CS 4620 Preliminary Exam #1

Tuesday 5 October 2010—50 minutes

Explain your reasoning for full credit.
You are permitted a double-sided sheet of notes.
Calculators are allowed but unnecessary.

Problem 1: 2D Transformations (15 pts)

(i) Estimate the 2D affine transformation matrix, $\mathbf{T} = \begin{bmatrix} \boldsymbol{F} & \boldsymbol{v} \\ 0^T & 1 \end{bmatrix} \in \mathbb{R}^{\mathbf{3}\times\mathbf{3}}$, given its action on three homogeneous points:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \xrightarrow{\boldsymbol{T}} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \xrightarrow{\boldsymbol{T}} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \qquad \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \xrightarrow{\boldsymbol{T}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

(ii) What kind of transformation does this matrix represent?

Problem 2: Affine Transformations (10 pts)

Show that affine transformations preserve parallel lines.
(Hint: Recall the explicit parameterization of a line.)

Problem 3: Quaternions (15 pts)

Rotate the point $\boldsymbol{p} = (1, 1, 1)$ using the rotation specified by the quaternion $q = \langle d; \boldsymbol{u} \rangle = \langle 1; 1, 1, 1 \rangle$.

Problem 4: SLERP (10 pts)

When interpolating with SLERP between two unit quaternions, $\boldsymbol{x}$ and $\boldsymbol{y}$, we use:

$$\text{SLERP}(\boldsymbol{x}, \boldsymbol{y}, \alpha), \text{ if } \boldsymbol{x} \cdot \boldsymbol{y} > 0, \text{ and SLERP}(\boldsymbol{x}, -\boldsymbol{y}, \alpha) \text{ otherwise.}$$

(i) Why is this method better than just $\text{SLERP}(\boldsymbol{x}, \boldsymbol{y}, \alpha)$? What is the difference between $+\boldsymbol{y}$ and $-\boldsymbol{y}$ here?

(ii) When interpolating unit normal vectors, $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$, for lighting calculations, should we also use

$$\text{SLERP}(\boldsymbol{n}_1, \boldsymbol{n}_2, \alpha), \text{ if } \boldsymbol{n}_1 \cdot \boldsymbol{n}_2 > 0, \text{ and SLERP}(\boldsymbol{n}_1, -\boldsymbol{n}_2, \alpha) \text{ otherwise?}$$