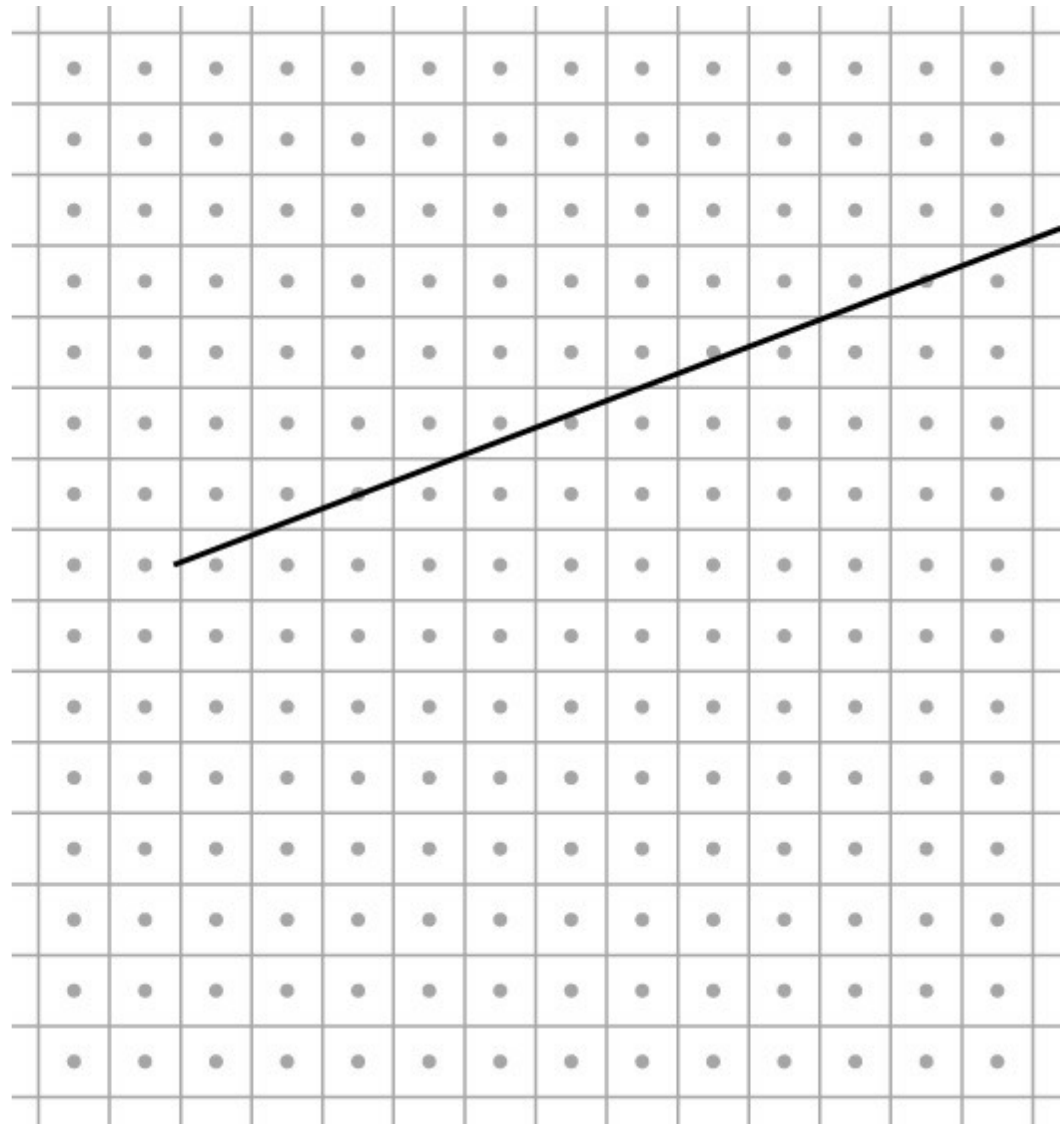
- **Points**
- **Line segments**
  – and chains of connected line segments
- **Triangles**
- **And that's all!**
  – Curves? Approximate them with chains of line segments
  – Polygons? Break them up into triangles
  – Curved surfaces? Approximate them with triangles
- **Trend over the decades: toward minimal primitives**
  – simple, uniform, repetitive: good for parallelism

Rasterization

- **First job: enumerate the pixels covered by a primitive**
  - simple, aliased definition: pixels whose centers fall inside
- **Second job: interpolate values across the primitive**
  - e.g. colors computed at vertices
  - e.g. normals at vertices
  - e.g. texture coordinates

Rasterizing lines

- **Define line as a rectangle**
- **Specify by two endpoints**
- **Ideal image: black inside, white outside**



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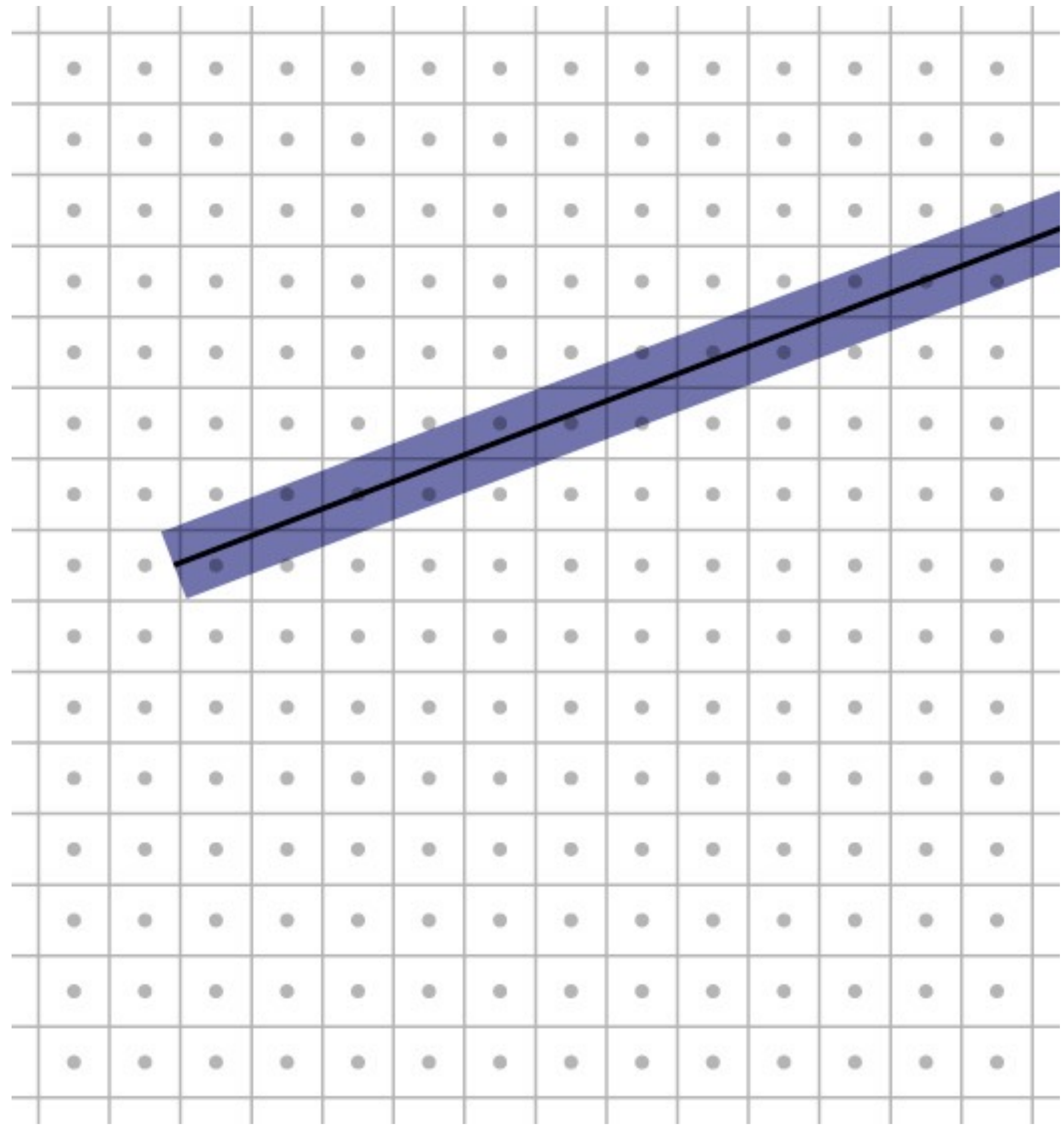
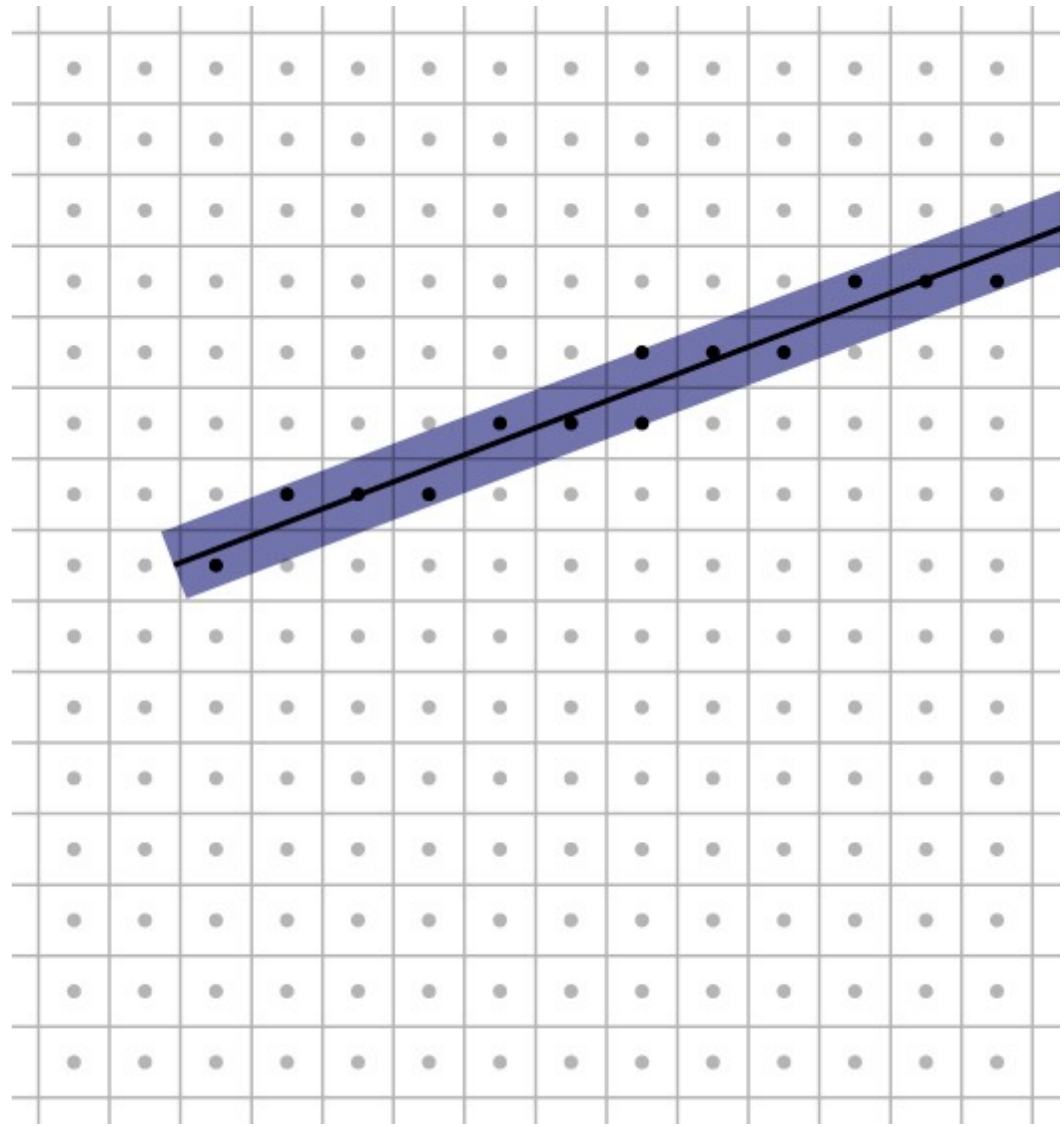


Point sampling

- **Approximate rectangle by drawing all pixels whose centers fall within the line**
- **Problem: sometimes turns on adjacent pixels**



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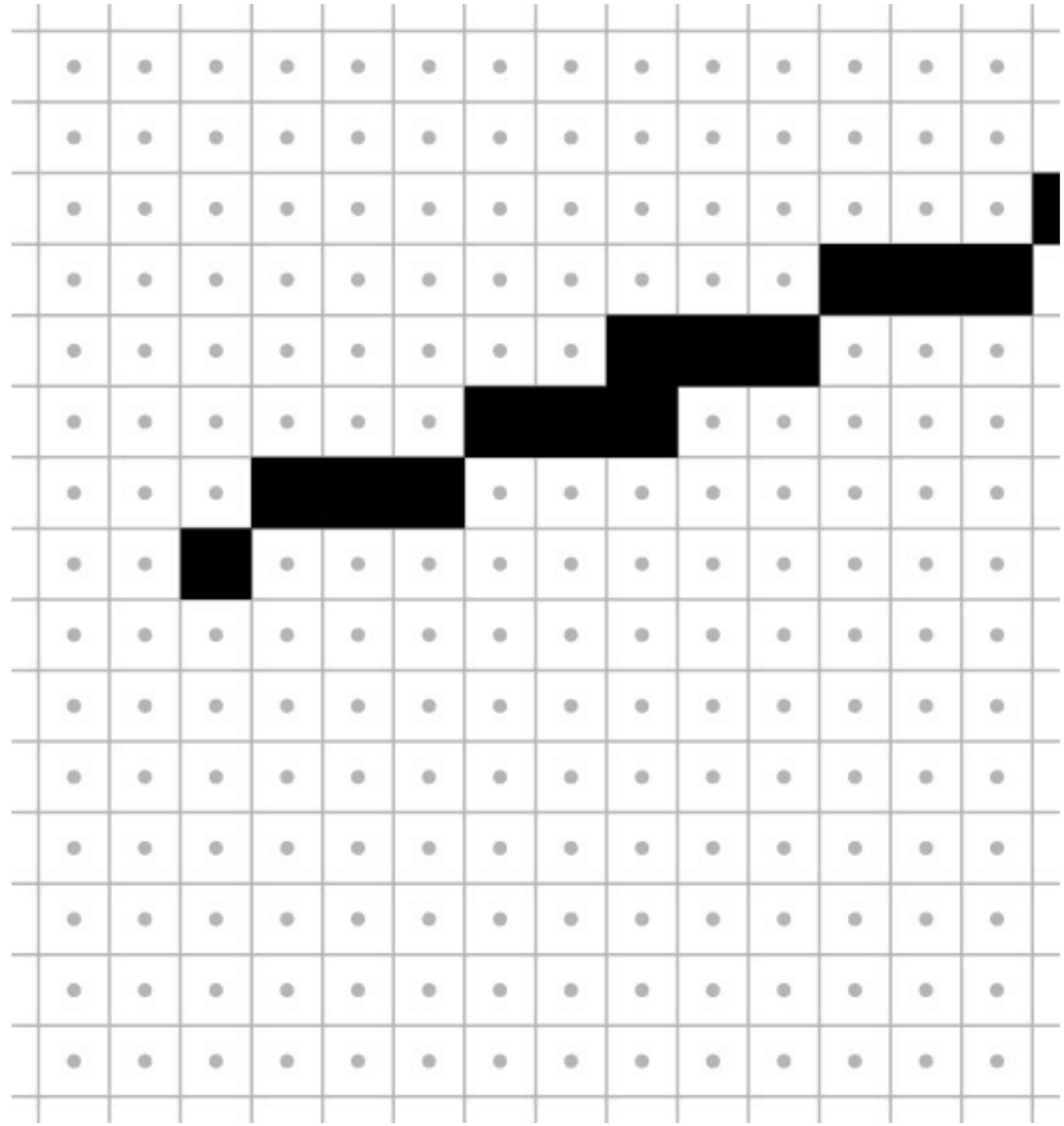
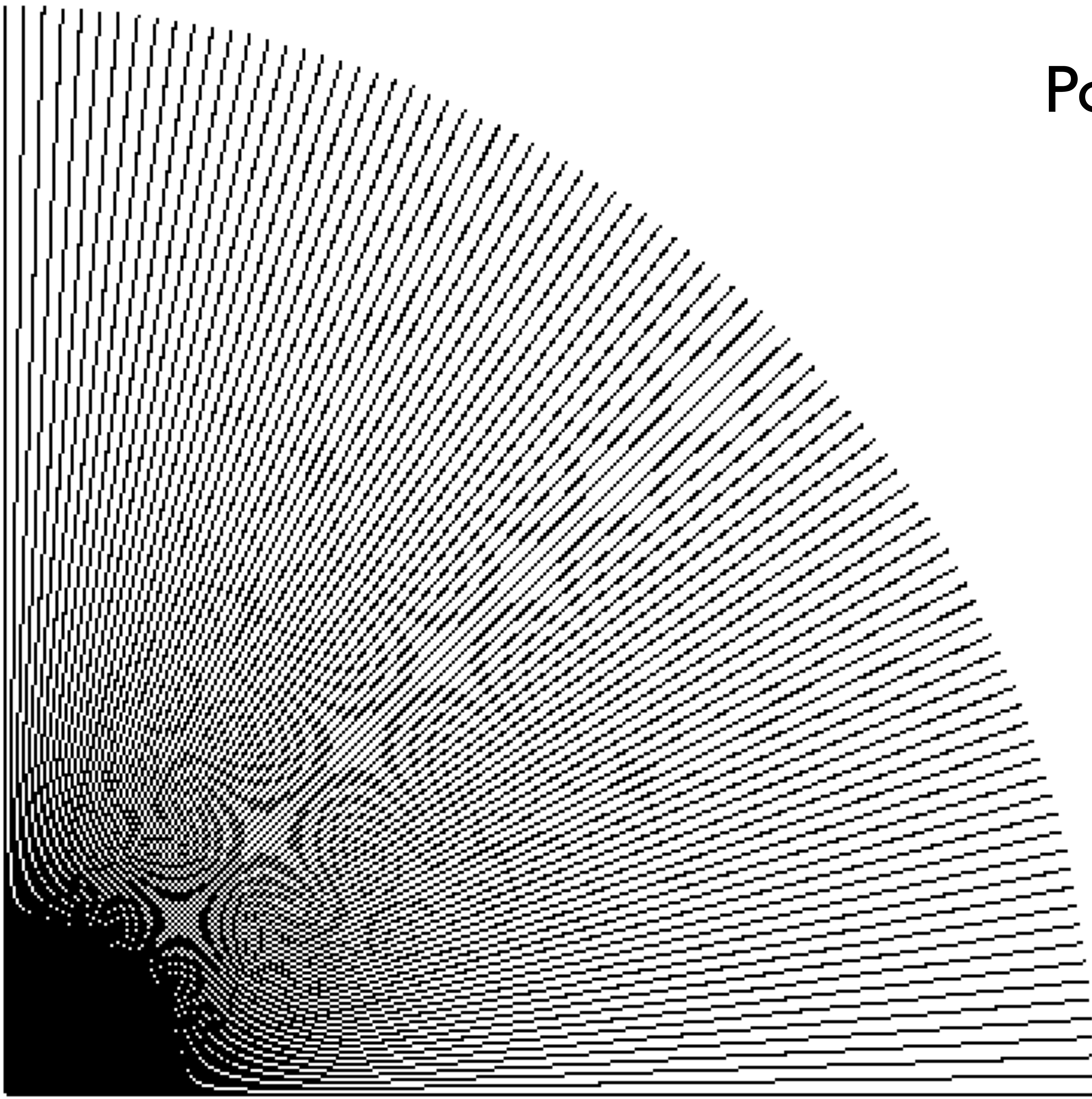
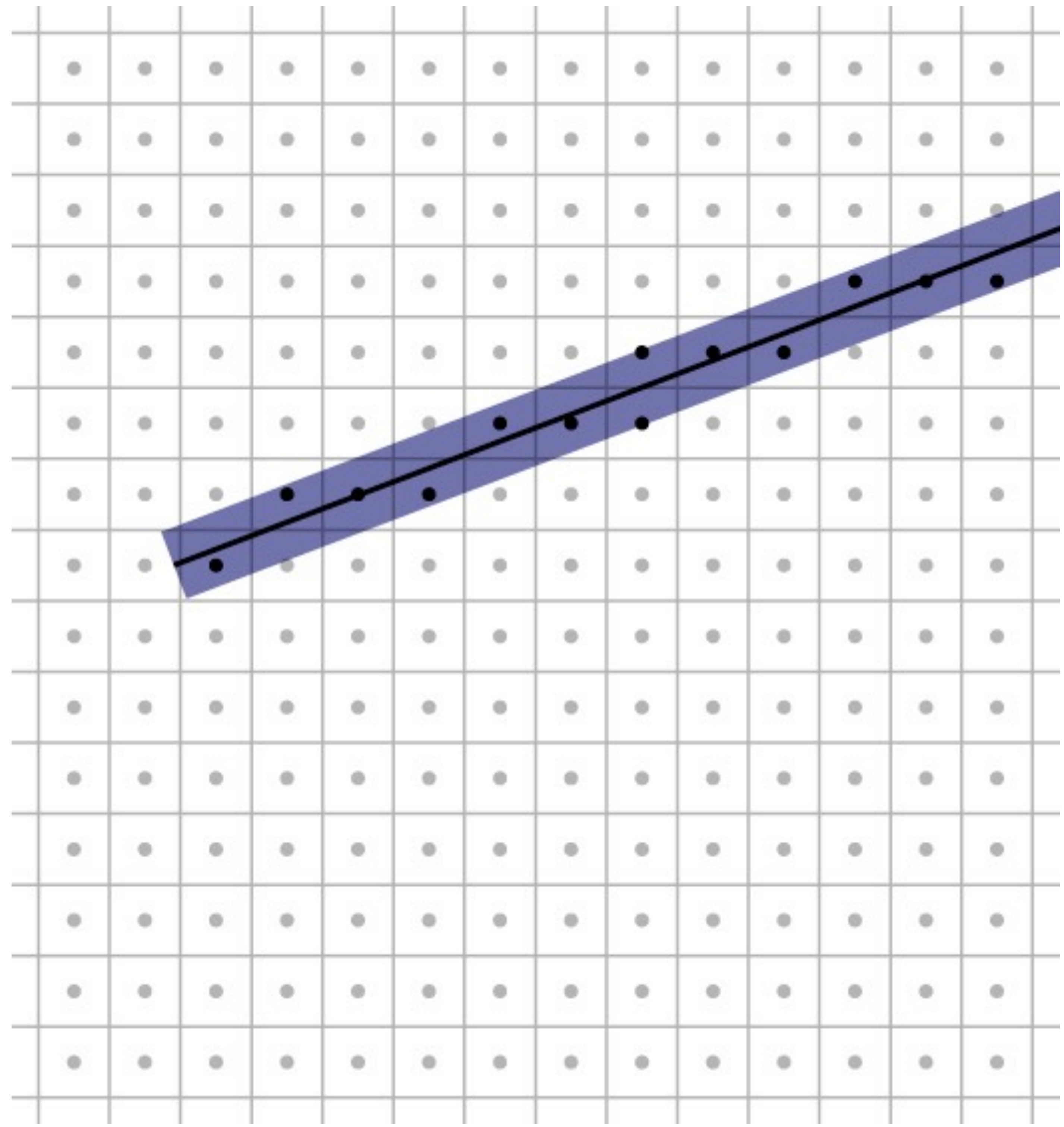




Point sampling in action

Bresenham lines (midpoint alg.)

- **Point sampling unit width rectangle leads to uneven line width**
- **Define line width parallel to pixel grid**
- **That is, turn on the single nearest pixel in each column**
- **Note that 45º lines are now thinner**



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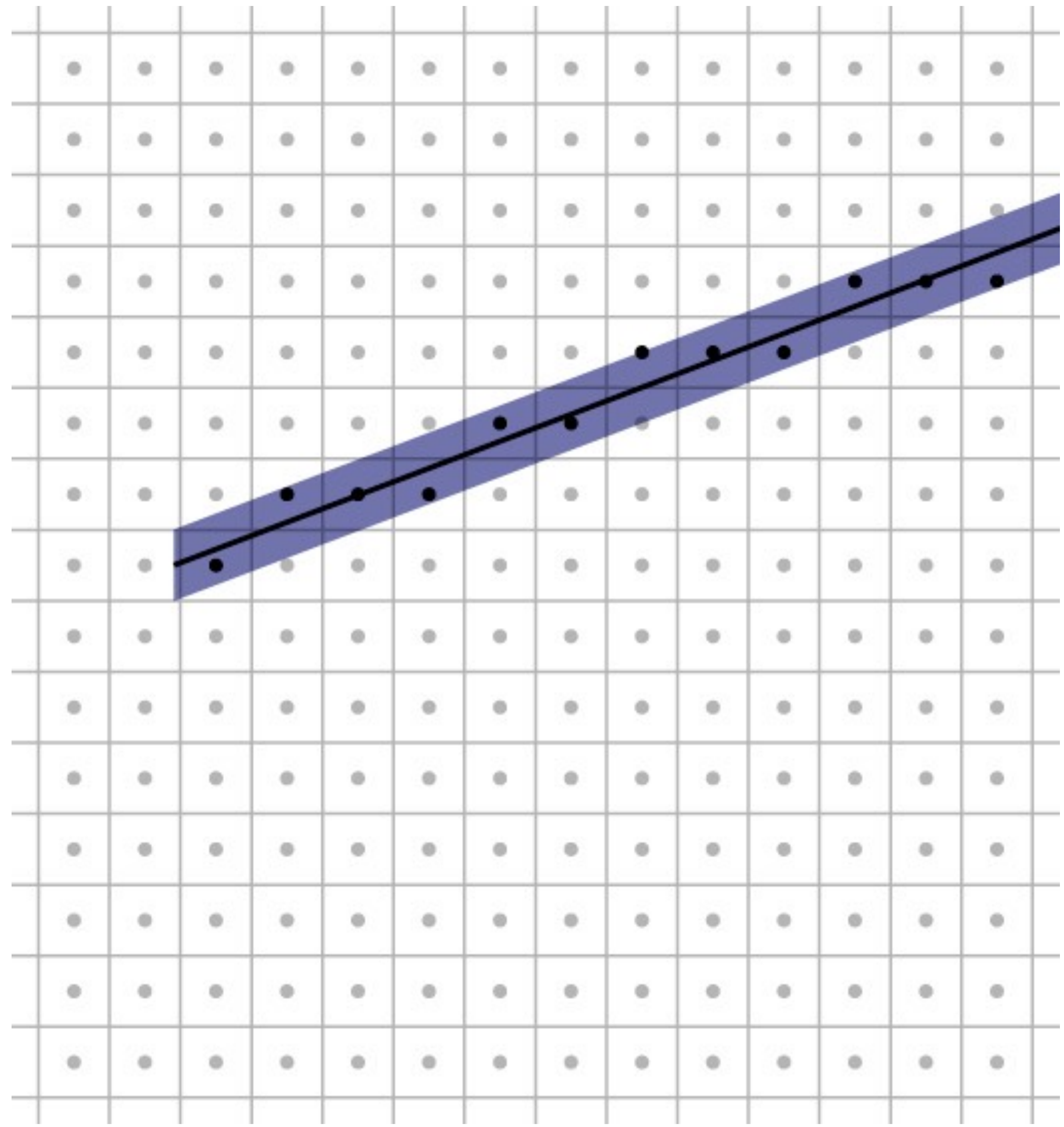

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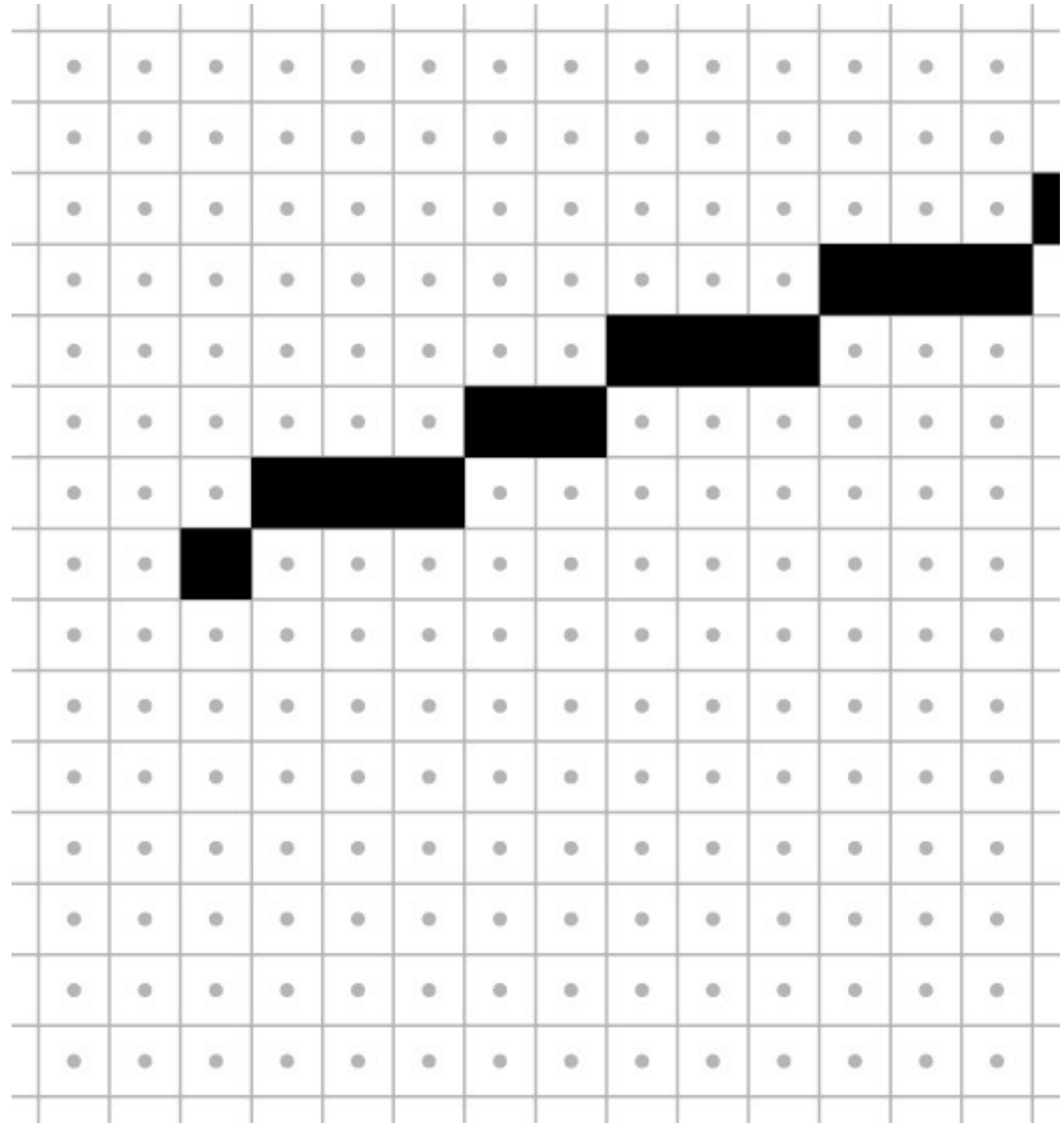
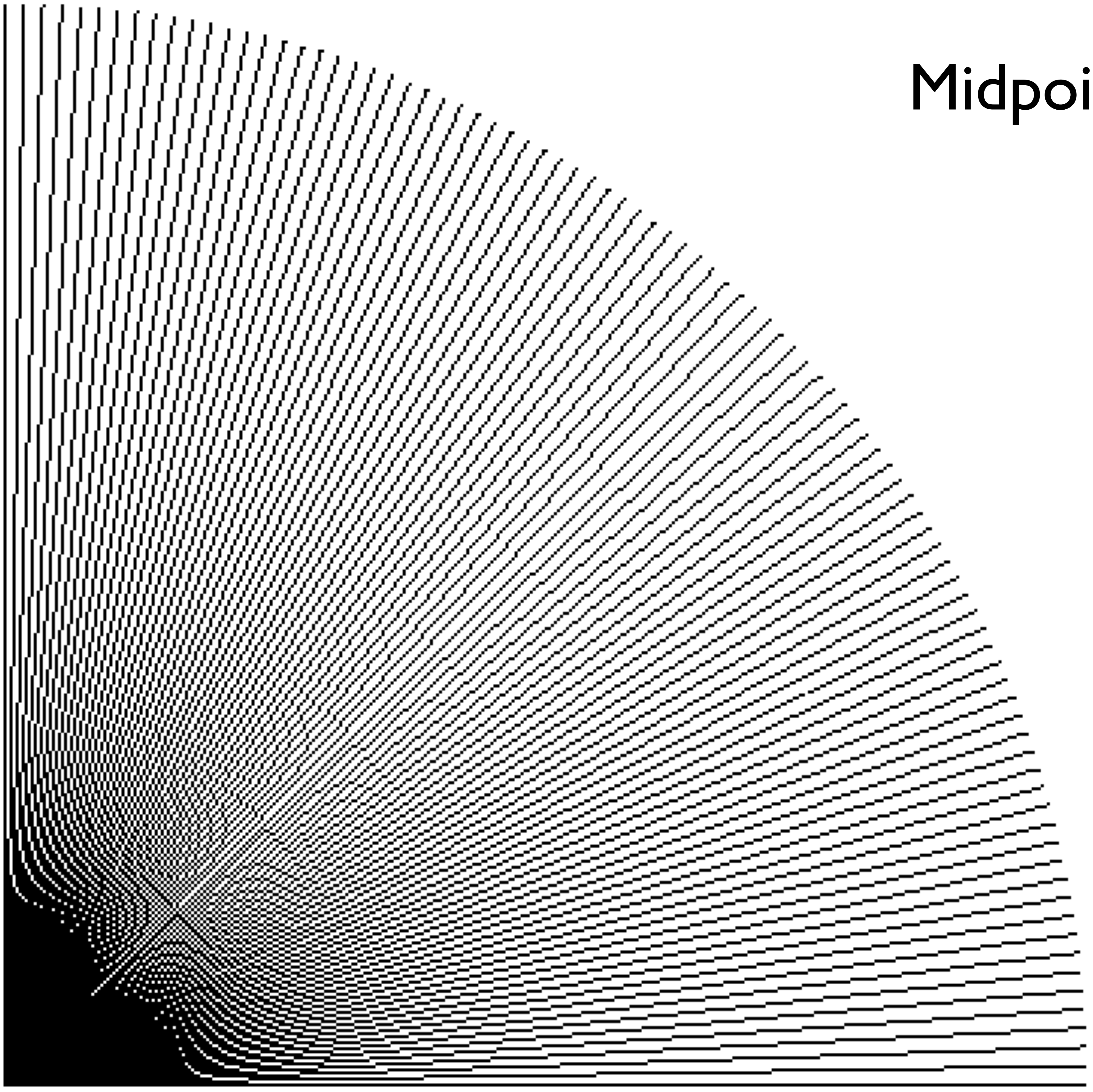
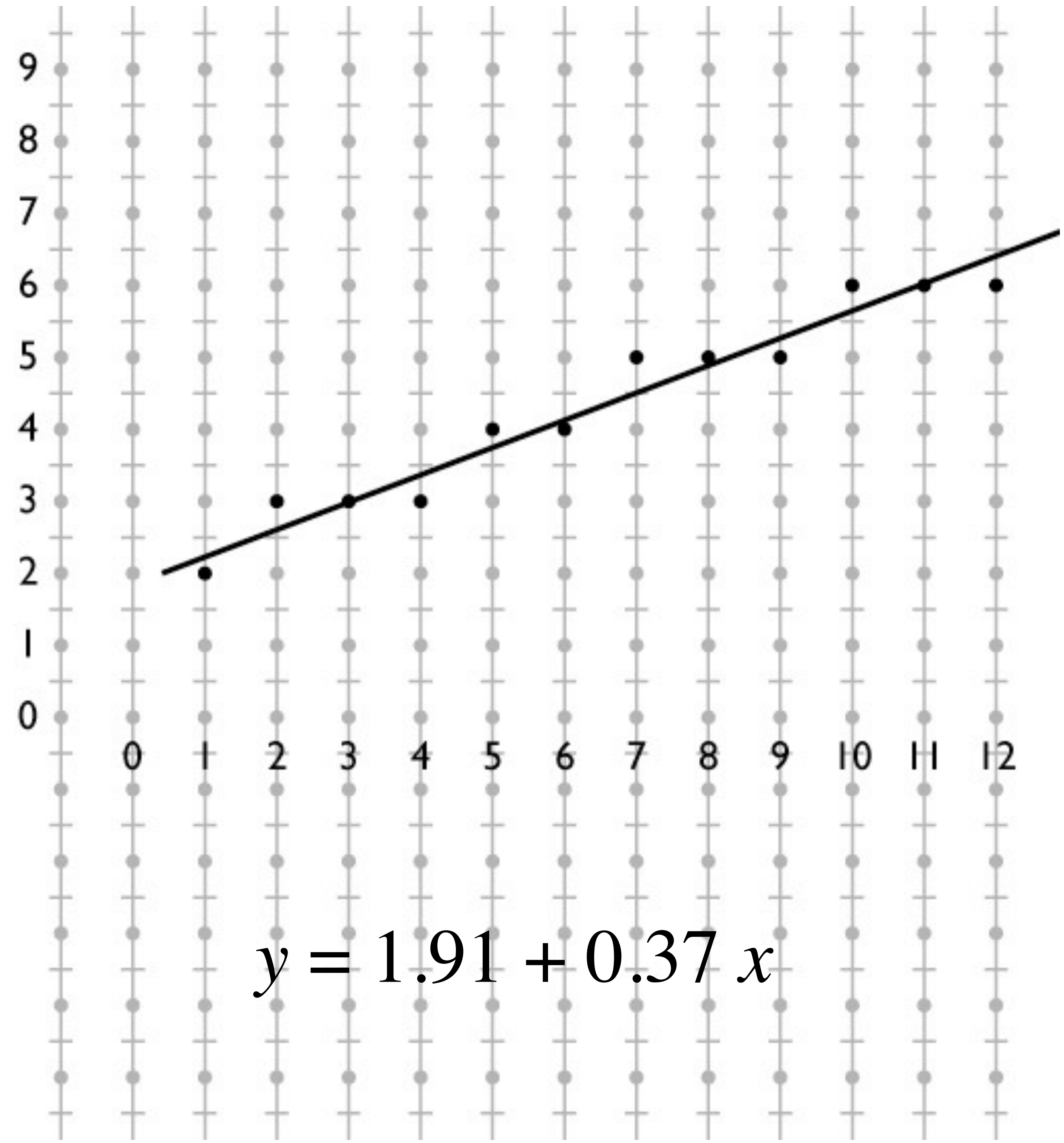


Midpoint algorithm in action



Algorithms for drawing lines

- **line equation:**
  $y = b + m\,x$
- **Simple algorithm: evaluate line equation per column**
- **W.l.o.g.** $x_0 < x_1$;
  $0 \leq m \leq 1$

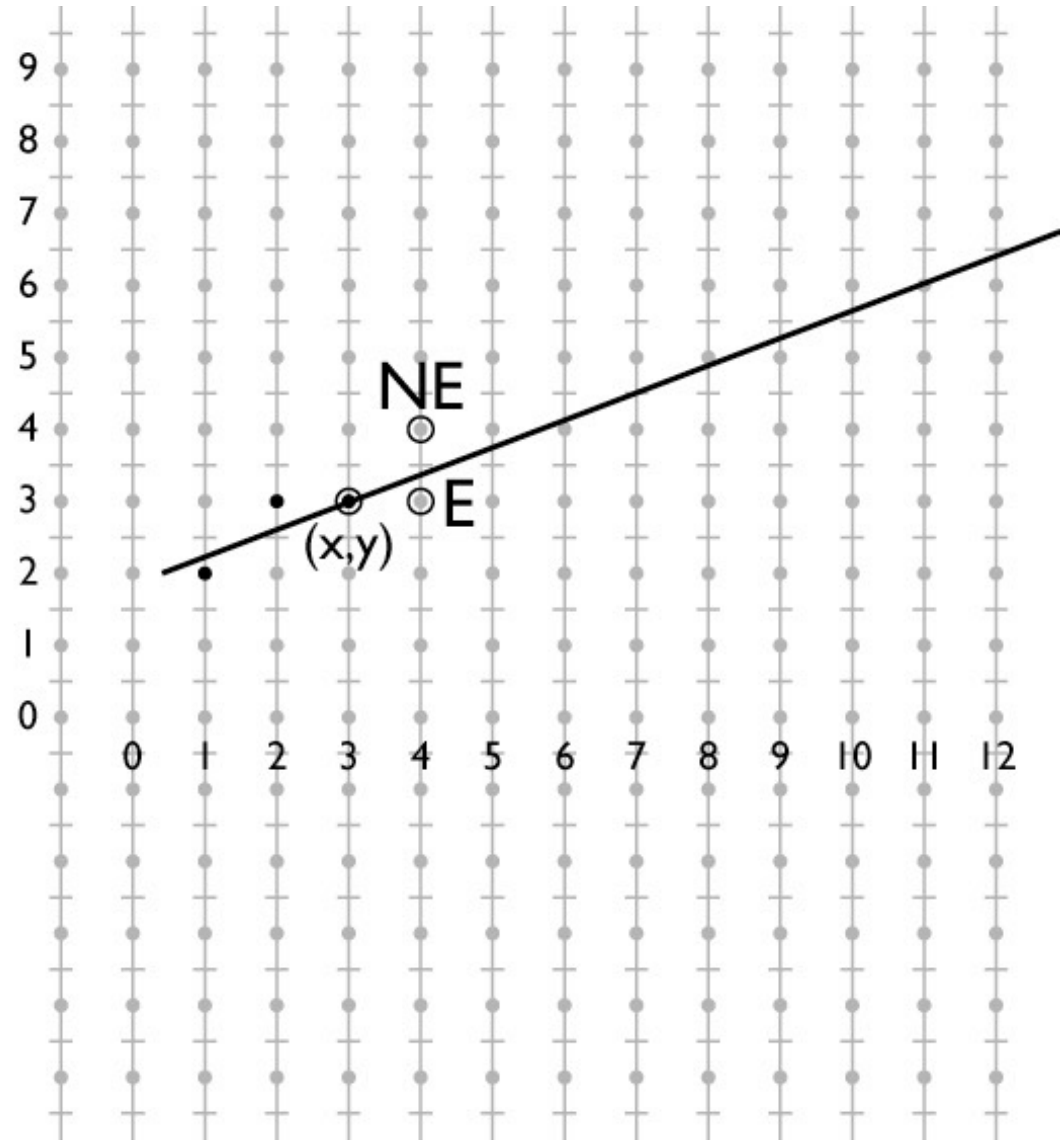



```
for x = ceil(x0) to floor(x1)
    y = b + m*x
    output(x, round(y))
```

$$y = 1.91 + 0.37\,x$$

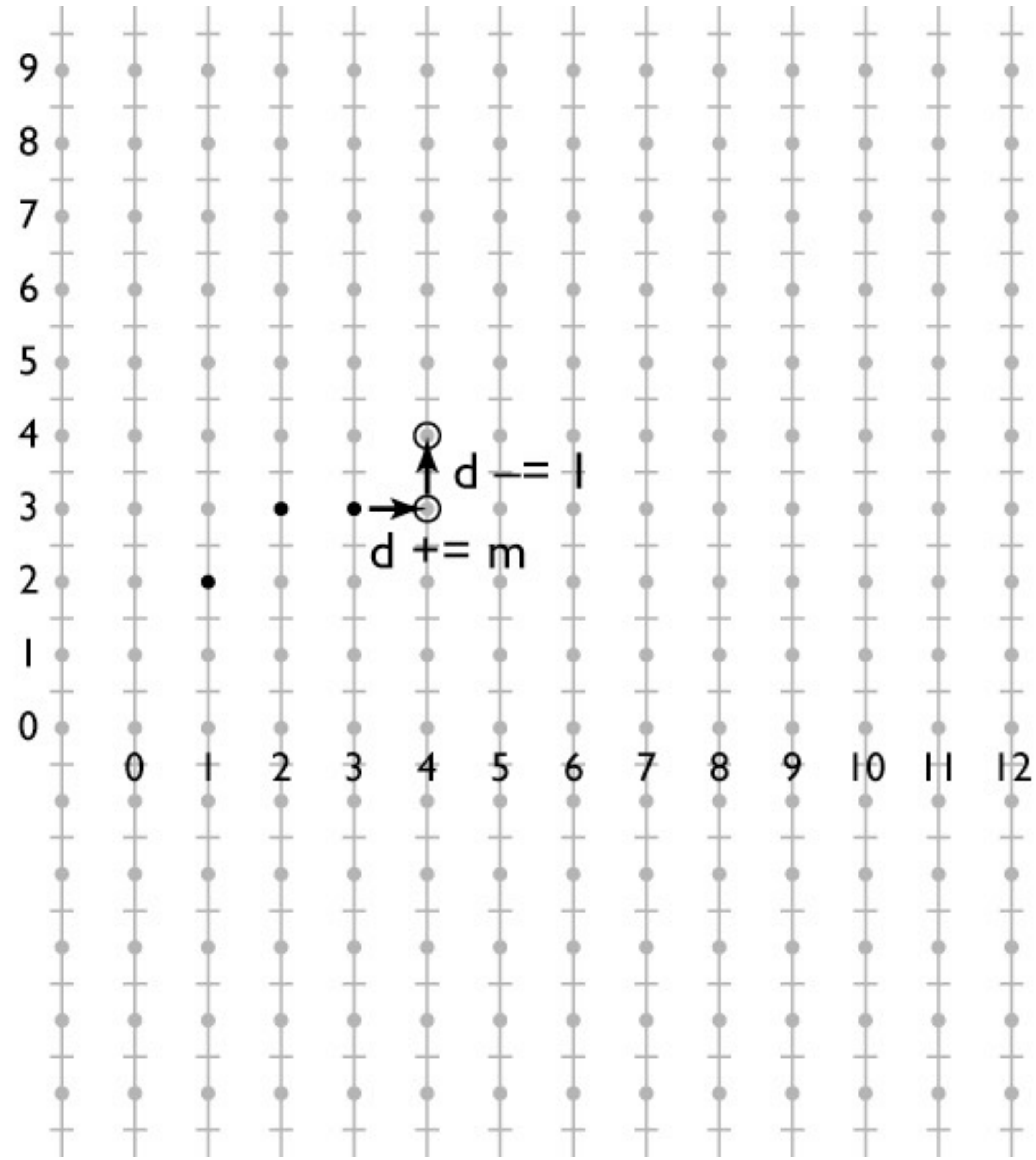
Optimizing line drawing

- **Multiplying and rounding is slow**
- **At each pixel the only options are E and NE**
- $d = m(x + 1) + b - y$
- $d > 0.5$ **decides between E and NE**



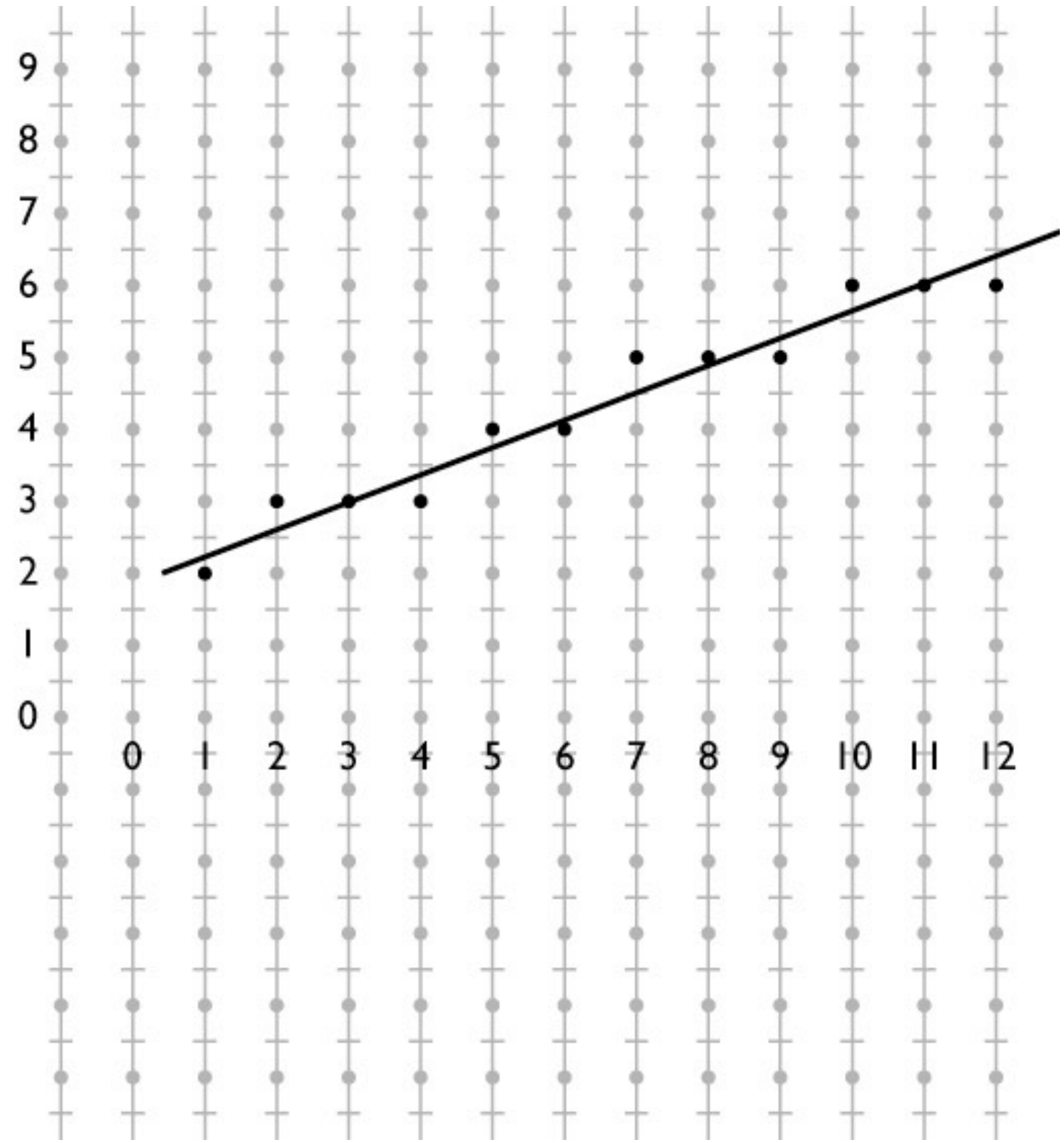
Optimizing line drawing

- $d = m(x + 1) + b - y$
- **Only need to update d for integer steps in x and y**
- **Do that with addition**
- **Known as "DDA" (digital differential analyzer)**



Midpoint line algorithm

```
x = ceil(x0)
y = round(m*x + b)
d = m*(x + 1) + b − y
while x < floor(x1)
    if d > 0.5
        y += 1
        d −= 1
    x += 1
    d += m
    output(x, y)
```

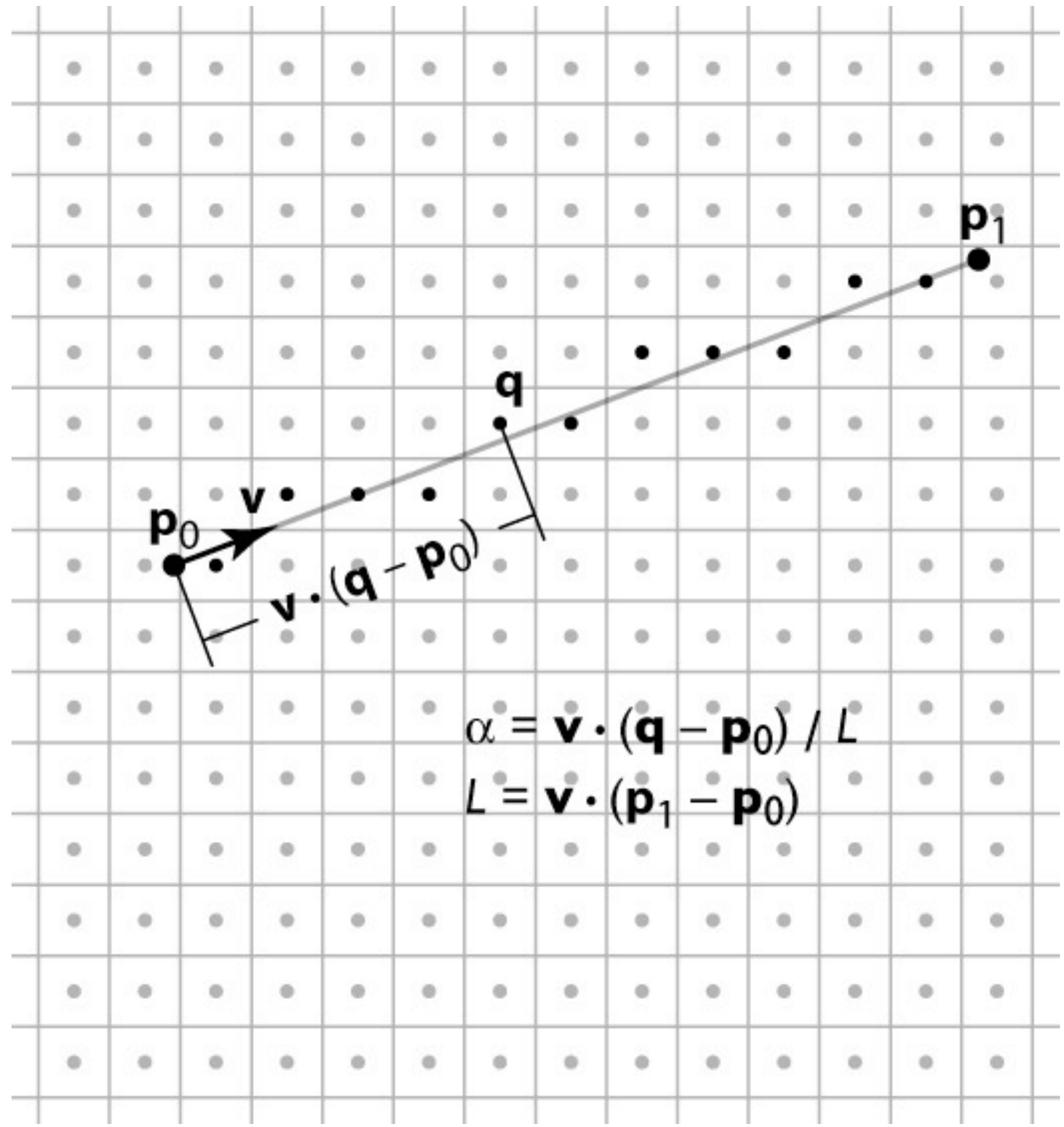


Linear interpolation

- **We often attach attributes to vertices**
  - e.g. computed diffuse color of a hair being drawn using lines
  - want color to vary smoothly along a chain of line segments
- **Recall basic definition**
  - 1D: $f(x) = (1 - \alpha)\, y_0 + \alpha\, y_1$
  - where $\alpha = (x - x_0) / (x_1 - x_0)$
- **In the 2D case of a line segment, alpha is just the fraction of the distance from (x_0, y_0) to (x_1, y_1)**

Linear interpolation

- **Pixels are not exactly on the line**
- **Define 2D function by projection on line**
  - this is linear in 2D
  - therefore can use DDA to interpolate



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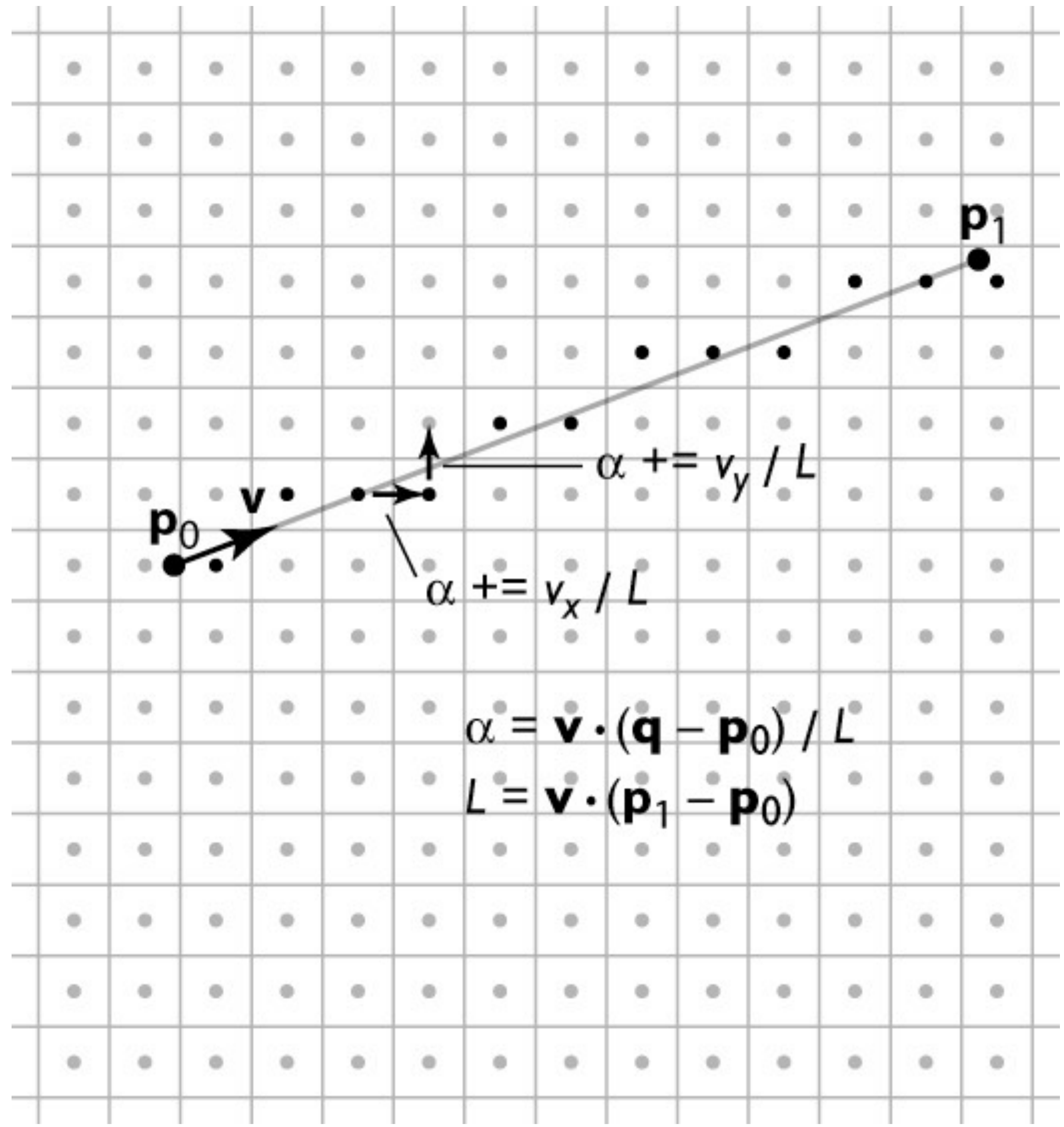

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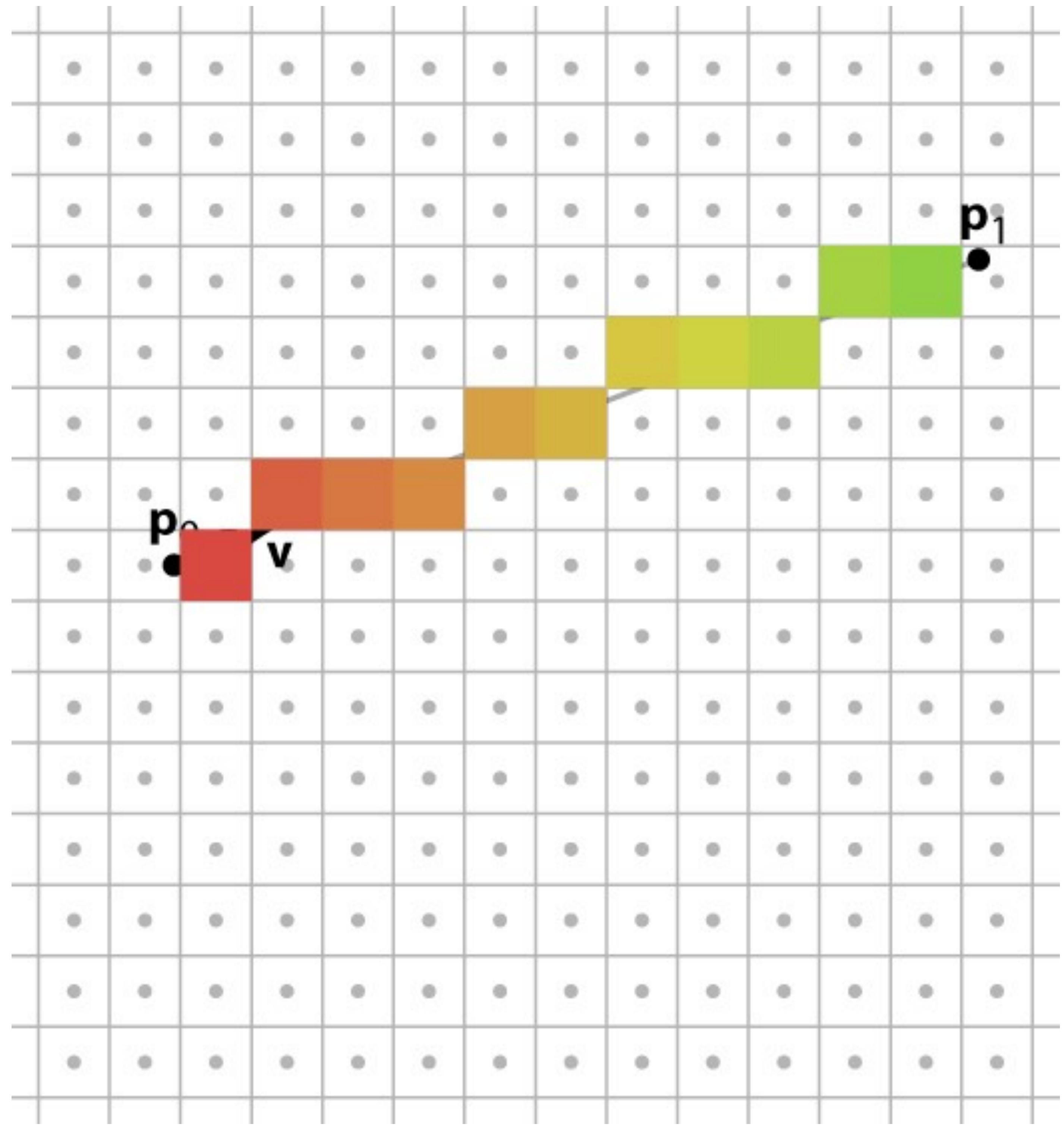


Alternate interpretation

- **We are updating d and α as we step from pixel to pixel**
  - d tells us how far from the line we are
  - α tells us how far along the line we are
- **So d and α are coordinates in a coordinate system oriented to the line**

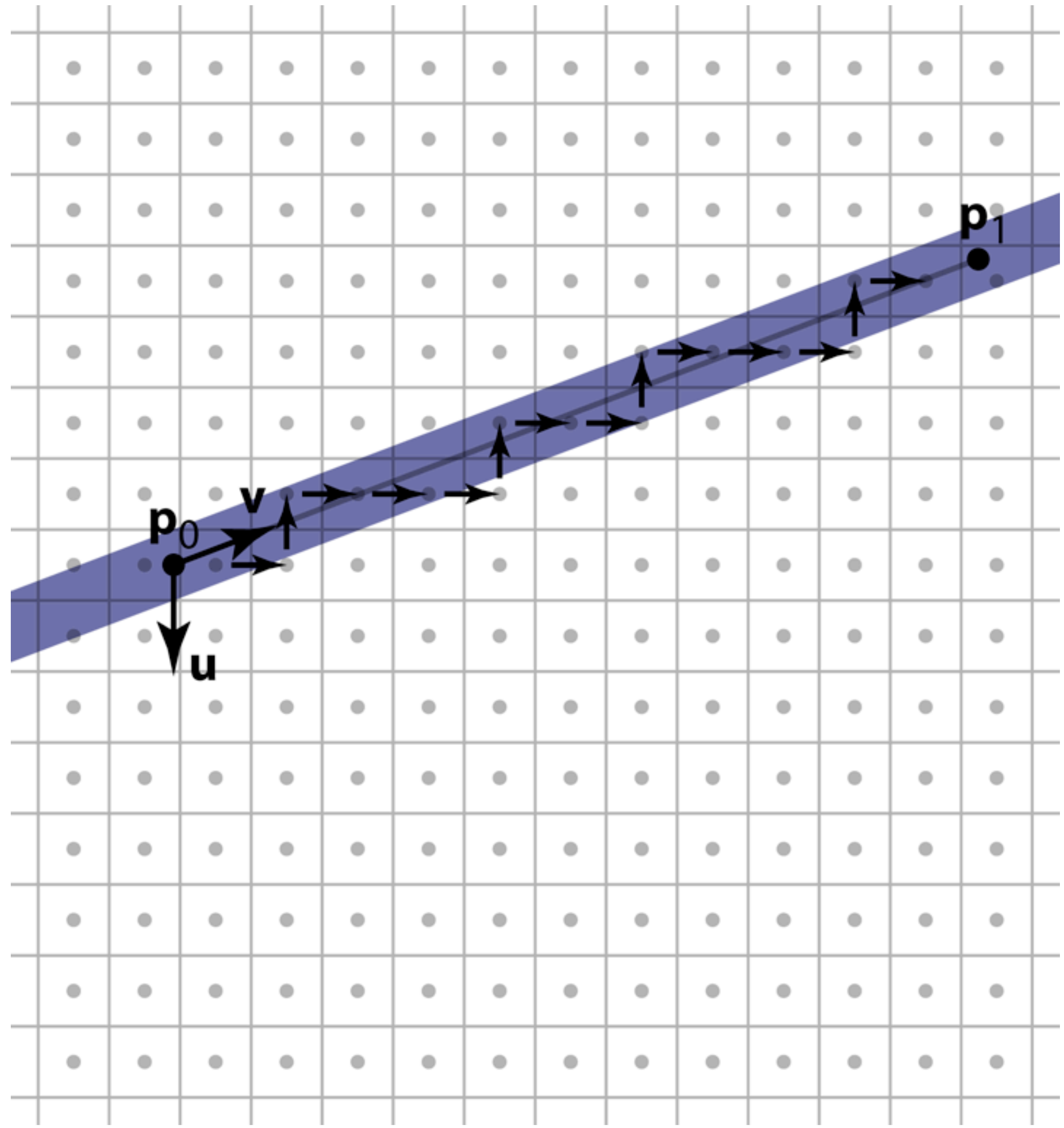
Alternate interpretation

- **View loop as visiting all pixels the line passes through**

  Interpolate d and α for each pixel

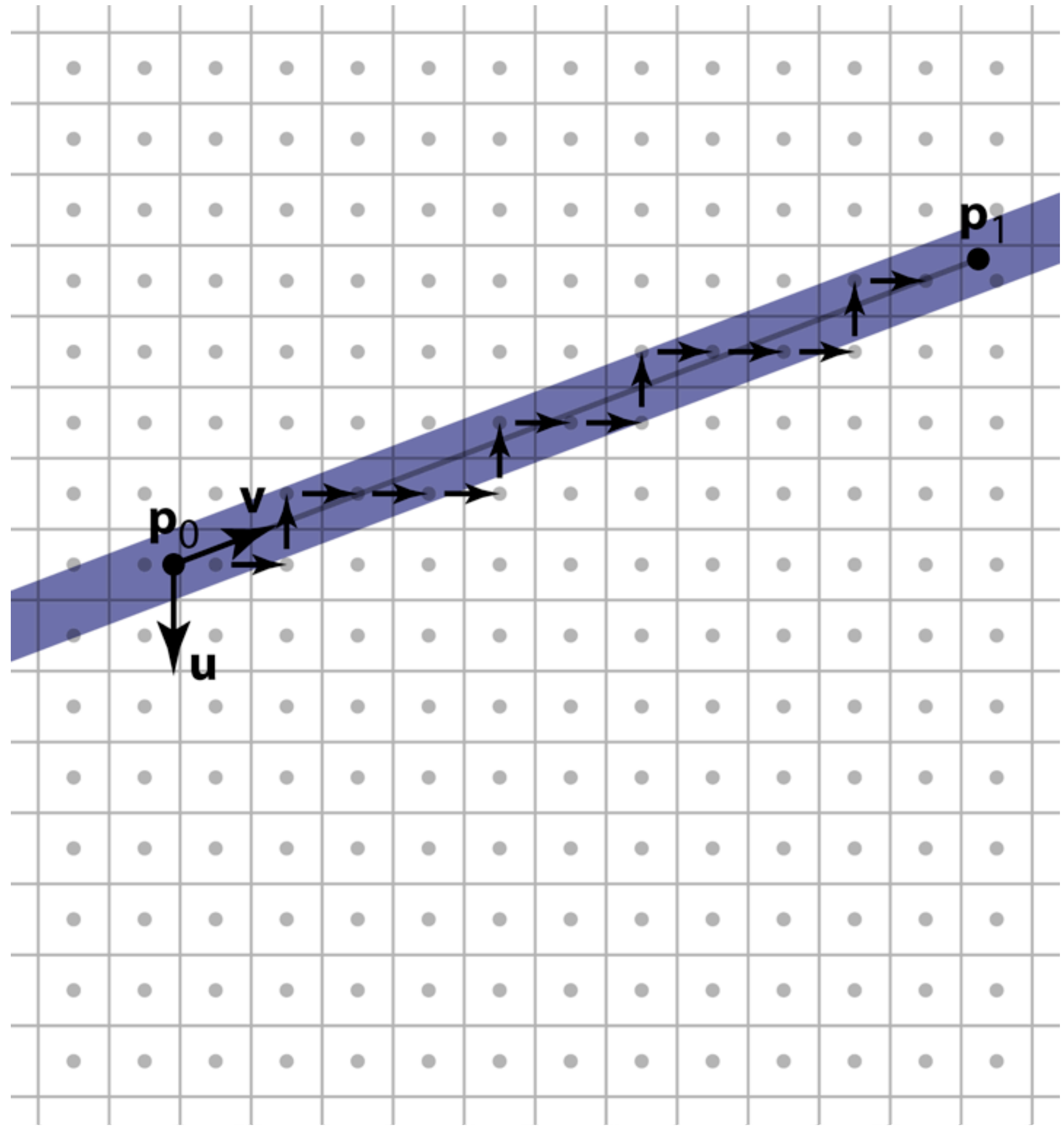
  Only output frag. if pixel is in band

- **This makes linear interpolation the primary operation**



Pixel-walk line rasterization

```
x = ceil(x0)
y = round(m*x + b)
d = m*x + b – y
while x < floor(x1)
    if d > 0.5
        y += 1; d –= 1;
    else
        x += 1; d += m;
    if –0.5 < d ≤ 0.5
        output(x, y)
```



Rasterizing triangles

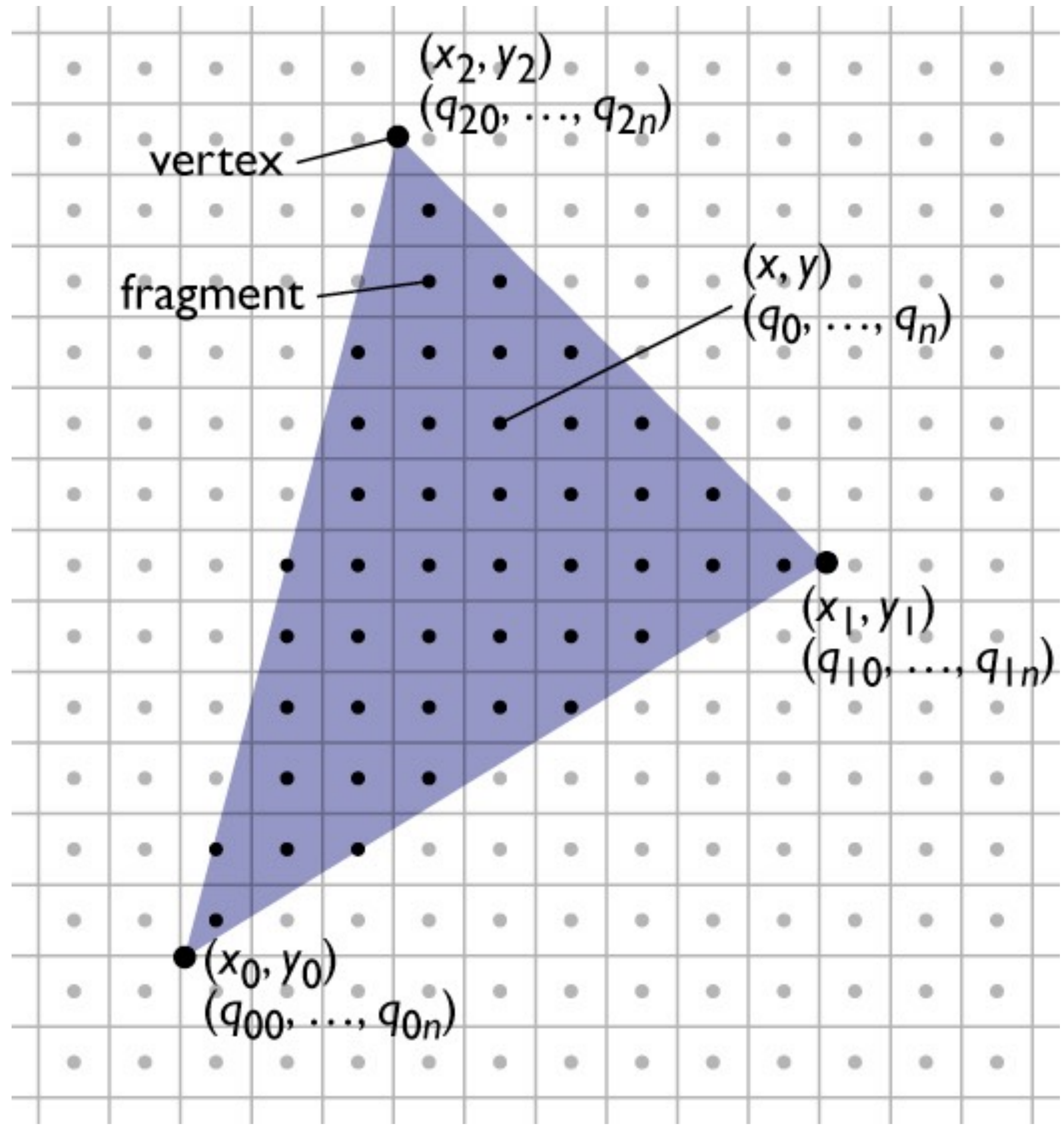
- **The most common case in most applications**
  – with good antialiasing can be the only case
  – some systems render a line as two skinny triangles
- **Triangle represented by three vertices**
- **Simple way to think of algorithm follows the pixel-walk interpretation of line rasterization**
  – walk from pixel to pixel over (at least) the polygon's area
  – evaluate linear functions as you go
  – use those functions to decide which pixels are inside

Rasterizing triangles

- **Input:**
  - three 2D points (the triangle's vertices in pixel space)
    - $(x_0, y_0); (x_1, y_1); (x_2, y_2)$
  - parameter values at each vertex
    - $q_{00}, \ldots, q_{0n}; q_{10}, \ldots, q_{1n}; q_{20}, \ldots, q_{2n}$
- **Output: a list of fragments, each with**
  - the integer pixel coordinates (x, y)
  - interpolated parameter values $q_0, \ldots, q_n$

Rasterizing triangles

- **Summary**
  1. evaluation of linear functions on pixel grid
  2. functions defined by parameter values at vertices
  3. using extra parameters to determine fragment set



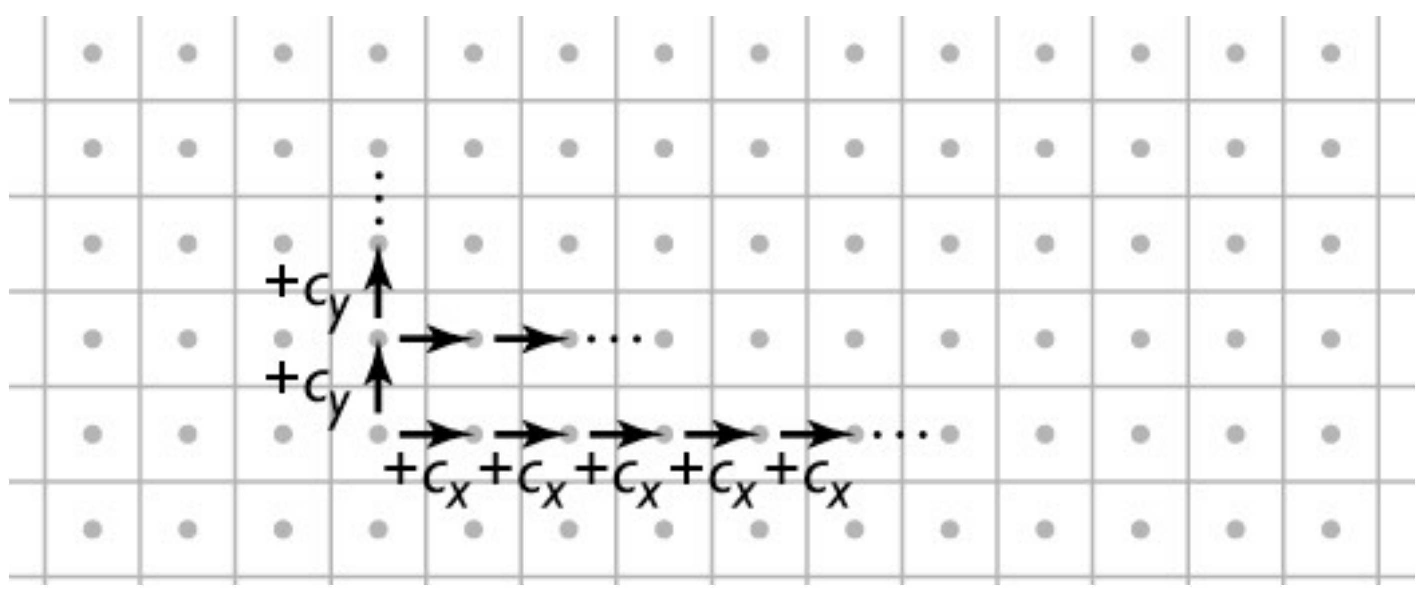
Incremental linear evaluation

- **A linear (affine, really) function on the plane is:**

$$q(x, y) = c_x x + c_y y + c_k$$

- **Linear functions are efficient to evaluate on a grid:**

$$q(x+1, y) = c_x(x+1) + c_y y + c_k = q(x, y) + c_x$$
$$q(x, y+1) = c_x x + c_y(y+1) + c_k = q(x, y) + c_y$$

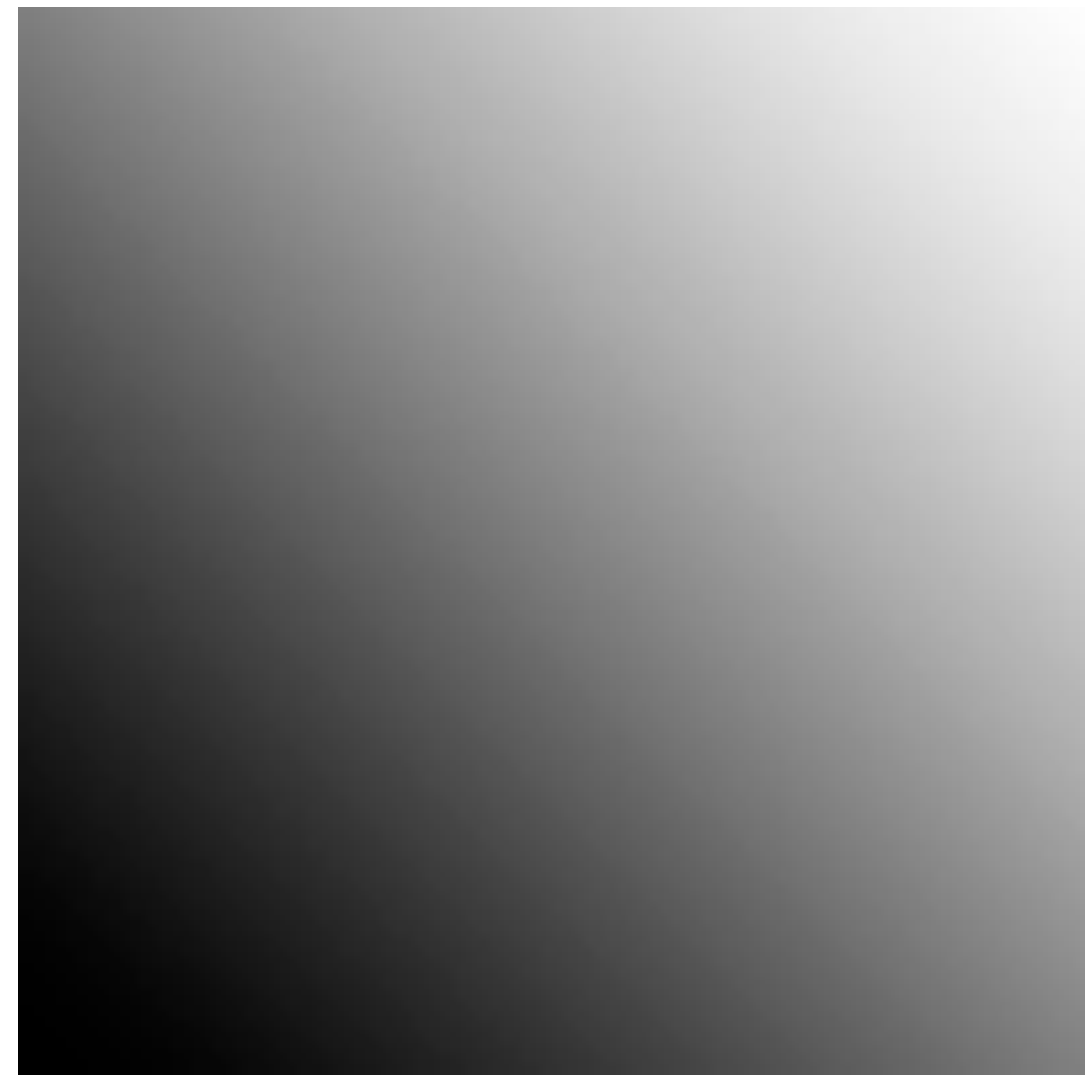


Incremental linear evaluation

```
linEval(xm, xM, ym, yM, cx, cy, ck) {

   // setup
   qRow = cx*xm + cy*ym + ck;

   // traversal
   for y = ym to yM {
      qPix = qRow;
      for x = xm to xM {
         output(x, y, qPix);
         qPix += cx;
      }
      qRow += cy;
   }
}
```



$c_x = .005; c_y = .005; c_k = 0$

(image size 100x100)

Rasterizing triangles

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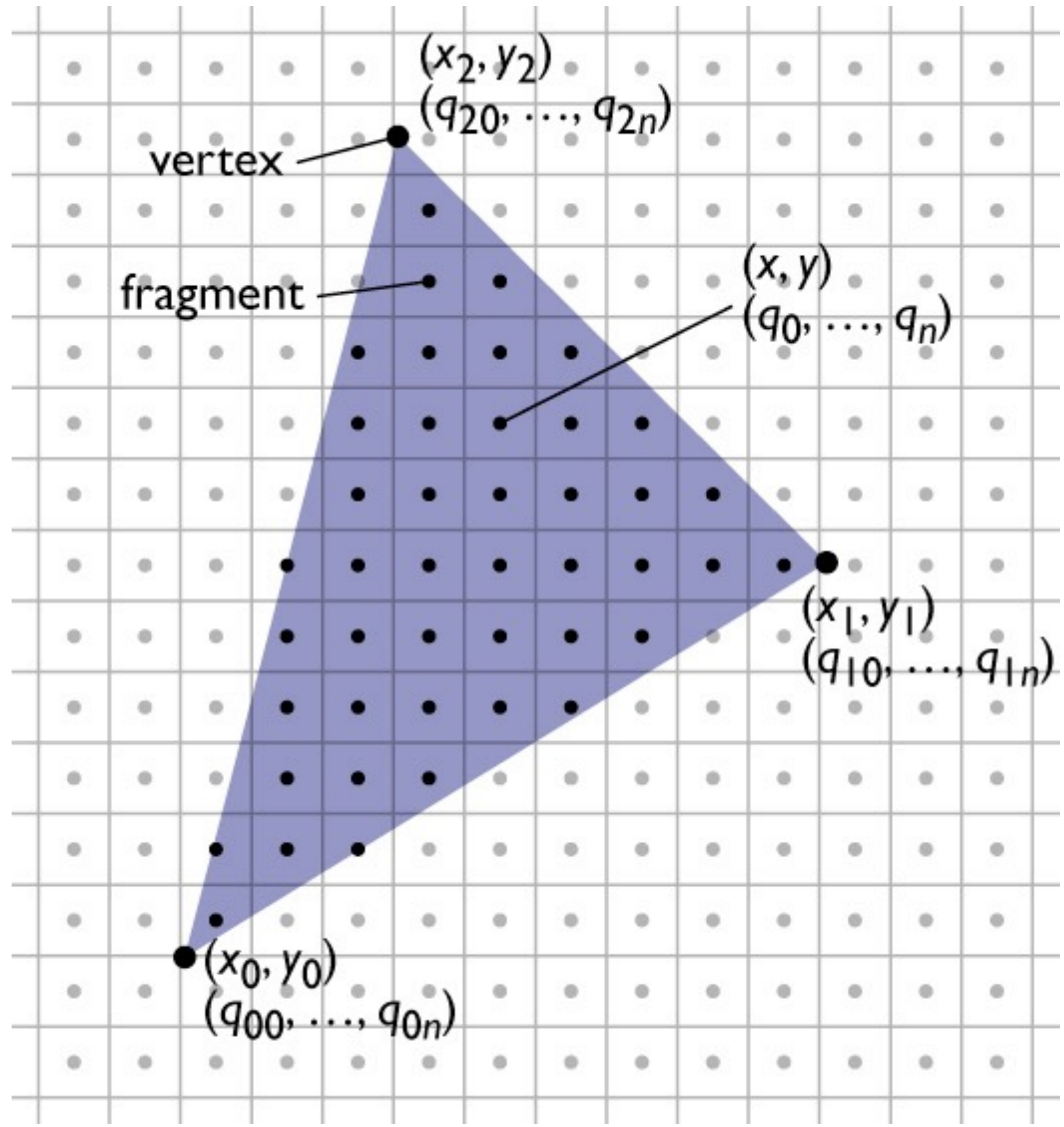


Defining parameter functions

- **To interpolate parameters across a triangle we need to find the c_x, c_y, and c_k that define the (unique) linear function that matches the given values at all 3 vertices**
  - this is 3 constraints on 3 unknown coefficients:

$$c_x x_0 + c_y y_0 + c_k = q_0$$
$$c_x x_1 + c_y y_1 + c_k = q_1$$
$$c_x x_2 + c_y y_2 + c_k = q_2$$

(each states that the function agrees with the given value at one vertex)

  - leading to a 3x3 matrix equation for the coefficients:

$$\begin{bmatrix} x_0 & y_0 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} c_x \\ c_y \\ c_k \end{bmatrix} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix}$$

(singular iff triangle is degenerate)

Defining parameter functions

- **More efficient version: shift origin to (x_0, y_0)**

$$q(x, y) = c_x(x - x_0) + c_y(y - y_0) + q_0$$
$$q(x_1, y_1) = c_x(x_1 - x_0) + c_y(y_1 - y_0) + q_0 = q_1$$
$$q(x_2, y_2) = c_x(x_2 - x_0) + c_y(y_2 - y_0) + q_0 = q_2$$

  – now this is a 2x2 linear system (since q_0 falls out):

$$\begin{bmatrix} (x_1 - x_0) & (y_1 - y_0) \\ (x_2 - x_0) & (y_2 - y_0) \end{bmatrix} \begin{bmatrix} c_x \\ c_y \end{bmatrix} = \begin{bmatrix} q_1 - q_0 \\ q_2 - q_0 \end{bmatrix}$$

  – solve using Cramer's rule (see Shirley):

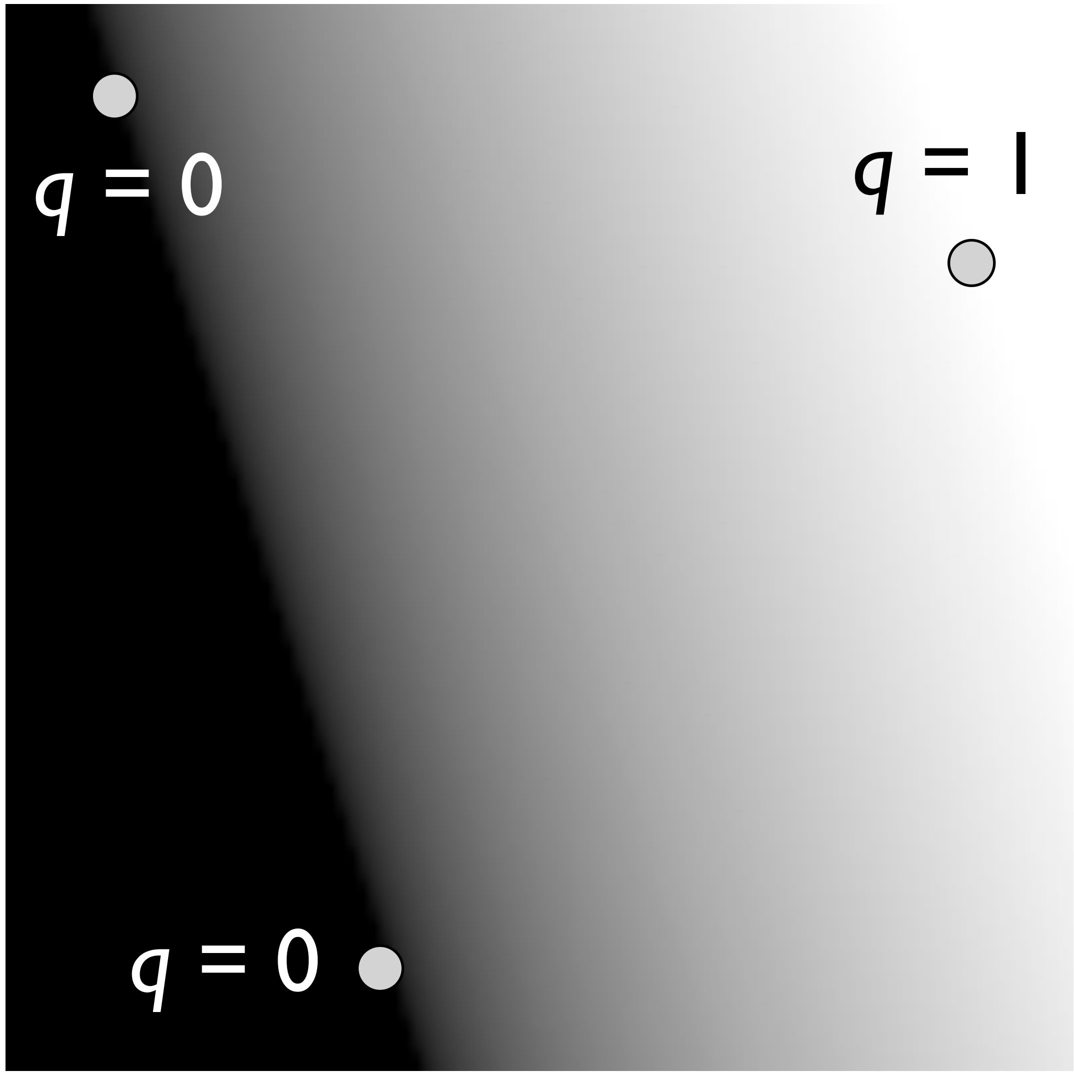
$$c_x = (\Delta q_1 \Delta y_2 - \Delta q_2 \Delta y_1)/(\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1)$$
$$c_y = (\Delta q_2 \Delta x_1 - \Delta q_1 \Delta x_2)/(\Delta x_1 \Delta y_2 - \Delta x_2 \Delta y_1)$$

Defining parameter functions

```
linInterp(xm, xM, ym, yM, x0, y0, q0,
        x1, y1, q1, x2, y2, q2) {

   // setup
   det = (x1–x0)*(y2–y0) – (x2–x0)*(y1–y0);
   cx = ((q1–q0)*(y2–y0) – (q2–q0)*(y1–y0)) / det;
   cy = ((q2–q0)*(x1–x0) – (q1–q0)*(x2–x0)) / det;
   qRow = cx*(xm–x0) + cy*(ym–y0) + q0;

   // traversal (same as before)
   for y = ym to yM {
      qPix = qRow;
      for x = xm to xM {
         output(x, y, qPix);
         qPix += cx;
      }
      qRow += cy;
   }
}
```

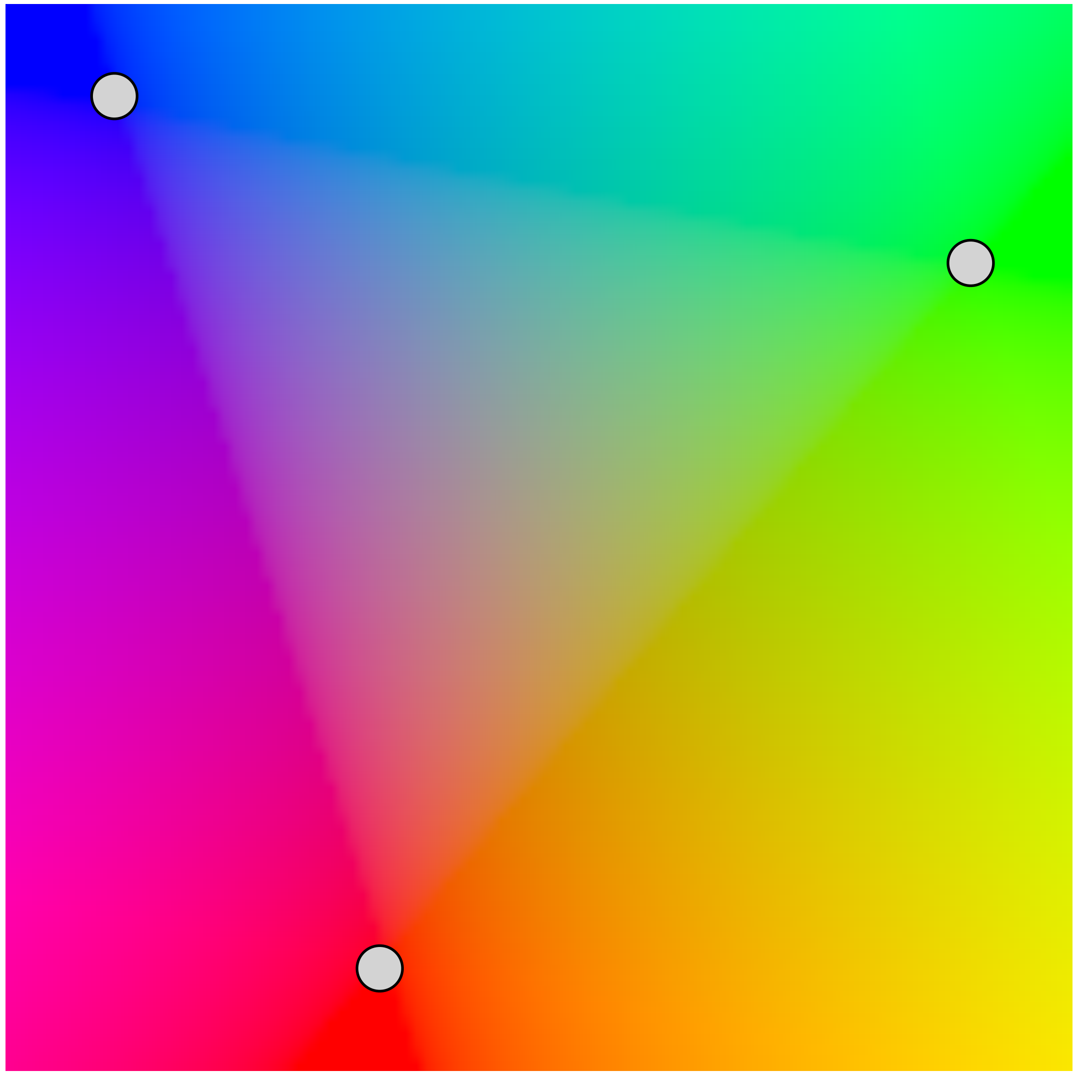


Interpolating several parameters

```
linInterp(xm, xM, ym, yM, n, x0, y0, q0[],
        x1, y1, q1[], x2, y2, q2[]) {

   // setup
   for k = 0 to n−1
      // compute cx[k], cy[k], qRow[k]
      // from q0[k], q1[k], q2[k]

   // traversal
   for y = ym to yM {
      for k = 1 to n, qPix[k] = qRow[k];
      for x = xm to xM {
         output(x, y, qPix);
         for k = 1 to n, qPix[k] += cx[k];
      }
      for k = 1 to n, qRow[k] += cy[k];
   }
}
```



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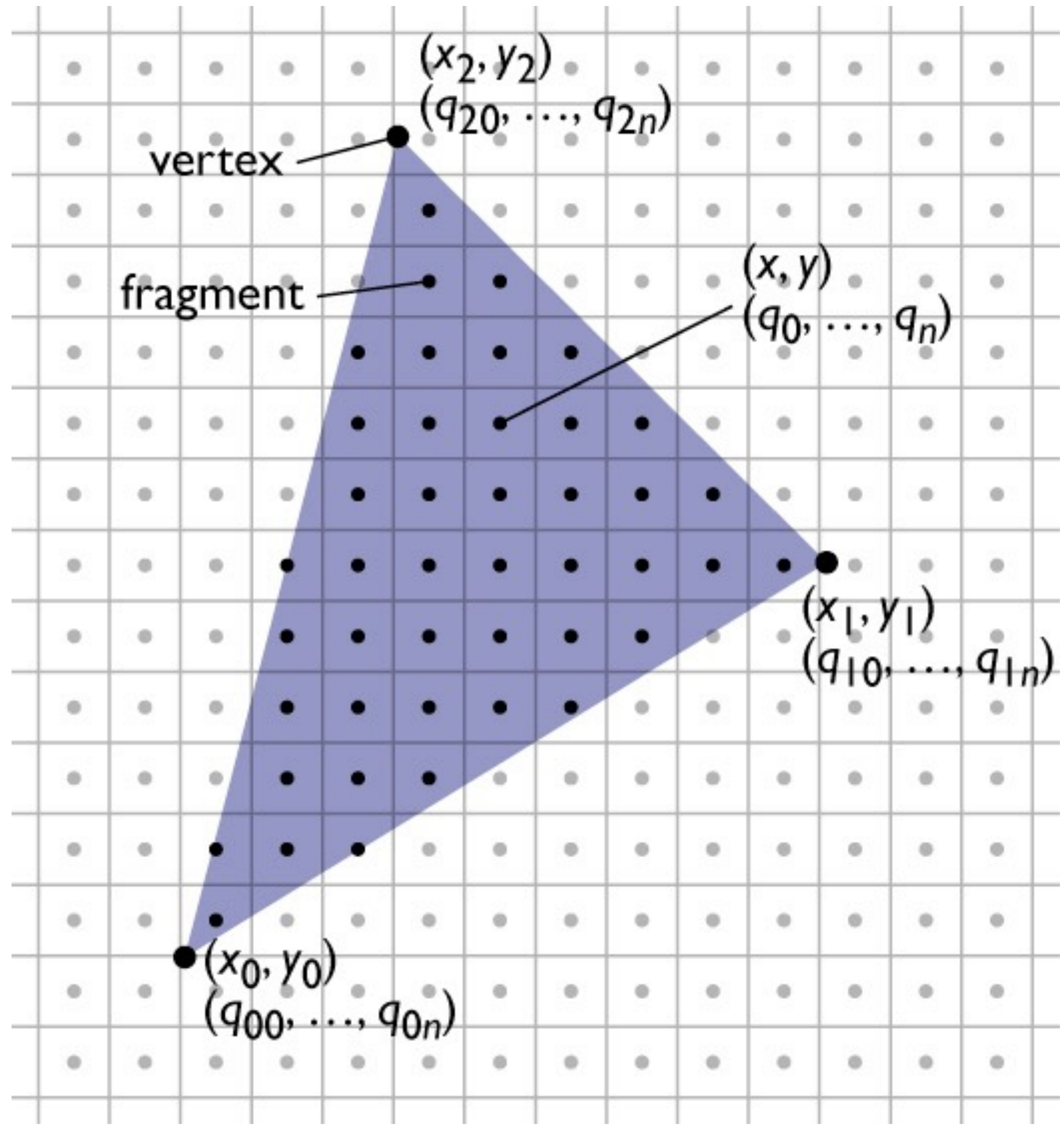
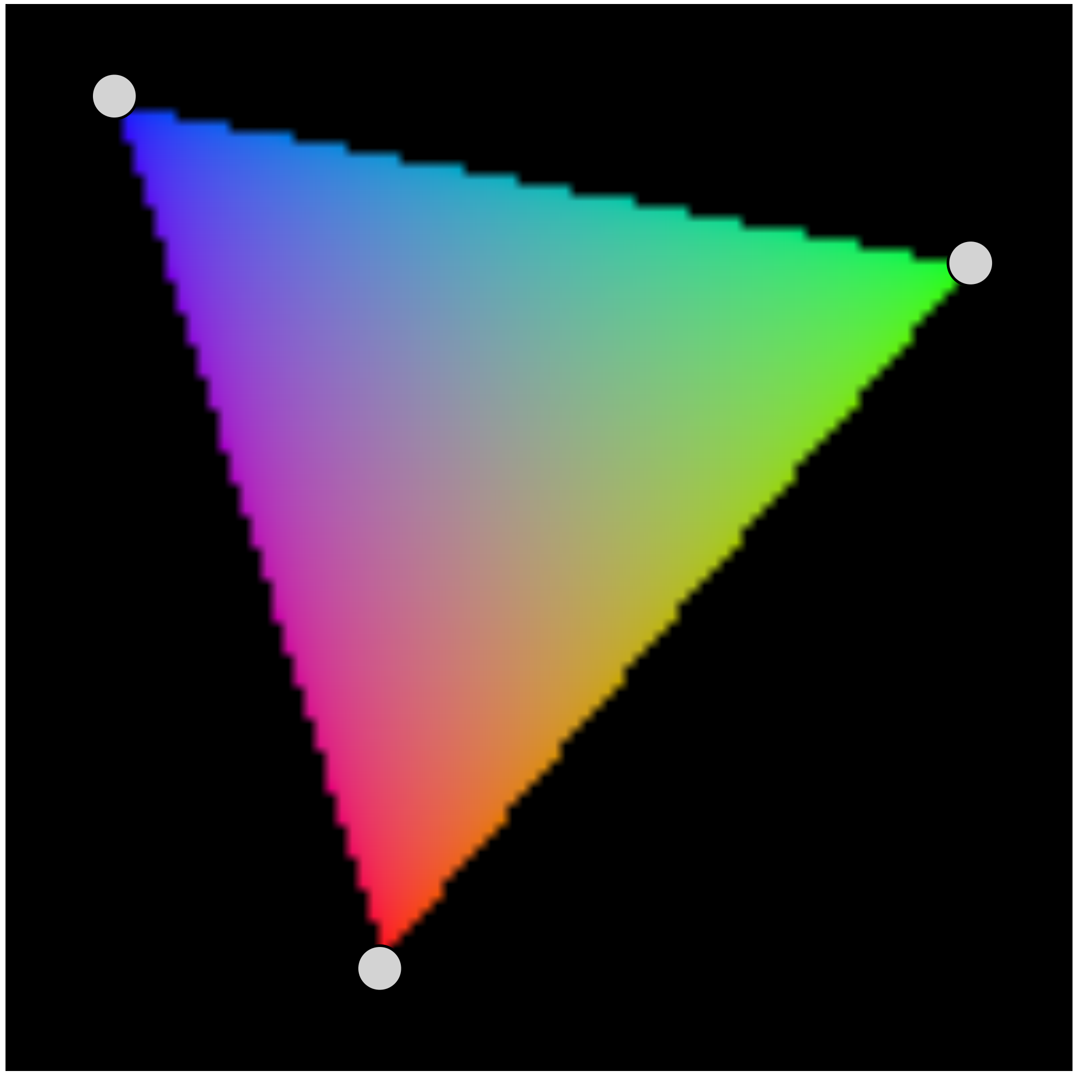


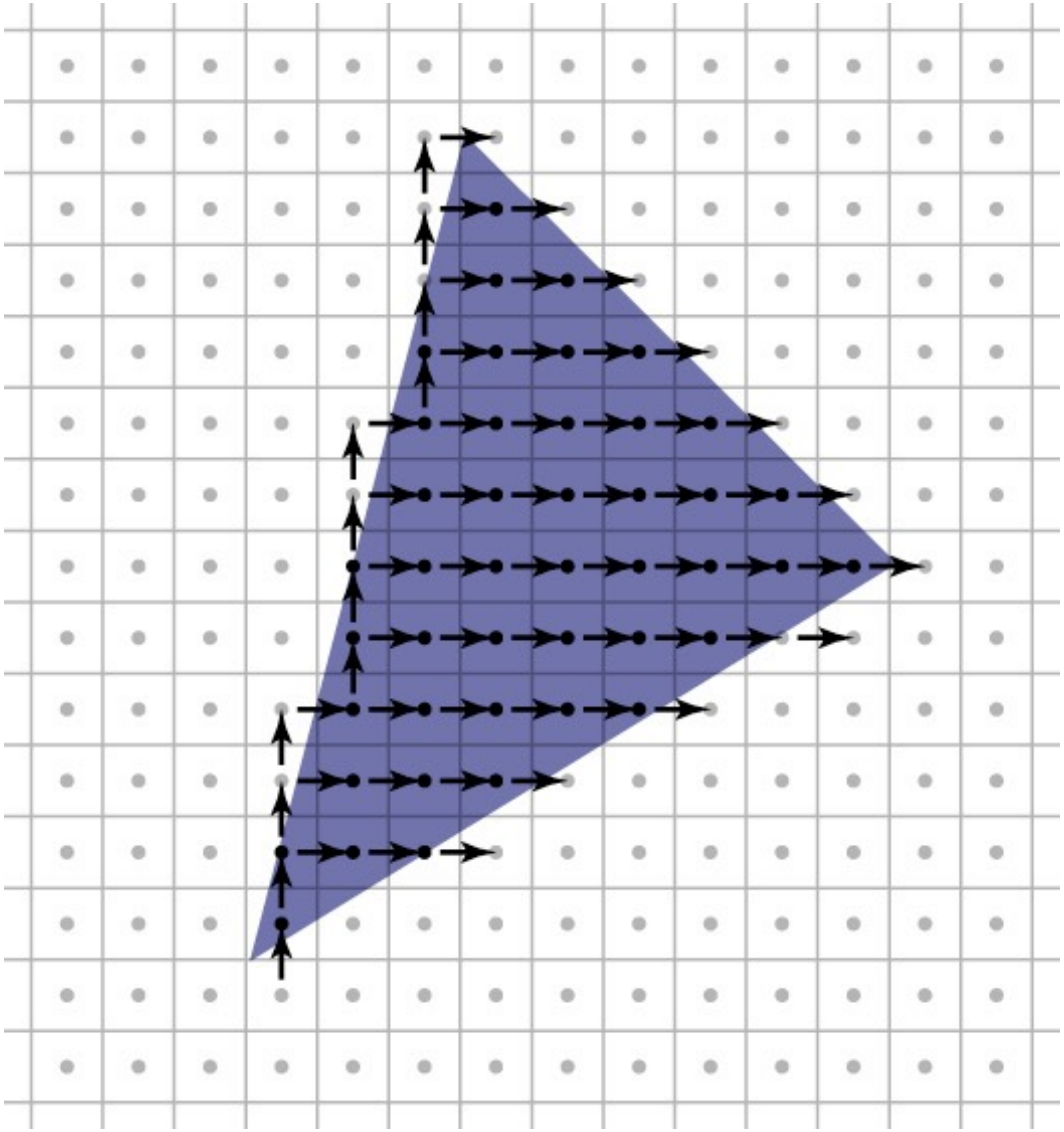
Clipping to the triangle

- **Interpolate three barycentric coordinates across the plane**
  - recall each barycentric coord is 1 at one vert. and 0 at the other two
- **Output fragments only when all three are > 0.**



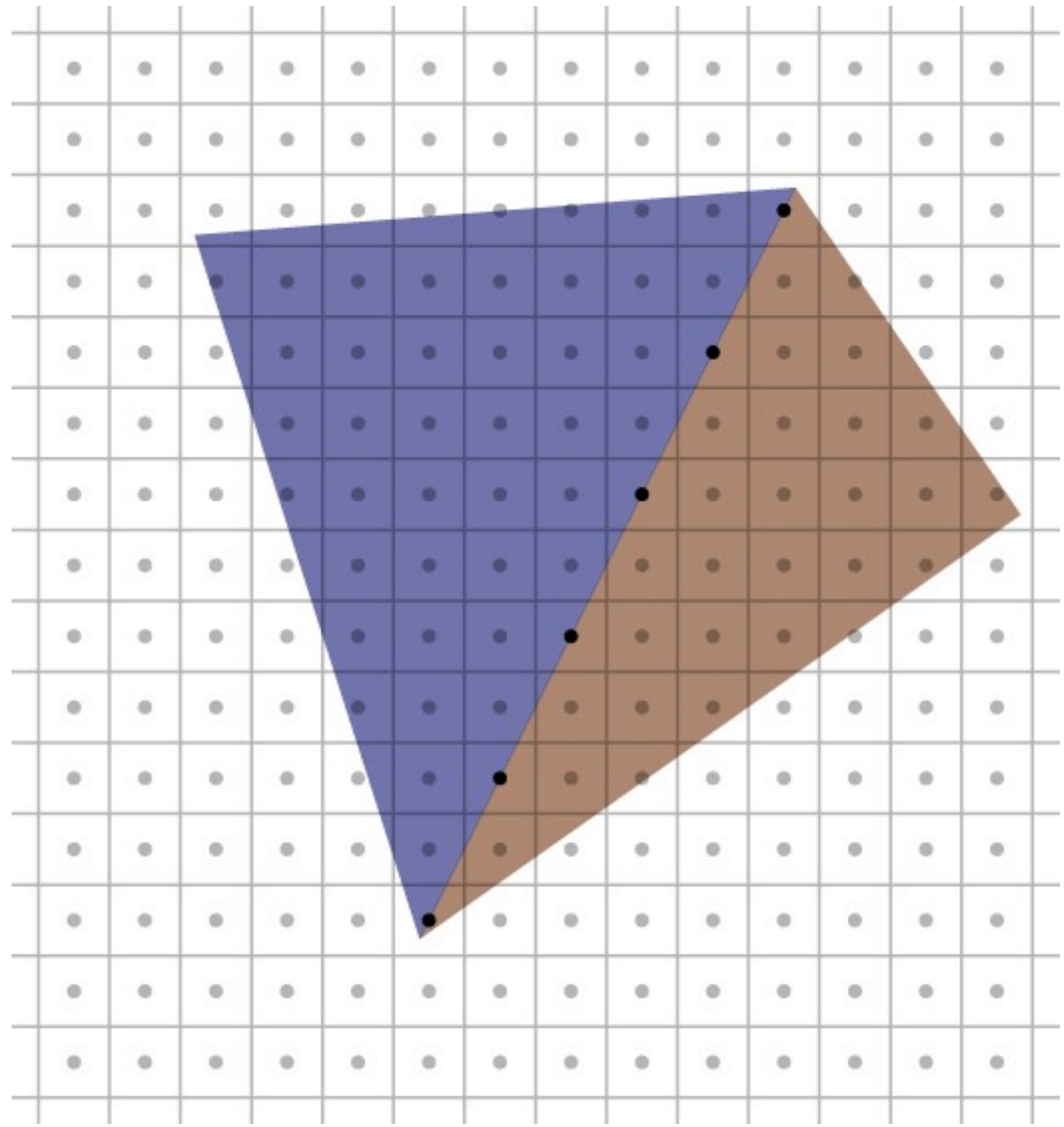
Pixel-walk (Pineda) rasterization

- **Conservatively visit a superset of the pixels you want**
- **Interpolate linear functions**
- **Use those functions to determine when to emit a fragment**



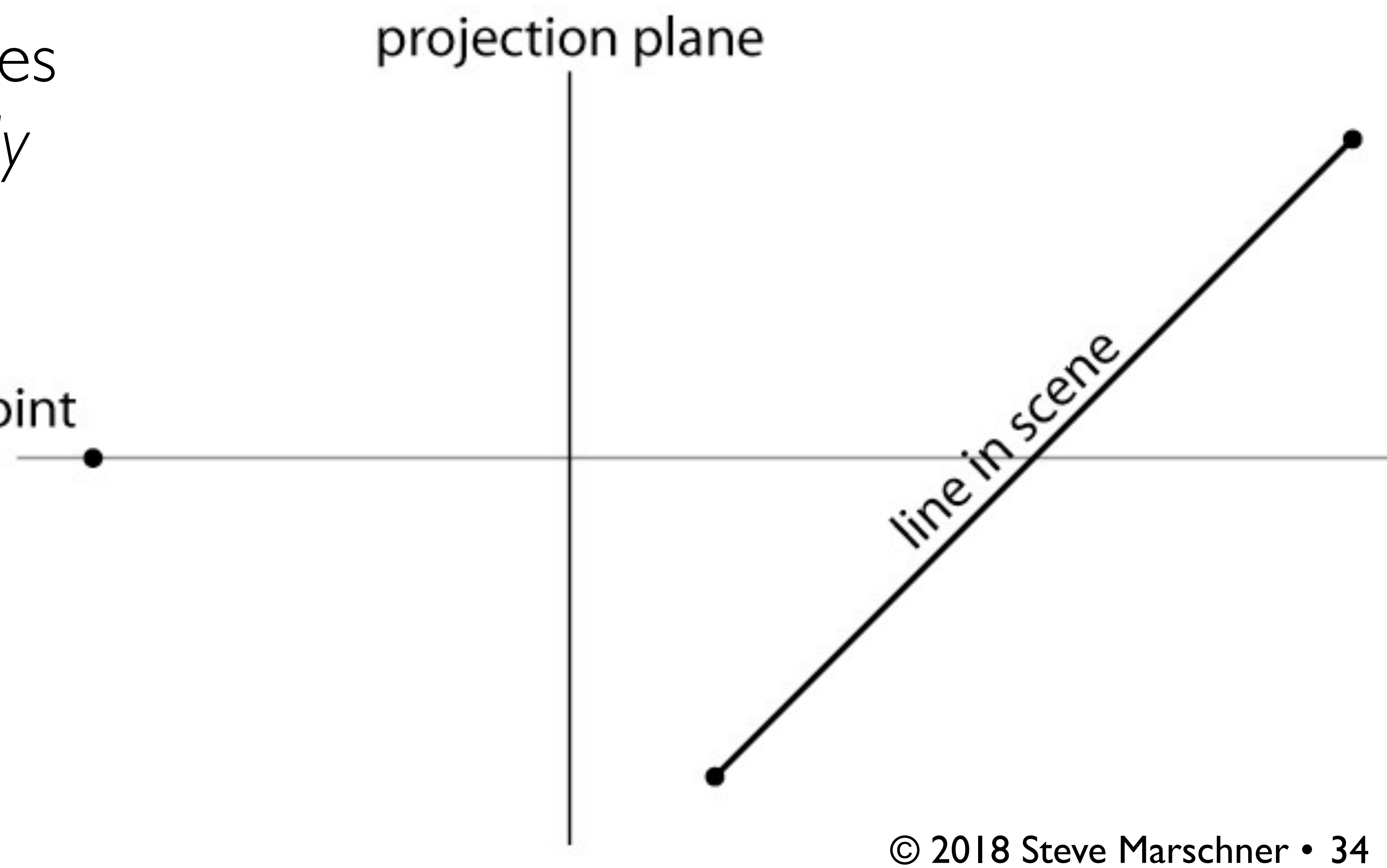
Rasterizing triangles

- **Exercise caution with rounding and arbitrary decisions**
  – need to visit these pixels once
  – but it's important not to visit them twice!



Perspective and interpolation

- **interpolating values in screen space is not the whole story**
  - often we are interpolating values that are supposed to vary linearly in the scene
  - because perspective projection does not preserve ratios of lengths, these values *should not vary linearly in screen space*



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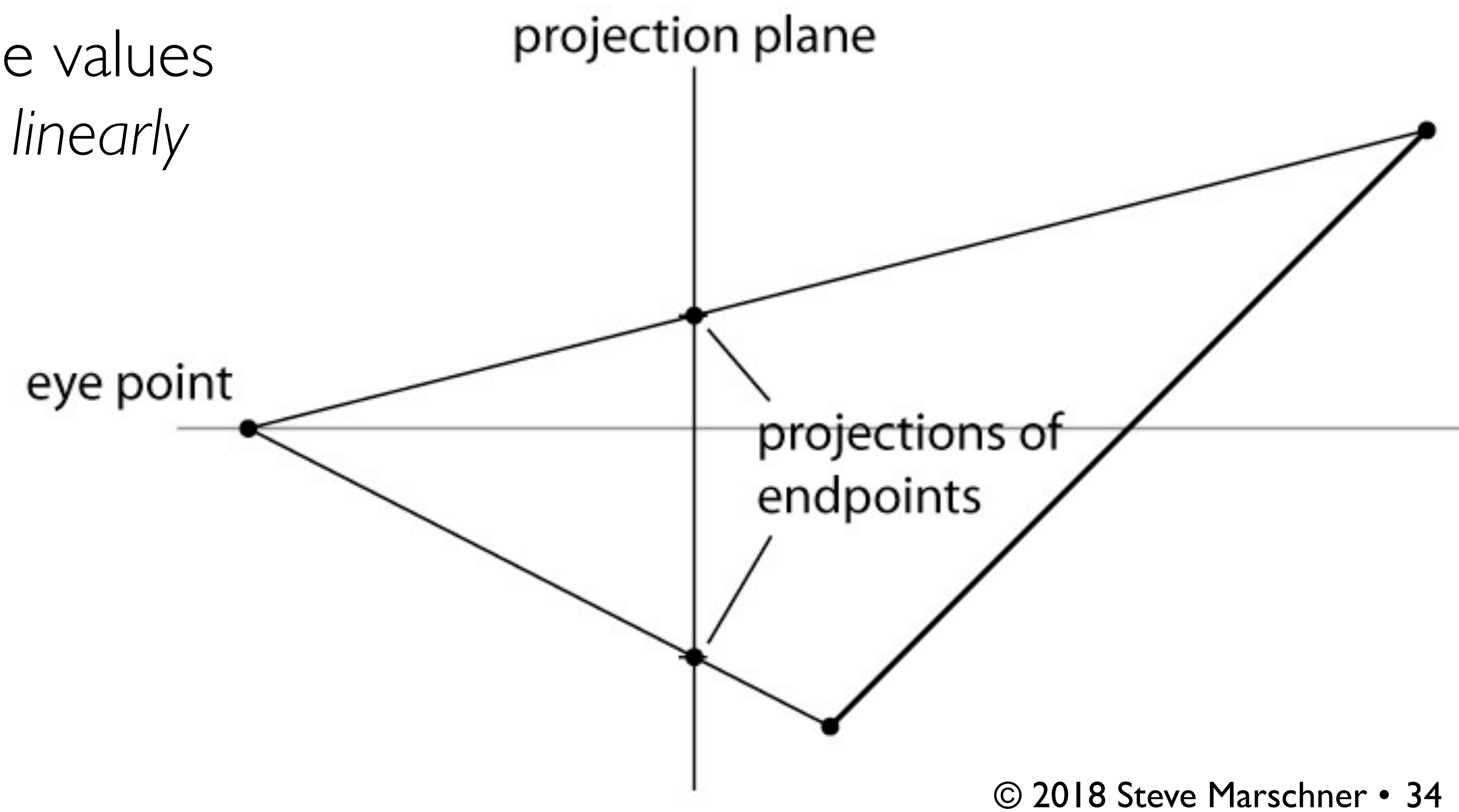

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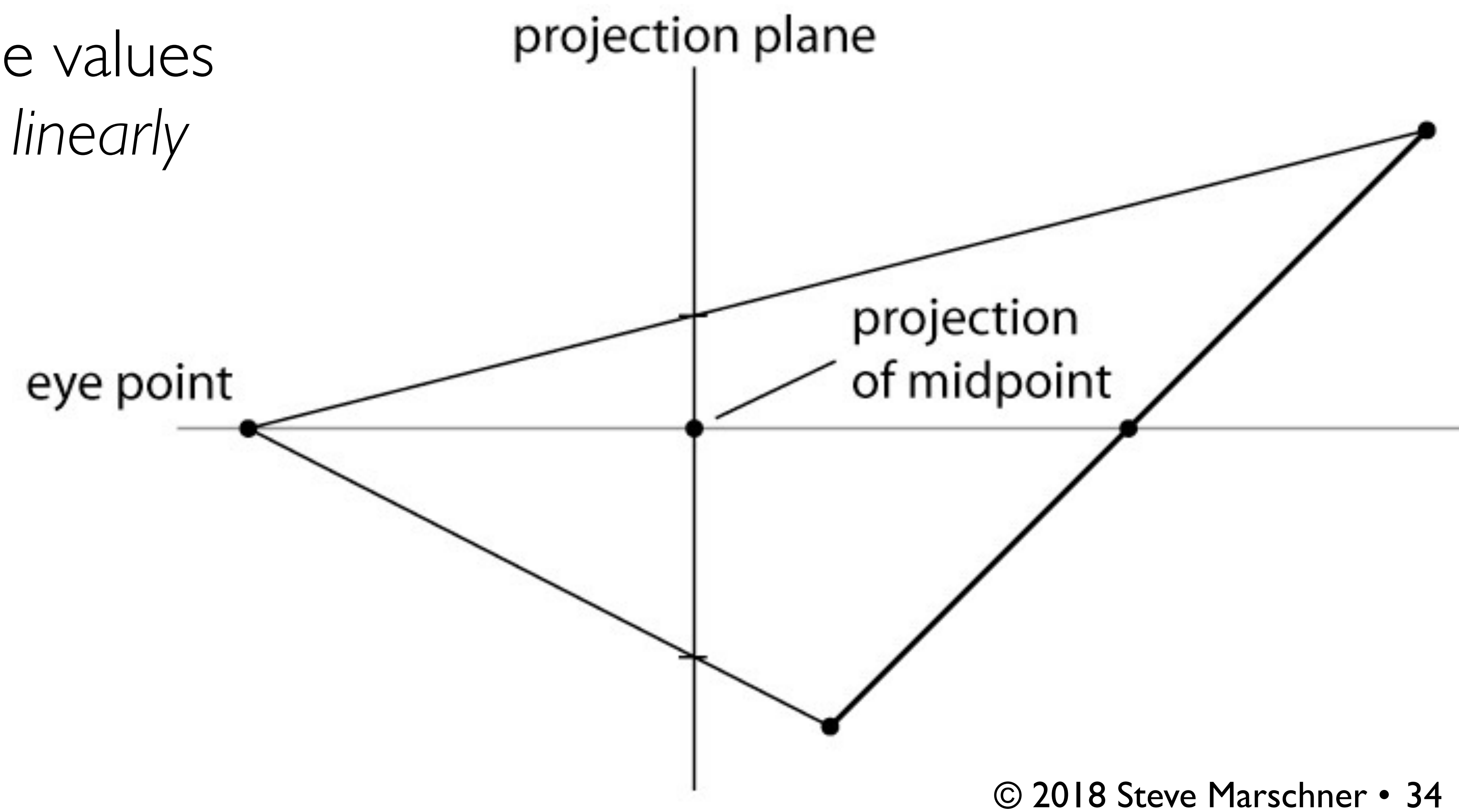
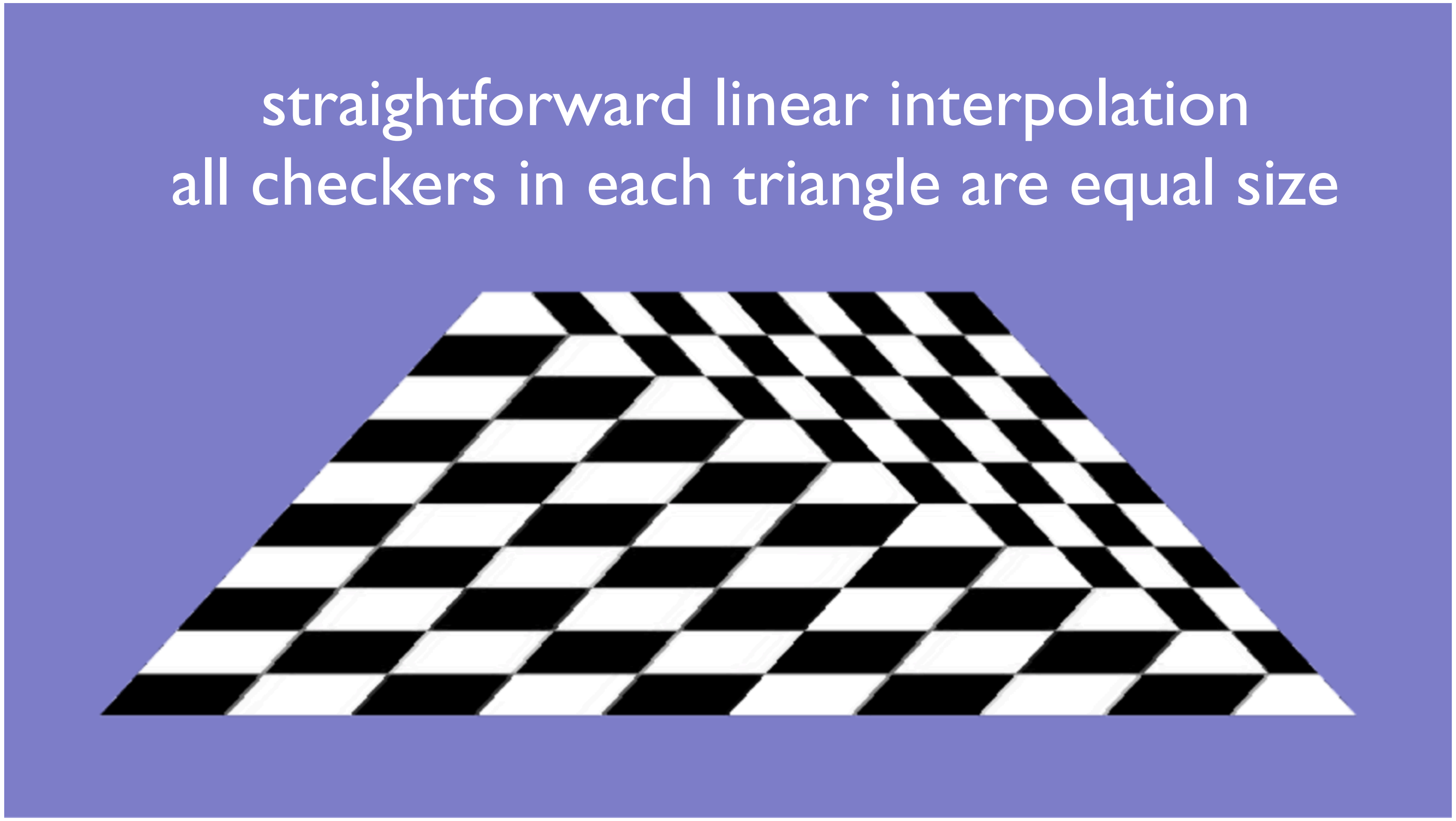


Perspective and interpolation

Demo

- **Texture coordinates are the canonical example**
  - equal steps in screen space are unequal steps in texture space



Perspective and interpolation

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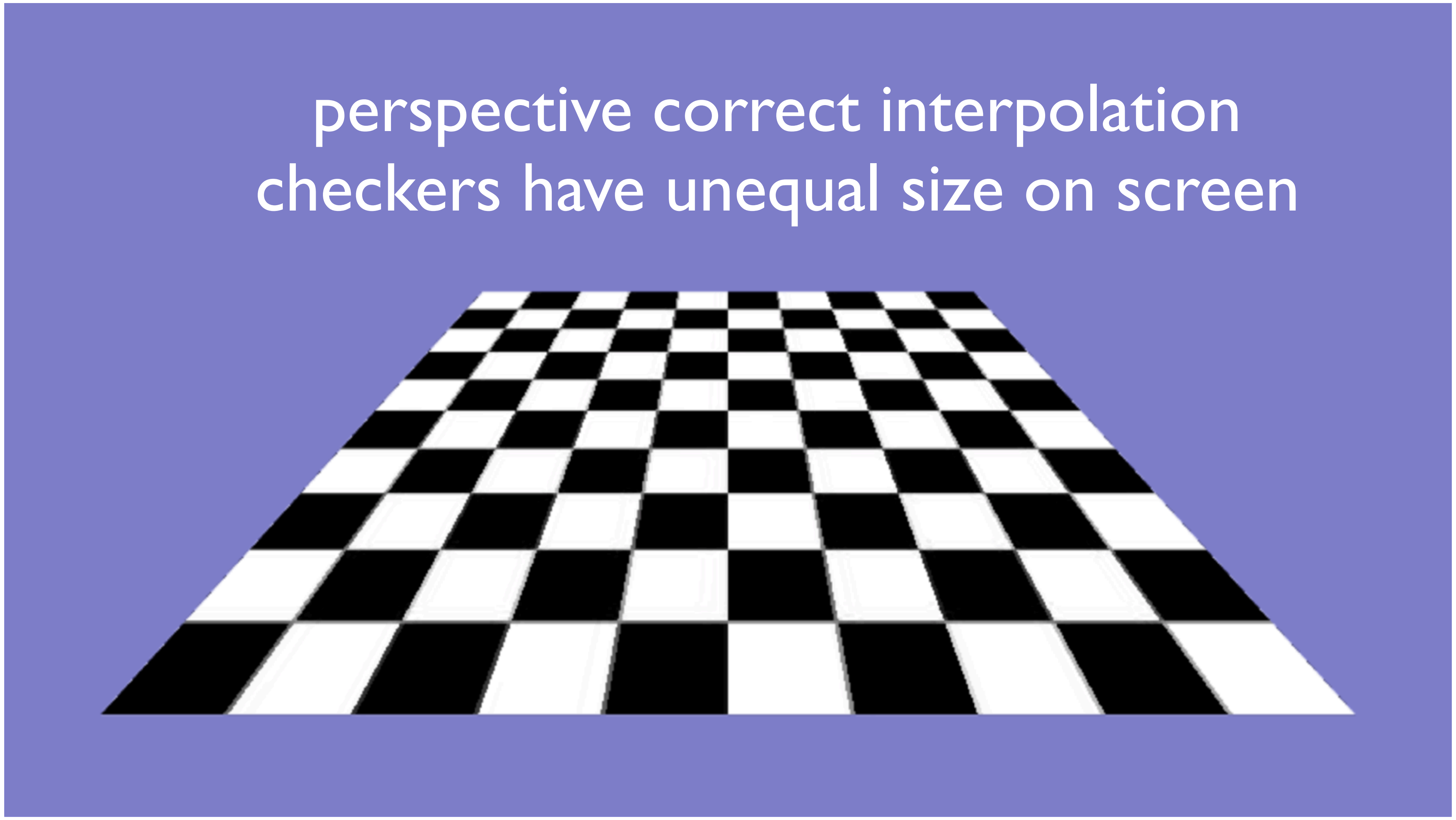
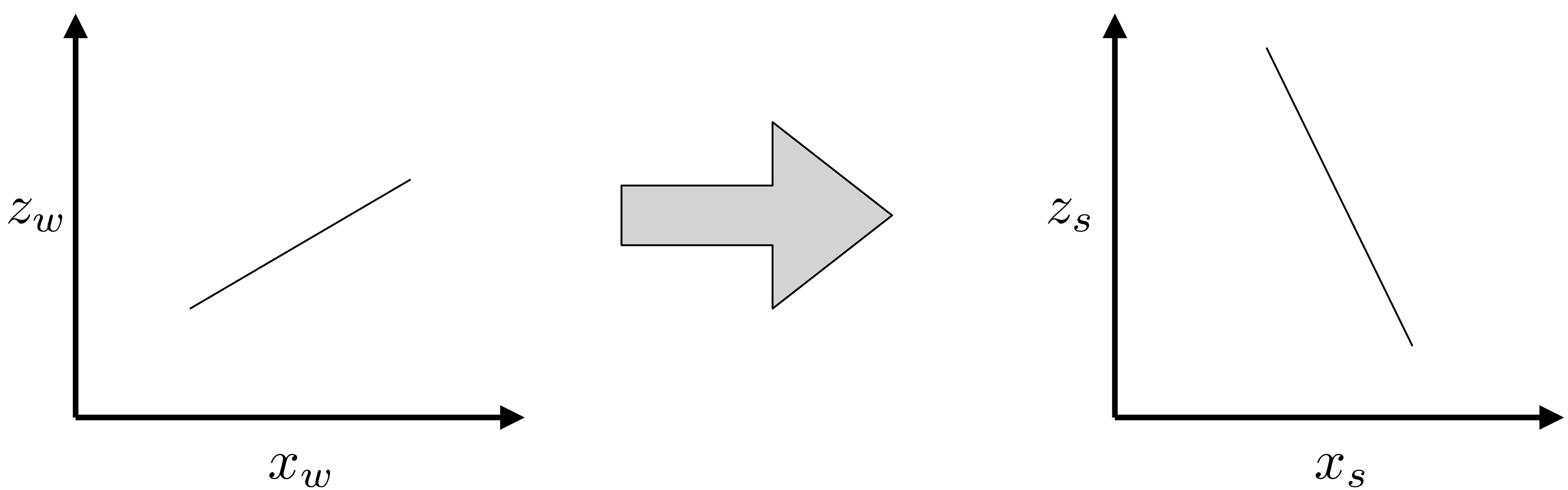


Perspective correct interpolation

- **Linear interpolation still suffices if we do it the right way**
  - remember projective transformations preserve straight lines

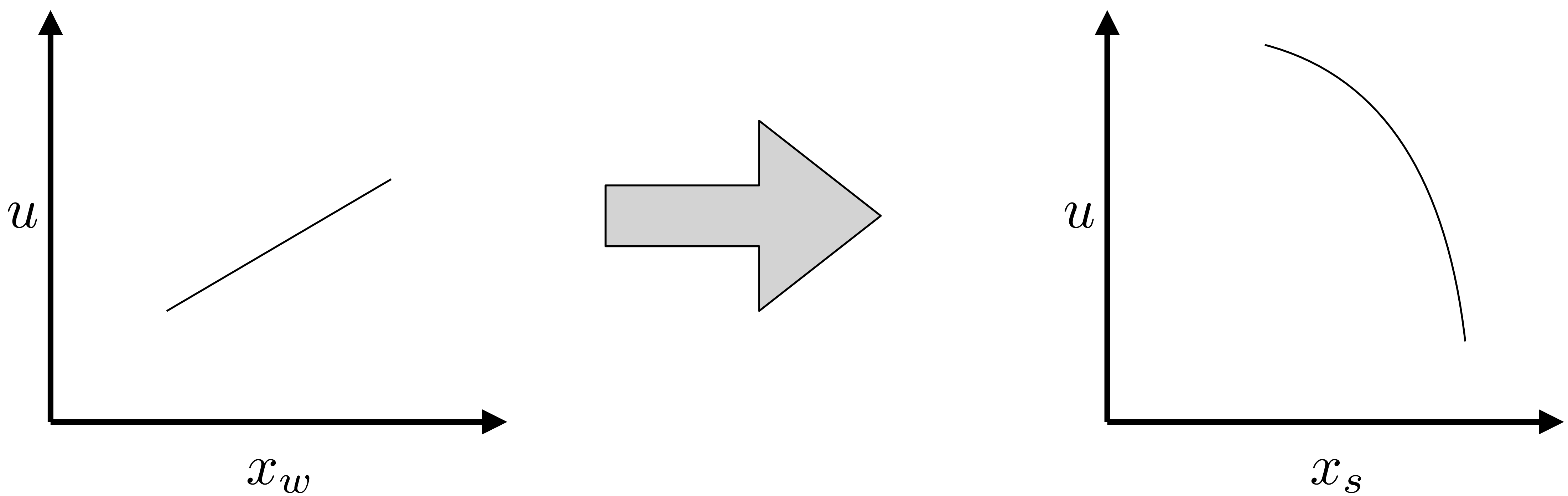
$$\begin{bmatrix} x_w \\ z_w \\ 1 \end{bmatrix} \to \begin{bmatrix} x_c \\ z_c \\ w_c \end{bmatrix} \to \begin{bmatrix} x_c/w_c \\ z_c/w_c \\ 1 \end{bmatrix} = \begin{bmatrix} x_s \\ z_s \\ 1 \end{bmatrix}$$



Perspective correct interpolation

- **Linear interpolation still suffices if we do it the right way**
  - remember projective transformations preserve straight lines
  - just carrying the tex, coord. along is not a projective transform.

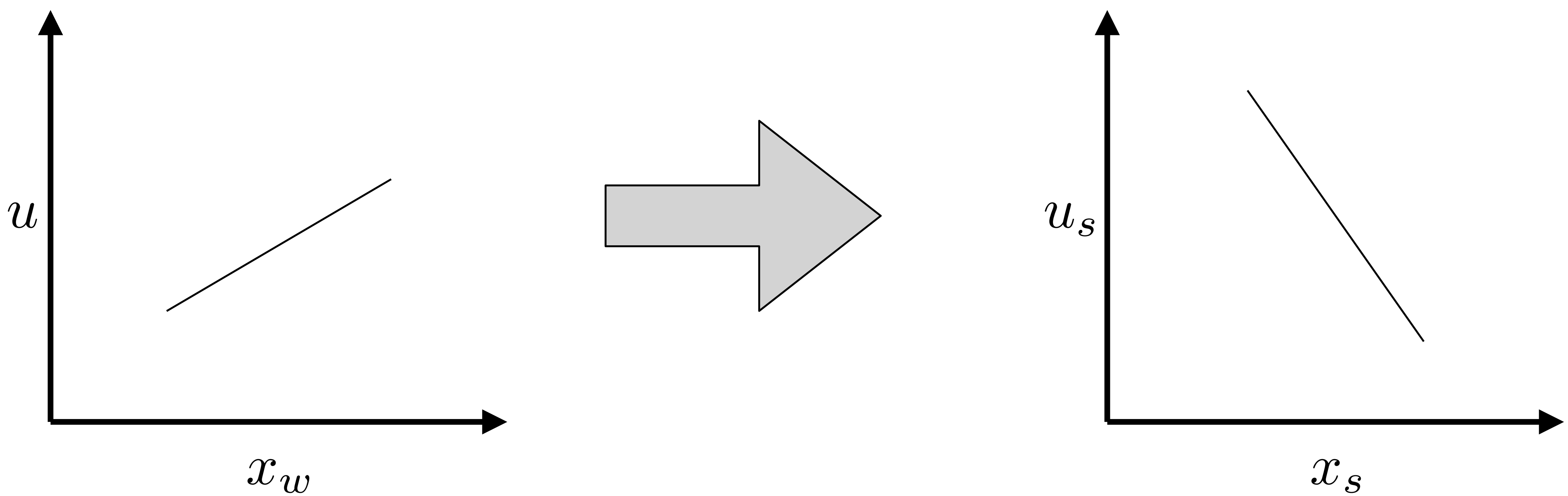
$$\begin{bmatrix} x_w \\ z_w \\ 1 \\ u \end{bmatrix} \to \begin{bmatrix} x_c \\ z_c \\ w_c \\ u \end{bmatrix} \to \begin{bmatrix} x_c/w_c \\ z_c/w_c \\ 1 \\ u \end{bmatrix} = \begin{bmatrix} x_s \\ z_s \\ 1 \\ u \end{bmatrix}$$



Perspective correct interpolation

- **Solution: treat u and v as additional coordinates in the projective transformation**
  – now the full transformation on (x, y, z, u, v) is projective

$$\begin{bmatrix} x_w \\ z_w \\ u \\ 1 \end{bmatrix} \to \begin{bmatrix} x_c \\ z_c \\ u \\ w_c \end{bmatrix} \to \begin{bmatrix} x_c/w_c \\ z_c/w_c \\ u/w_c \\ 1 \end{bmatrix} = \begin{bmatrix} x_s \\ z_s \\ u_s \\ 1 \end{bmatrix}$$



Perspective correct interpolation

- **Bottom line: treat all attributes the same as (x, y, z)**
  - divide them by w before interpolation
  - interpolate quantities u/w, etc., linearly across screen
  - also interpolate 1/w as an additional attribute
  - divide interpolated u/w by 1/w to recover u

Clipping

- **Rasterizer tends to assume triangles are on screen**
  - particularly problematic to have triangles crossing the plane $z = 0$
- **After projection, before perspective divide**
  - clip against the planes $x, y, z = 1, -1$ (6 planes)
  - primitive operation: clip triangle against axis-aligned plane

Clipping a triangle against a plane

- **4 cases, based on sidedness of vertices**
  - all in (keep)
  - all out (discard)
  - one in, two out (one clipped triangle)
  - two in, one out (two clipped triangles)