CS 465 Prelim 2

Tuesday 4 November 2003

Problem 1: Transformations (25 pts)

Identify the following matrices as implementing one of the following classes of transformation: (a) Uniform scale; (b) Nonuniform scale; (c) Clockwise rotation (by less than 180°) about $\mathbf{e}_1$, $\mathbf{e}_2$, or $\mathbf{e}_3$; (d) Counterclockwise rotation (by less than 180°) about $\mathbf{e}_1$, $\mathbf{e}_2$, or $\mathbf{e}_3$; (e) Translation; (f) Shear; (g) Reflection; (h) Perspective projection; (i) Parallel projection.

$$(1) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(5) \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad (7) \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 2: Ray tracing bugs (40 pts)

Briefly describe the visual effect of each of the following errors in implementing a ray tracer and explain why it happens

1. Forgetting to normalize the light direction before computing diffuse shading. Assume we are looking at a sphere illuminated by a point source 10 units away.
2. Starting shadow rays a $t = 0$, exactly at the shading point.
3. Forgetting to ignore shadow ray intersections that occur beyond the light source.
4. In the scene intersection loop, returning as soon as any object is hit, rather than finding the closest one.

5. Using the edges $\mathbf{p}_1 - \mathbf{p}_0$, $\mathbf{p}_2 - \mathbf{p}_1$, and $\mathbf{p}_2 - \mathbf{p}_0$ (rather than $\mathbf{p}_0 - \mathbf{p}_2$) in the point-in-triangle test performed as part of ray-triangle intersection. Assume the scene consists of a single equilateral triangle being viewed perpendicularly.
6. Forgetting to normalize the half vector in Blinn-Phong shading, leaving it as the average of the eye and light directions.

Problem 3: Viewing (35 pts)

Consider a camera with aspect ratio 1.0 and a field of view of $2\tan^{-1}\frac{1}{2}$. The camera is positioned at $(0, 5, 0)$ looking toward the origin with $-z$ up. Let world-space coordinates be denoted (x, y, z), eye-space coordinates be denoted (x_e, y_e, z_e), and image-space coordinates be denoted (x', y').

1. What is the camera's basis? (Give the unit vectors $\hat{\mathbf{u}}$, $\hat{\mathbf{v}}$, and $\hat{\mathbf{w}}$—recall that $\hat{\mathbf{w}}$ points *opposite* from the direction the camera is facing.)
2. If we consider the coordinates of the image to span the square $[-1, 1] \times [-1, 1]$, give an expression to generate the eye ray (origin and direction) for the image point (x', y').
3. Give the (4×4) viewing and (3×4) projection matrices for this camera.
4. Show that your answers are compatible by computing the following in sequence (showing your work):
   (a) Start with the image-space point $(\frac{1}{2}, \frac{1}{3})$ and use the answer to part 2 to generate the corresponding ray (give the origin and direction of the ray in world coordinates).
   (b) Intersect the ray with the plane $y = -7$ to produce a world-space 3D point.
   (c) Use the answer to part 3 to transform that point into eye space.
   (d) Use the answer to part 3 to transform the eye space point into image space, and verify that you're back where you started.