

3D Viewing

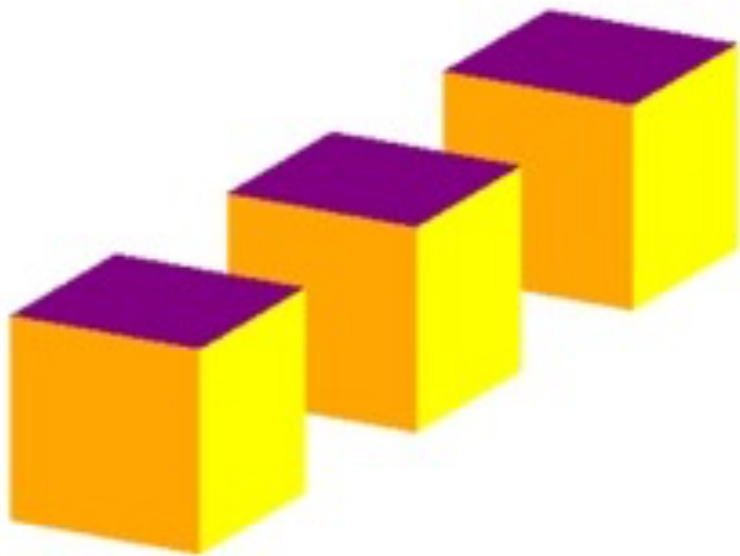
CS 4620 Lecture 13

Announcements

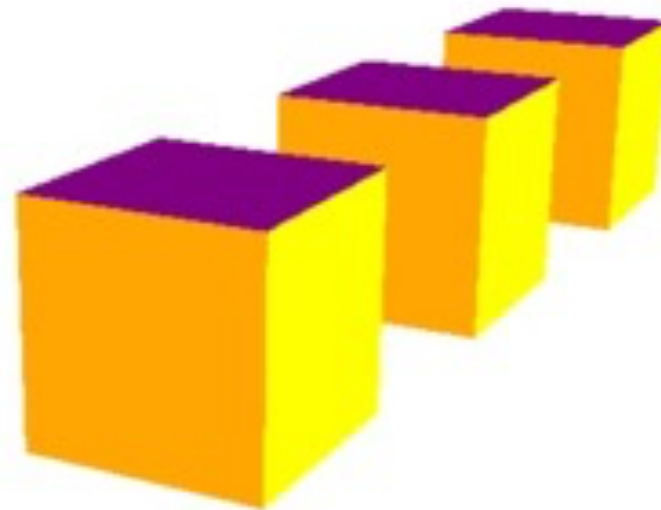
- Demos tonight
 - If you can't make it, send mail to cs4620-staff-1@cornell.edu

- A3 out
- PPA 1 out (4621)

- Wed: OpenGL
- Fri: rest of perspective



Orthographic Projection

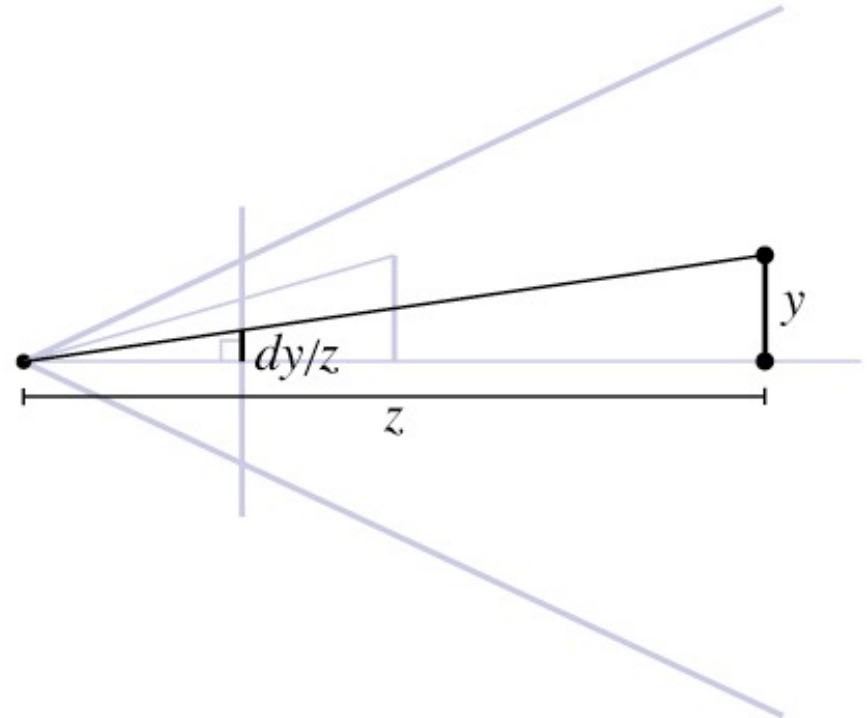


Perspective Projection

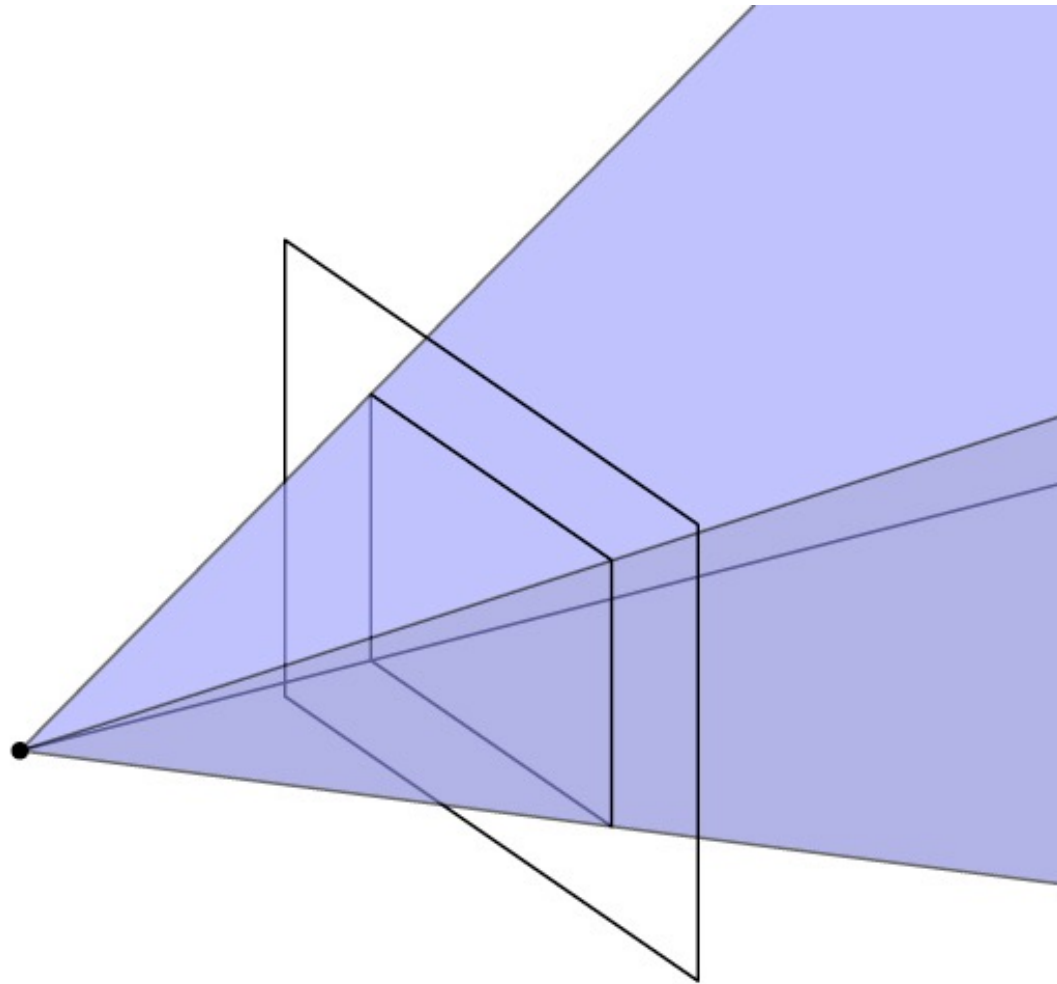
<http://i.msdn.microsoft.com/dynimg/C15161.jpg>

Perspective projection (normal)

- Perspective is projection by lines through a point;
- “normal” = plane perpendicular to view direction
 - magnification determined by:
 - image height
 - object depth
 - image plane distance
 - f.o.v. $\alpha = 2 \operatorname{atan}(h/(2d))$
 - $y' = d y / z$
 - “normal” case corresponds to common types of cameras



View volume: perspective



Field of view (or f.o.v.)

- The angle between the rays corresponding to opposite edges of a perspective image
 - simpler to compute for “normal” perspective
 - have to decide to measure vert., horiz., or diag.
- In cameras, determined by focal length
 - confusing because of many image sizes
 - for 35mm format (36mm by 24mm image)
 - 18mm = 67° v.f.o.v. — super-wide angle
 - 28mm = 46° v.f.o.v. — wide angle
 - 50mm = 27° v.f.o.v. — “normal”
 - 100mm = 14° v.f.o.v. — narrow angle (“telephoto”)

Choice of field of view

- In photography, wide angle lenses are specialty tools
 - “hard to work with”
 - easy to create weird-looking perspective effects
- In graphics, you can type in whatever f.o.v. you want
 - and people often type in big numbers!



[Ken Perlin]

Field of view

- Determines “strength” of perspective effects



close viewpoint
wide angle
prominent foreshortening

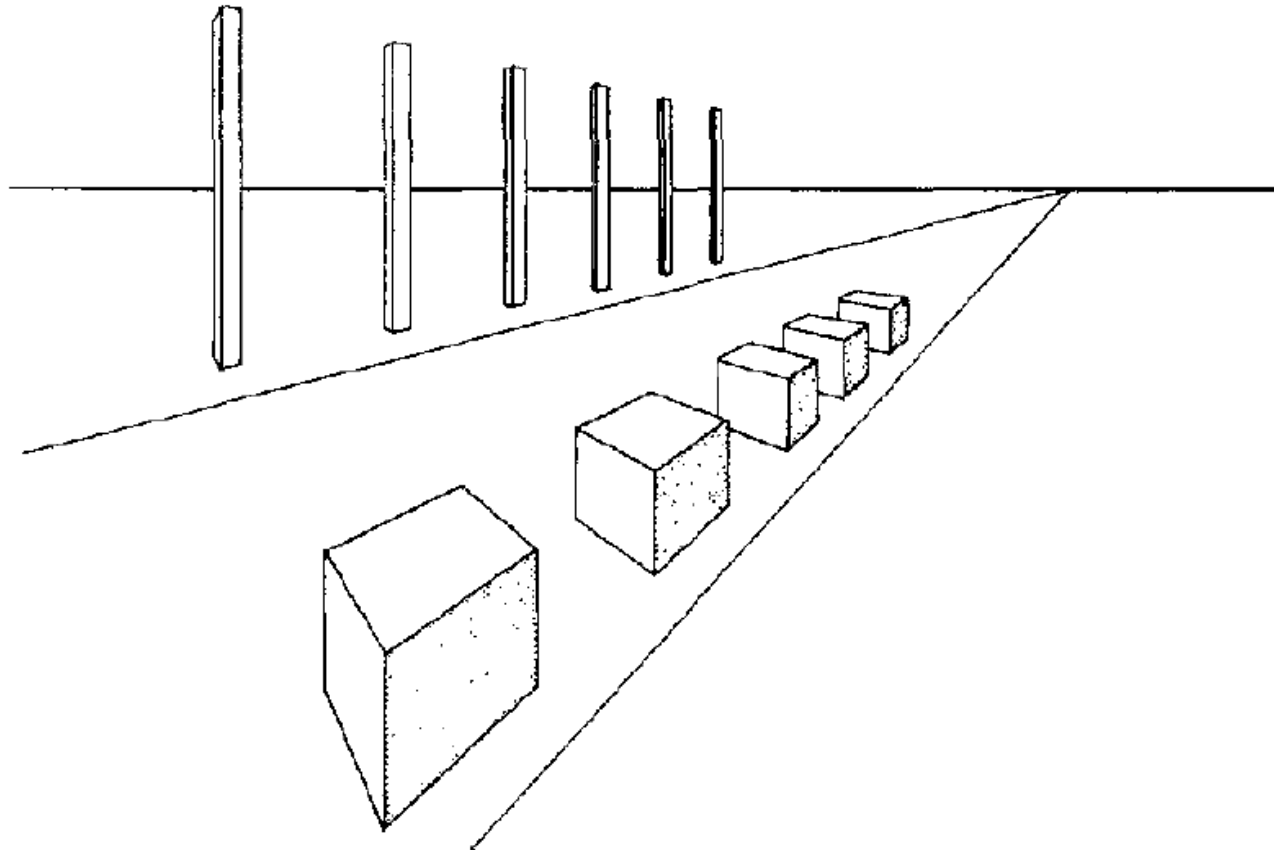


far viewpoint
narrow angle
little foreshortening

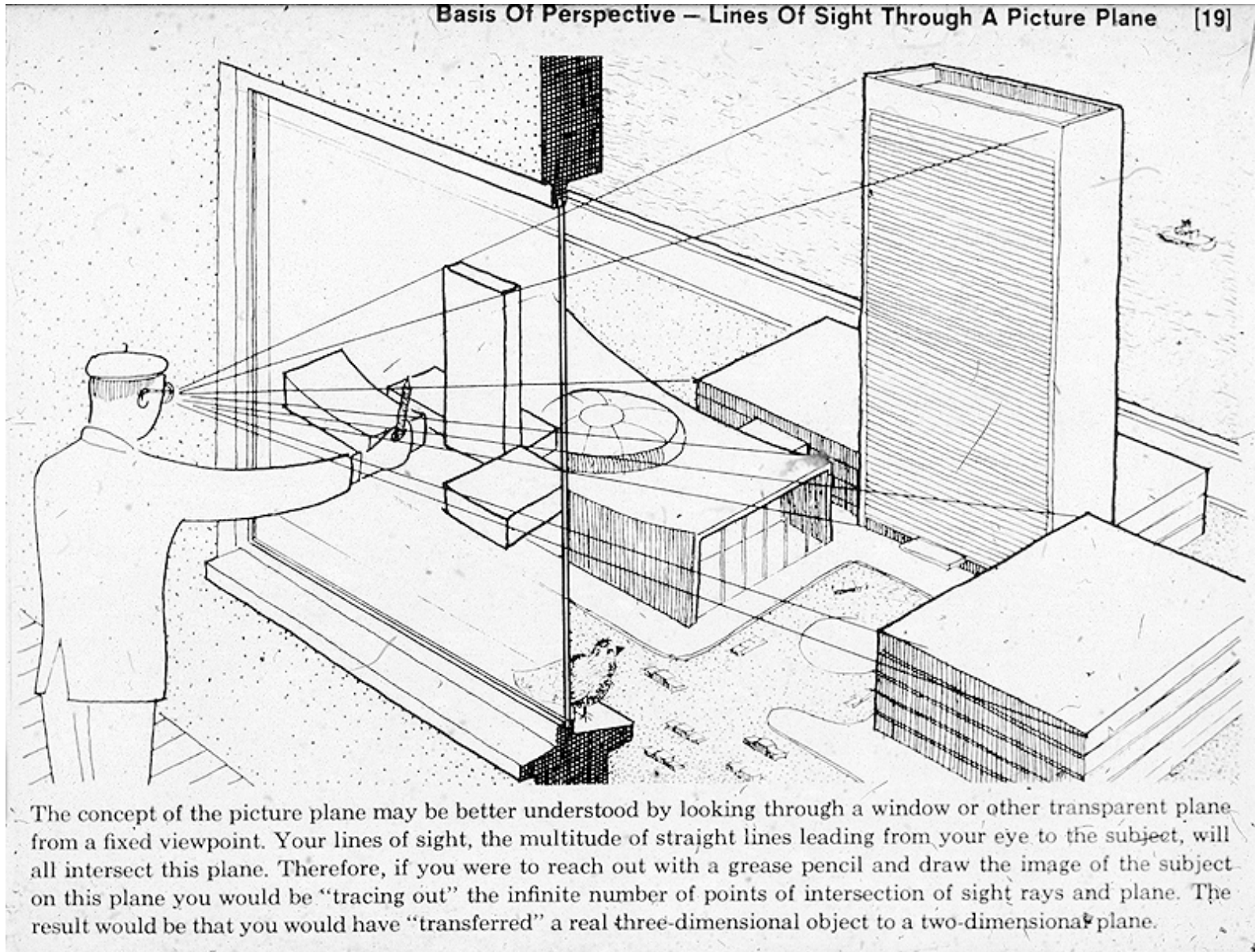
[Ansel Adams]

Perspective distortions

- Lengths, length ratios



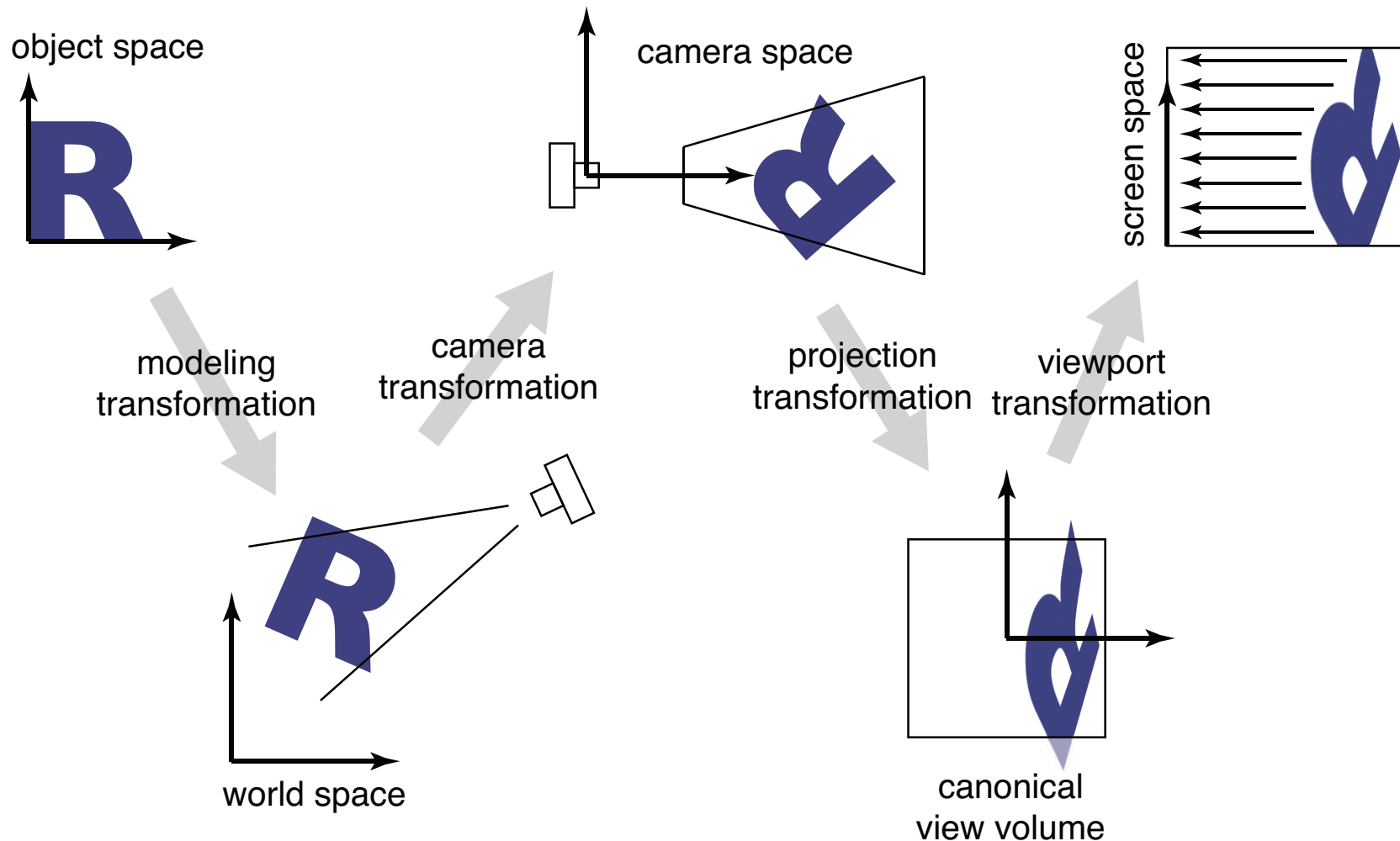
Plane projection in drawing



source unknown

Pipeline of transformations

- Standard sequence of transforms



Viewing, backward and forward

- Ray tracing: backward approach to viewing
 - start from pixel
 - ask what part of scene projects to pixel
 - explicitly construct the ray corresponding to the pixel
- GPU: forward approach
 - start from a point in 3D
 - compute its projection into the image
- Central tool is matrix transformations
 - combines seamlessly with coordinate transformations used to position camera and model
 - ultimate goal: single matrix operation to map any 3D point to its correct screen location.

Mathematics of projection

- Always work in eye coords
 - assume eye point at $\mathbf{0}$ and plane perpendicular to z
- Orthographic case
 - a simple projection: just toss out z
- Perspective case: scale diminishes with z
 - and increases with d

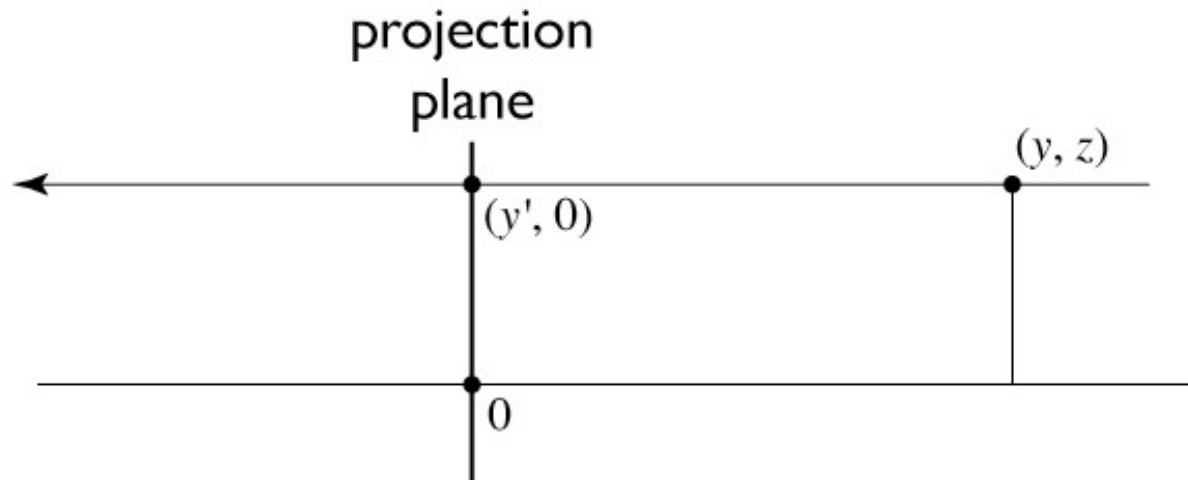
Orthographic transformation chain

- Start with coordinates in object's local coordinates
- Transform into world coords (modeling transform, M_m)
- Transform into eye coords (camera xf., $M_{cam} = F_c^{-1}$)
- Orthographic projection, M_{orth}
- Viewport transform, M_{vp}

$$\mathbf{p}_s = M_{vp} M_{orth} M_{cam} M_m \mathbf{p}_o$$

$$\begin{bmatrix} x_s \\ y_s \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} & \mathbf{e} \end{bmatrix}^{-1} M_m \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

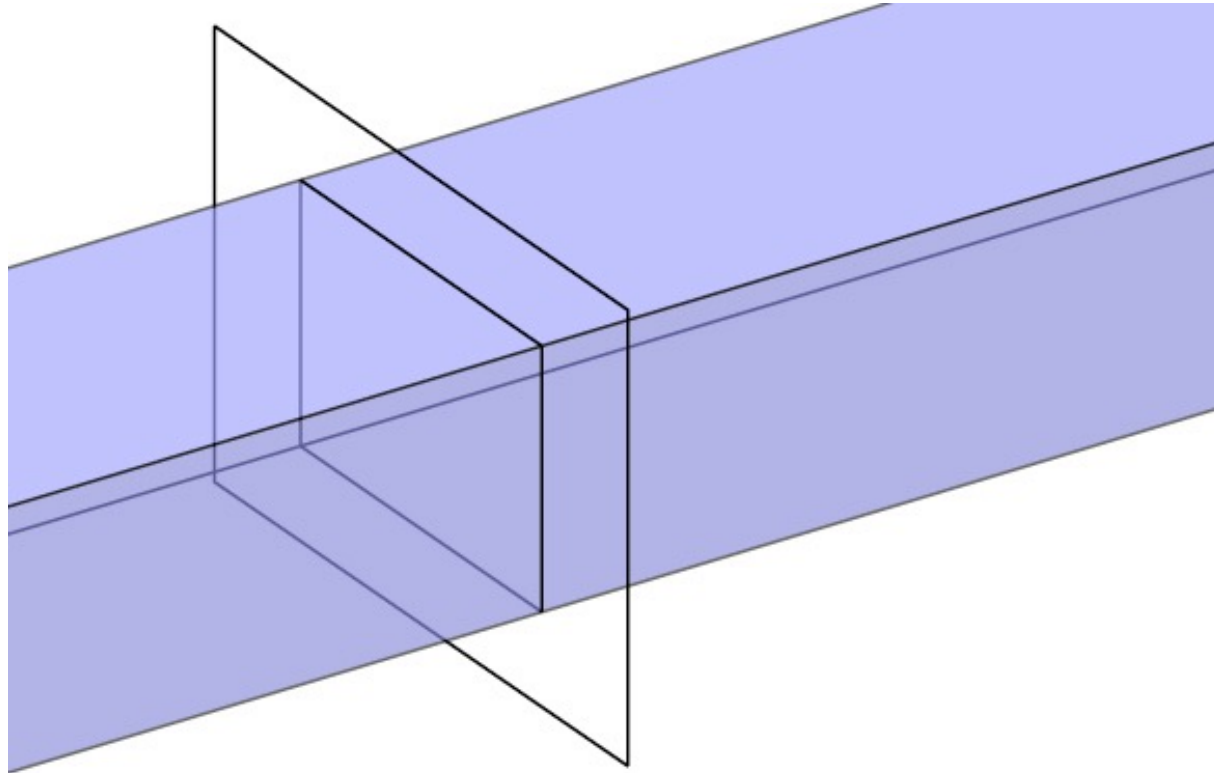
Parallel projection: orthographic



to implement orthographic, just toss out z:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

View volume: orthographic

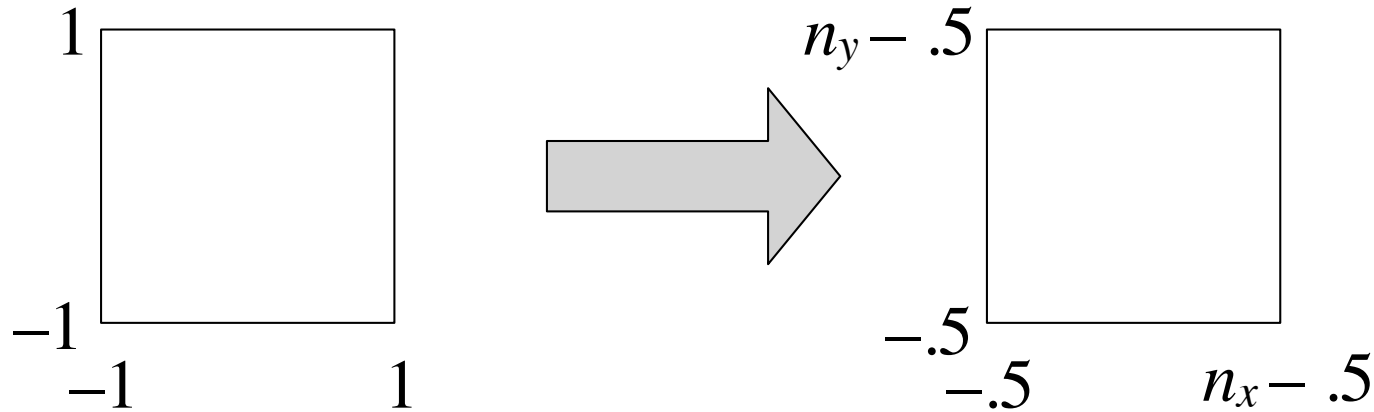


Viewing a cube of size 2

- Start by looking at a restricted case: the *canonical view volume*
- It is the cube $[-1, 1]^3$, viewed from the z direction

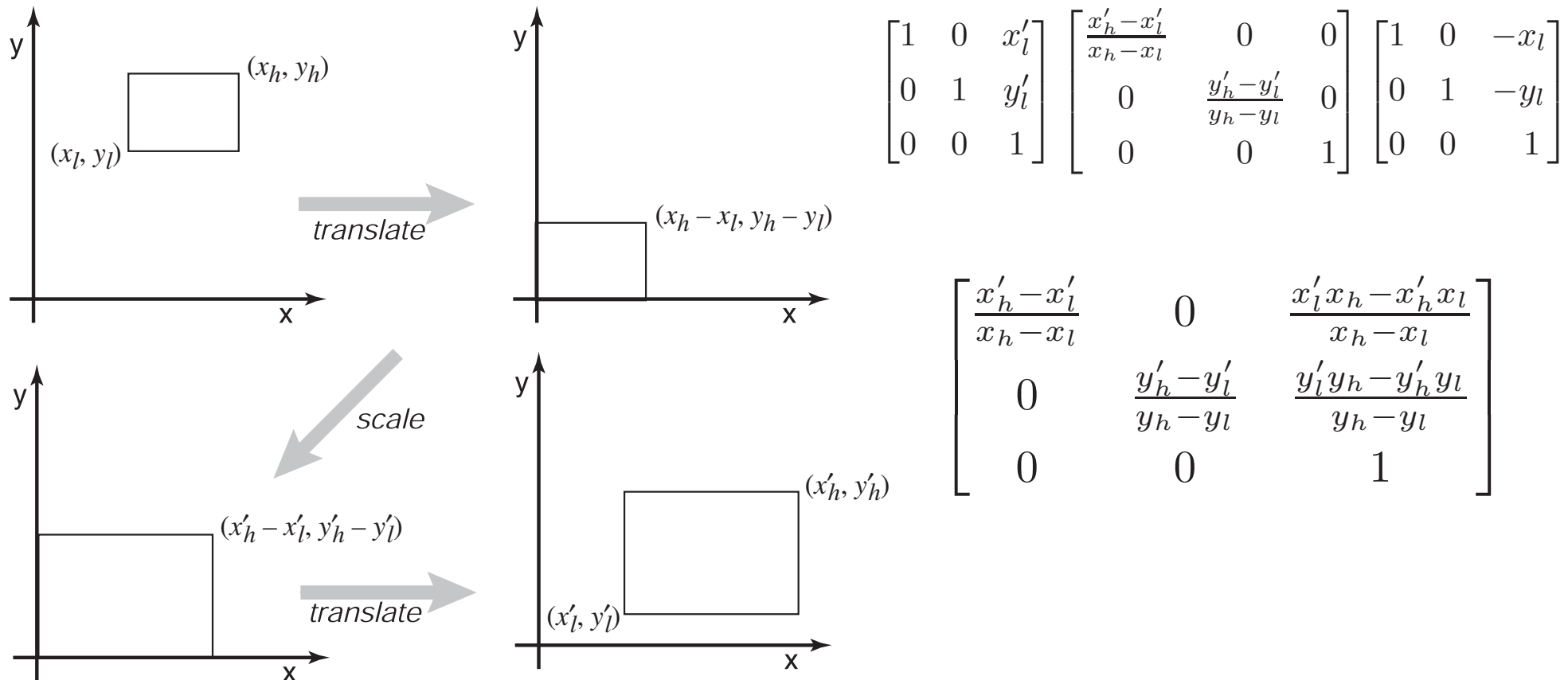
Viewing a cube of size 2

- To draw in image, need coordinates in pixel units, though
- Exactly the opposite of mapping (i,j) to (u,v) in ray generation



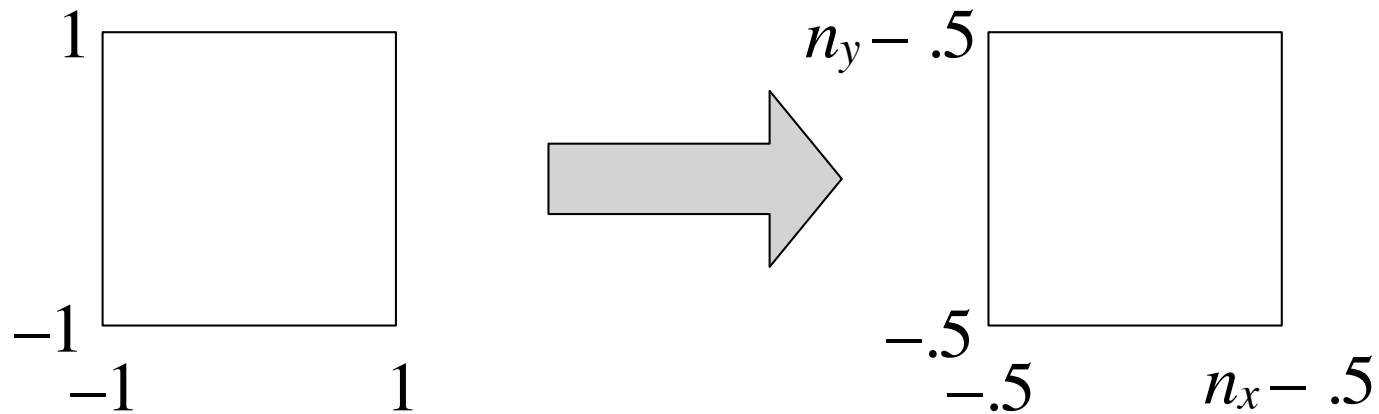
Windowing transforms

- This transformation is worth generalizing: take one axis-aligned rectangle or box to another
 - a useful, if mundane, piece of a transformation chain



[Shirley3e f. 6-16; eq. 6-6]

Viewport transformation



$$\begin{bmatrix} x_{\text{screen}} \\ y_{\text{screen}} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & \frac{n_x - 1}{2} \\ 0 & \frac{n_y}{2} & \frac{n_y - 1}{2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{\text{canonical}} \\ y_{\text{canonical}} \\ 1 \end{bmatrix}$$

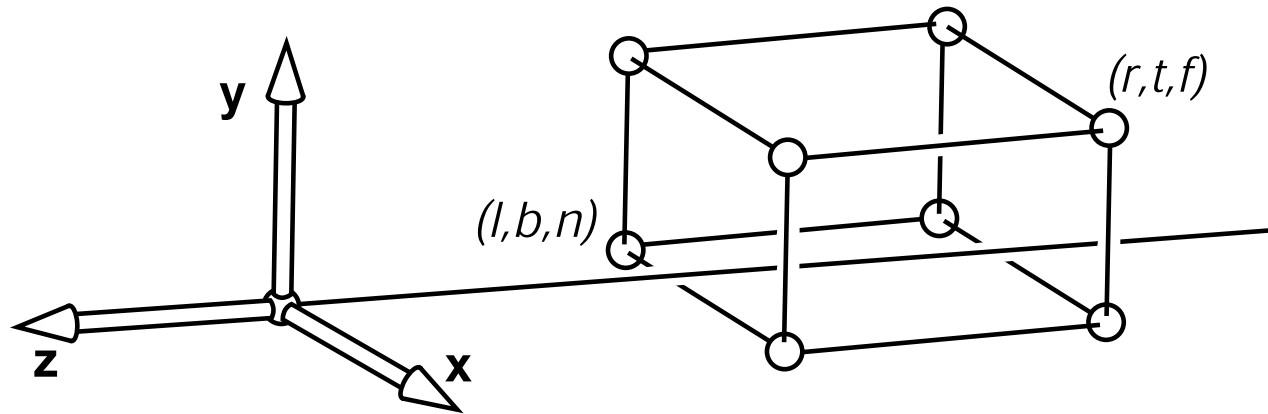
Viewport transformation

- In 3D, carry along z for the ride
 - one extra row and column

$$\mathbf{M}_{vp} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x - 1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y - 1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Orthographic projection

- First generalization: different view rectangle
 - retain the minus-z view direction
-



- specify view by left, right, top, bottom (as in RT)
- also near, far

Clipping planes

- In object-order systems we always use at least two *clipping planes* that further constrain the view volume
 - near plane: parallel to view plane; things between it and the viewpoint will not be rendered
 - far plane: also parallel; things behind it will not be rendered
- These planes are:
 - partly to remove unnecessary stuff (e.g. behind the camera)
 - but really to constrain the range of depths
(we'll see why later)

Orthographic projection

- We can implement this by mapping the view volume to the canonical view volume.
- This is just a 3D windowing transformation!

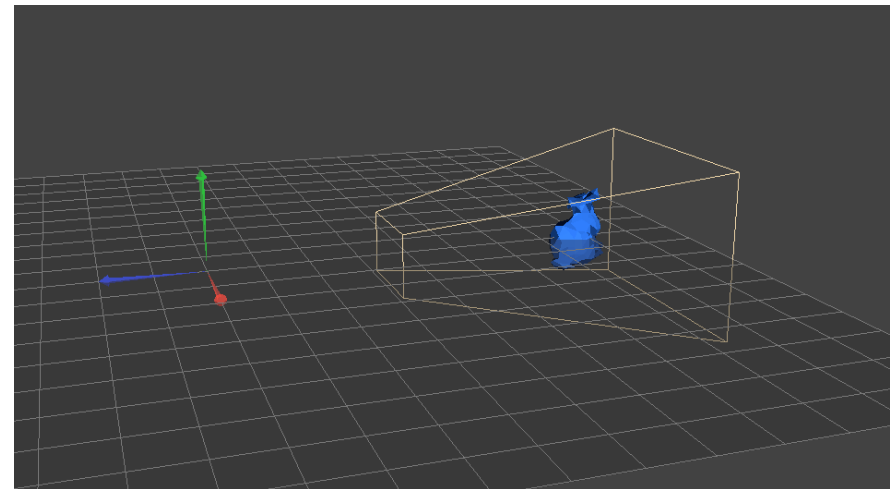
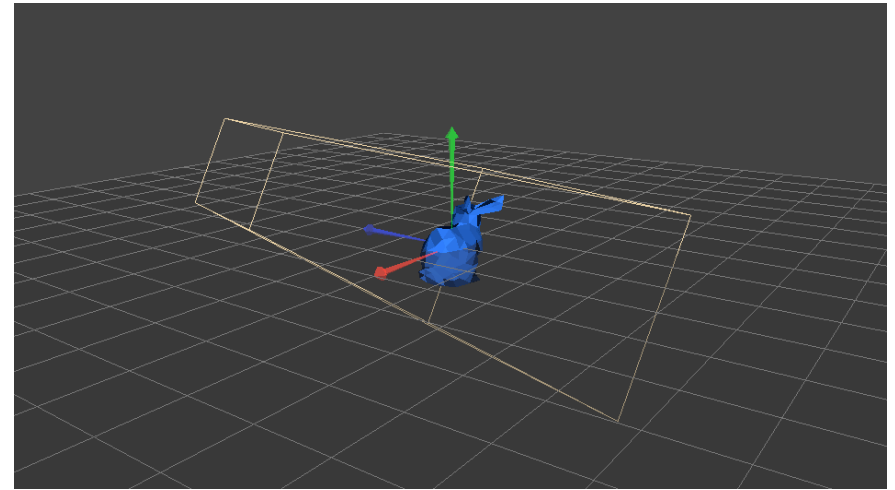
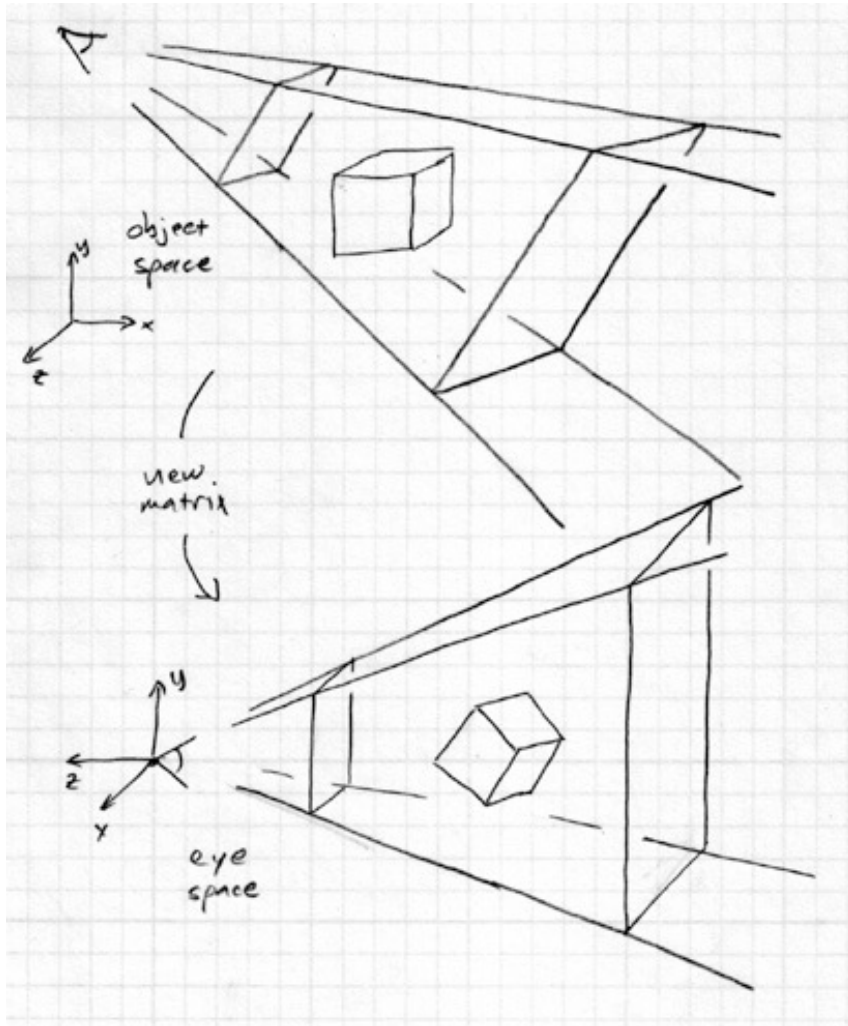
$$\mathbf{M}_{\text{orth}} = \begin{bmatrix} \frac{x'_h - x'_l}{x_h - x_l} & 0 & 0 & \frac{x'_l x_h - x'_h x_l}{x_h - x_l} \\ 0 & \frac{y'_h - y'_l}{y_h - y_l} & 0 & \frac{y'_l y_h - y'_h y_l}{y_h - y_l} \\ 0 & 0 & \frac{z'_h - z'_l}{z_h - z_l} & \frac{z'_l z_h - z'_h z_l}{z_h - z_l} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Camera and modeling matrices

- We worked out all the preceding transforms starting from eye coordinates
 - before we do any of this stuff we need to transform into that space
- Transform from world (canonical) to eye space is traditionally called the *viewing matrix*
 - it is the canonical-to-frame matrix for the camera frame
 - that is, F_c^{-1}
- Geometry would originally be in object's local coordinates
 - Transform into world coordinates by *modeling matrix*, M_m
- Note many programs combine the two into a *modelview* matrix and just skip world coordinates

Viewing transformation



the camera matrix rewrites all coordinates in eye space

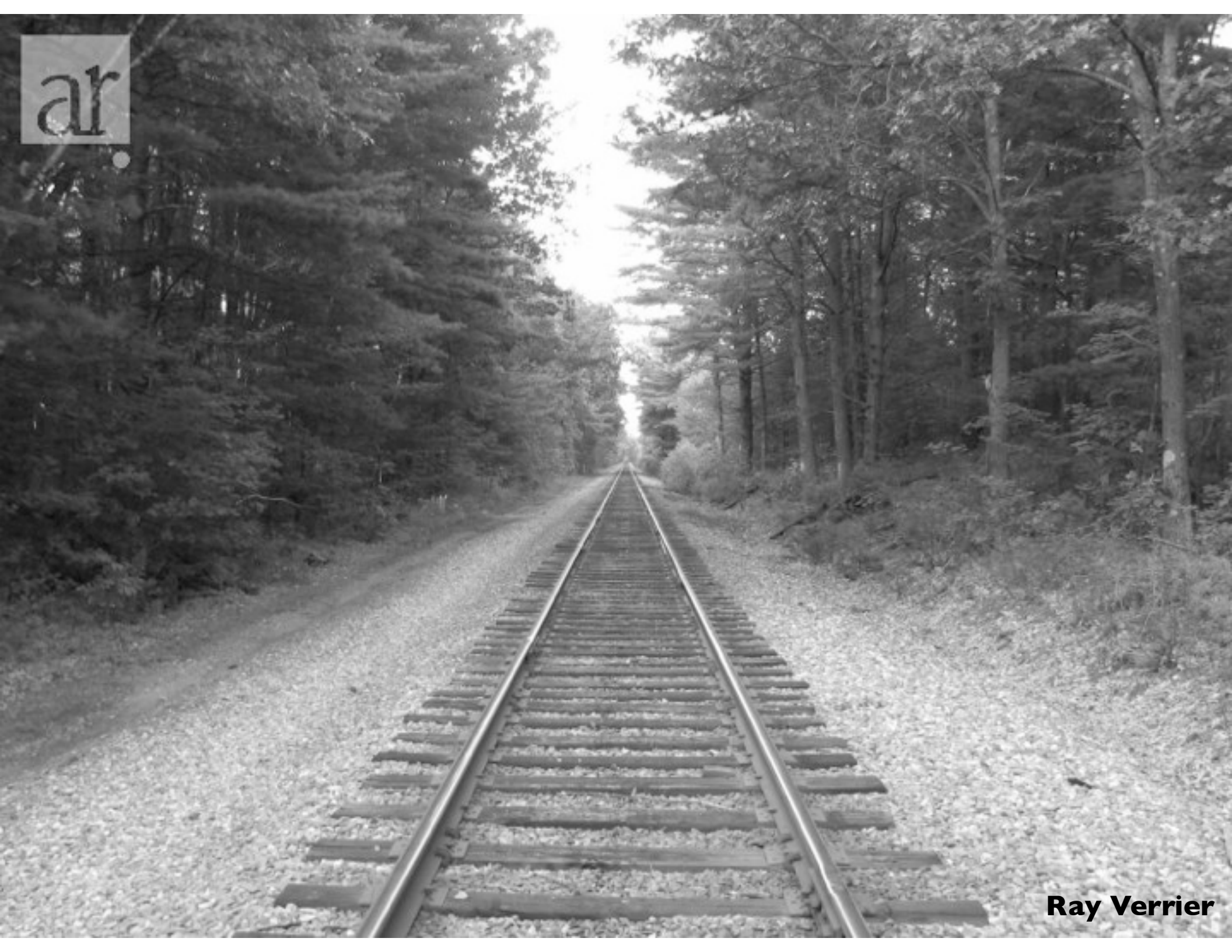
Orthographic transformation chain

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$$\mathbf{p}_s = M_{vp} M_{orth} M_{cam} M_m \mathbf{p}_o$$

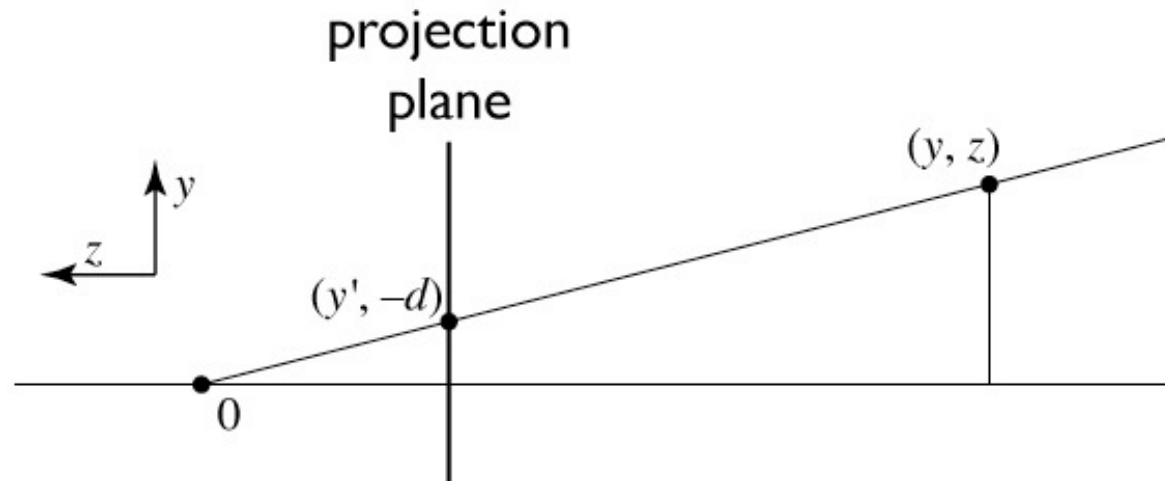
$$\begin{bmatrix} x_s \\ y_s \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & \frac{n_x-1}{2} \\ 0 & \frac{n_y}{2} & 0 & \frac{n_y-1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{u} & \mathbf{v} & \mathbf{w} & \mathbf{e} \end{bmatrix}^{-1} M_m \begin{bmatrix} x_o \\ y_o \\ z_o \\ 1 \end{bmatrix}$$

ar



Ray Verrier

Perspective projection



similar triangles:

$$\frac{y'}{d} = \frac{y}{-z}$$

$$y' = -dy/z$$

Homogeneous coordinates revisited

- Perspective requires division
 - that is not part of affine transformations
 - in affine, parallel lines stay parallel
 - therefore not vanishing point
 - therefore no rays converging on viewpoint
- “True” purpose of homogeneous coords: projection

Homogeneous coordinates revisited

- Introduced $w = 1$ coordinate as a placeholder

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

– used as a convenience for unifying translation with linear

- Can also allow arbitrary w

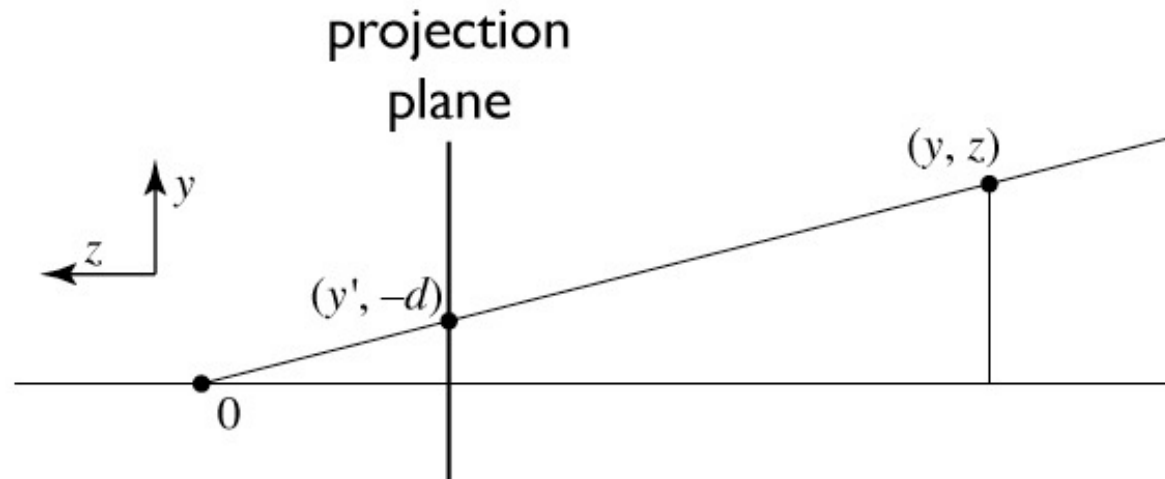
$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \sim \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

Implications of w

- All scalar multiples of a 4-vector are equivalent
- When w is not zero, can divide by w
 - therefore these points represent “normal” affine points
- When w is zero, it’s a point at infinity, a.k.a. a direction
 - this is the point where parallel lines intersect
 - can also think of it as the vanishing point
- Projective space: CS 4670

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \sim \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

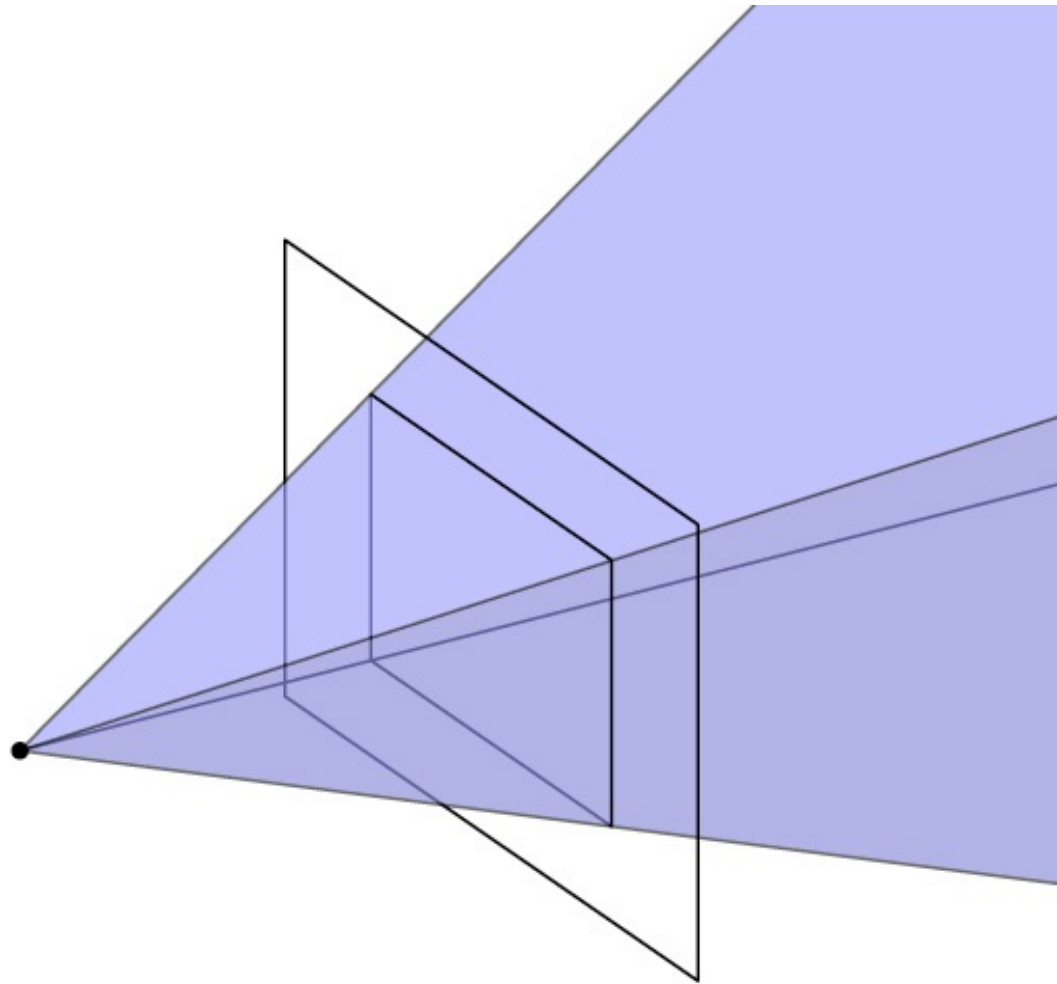
Perspective projection



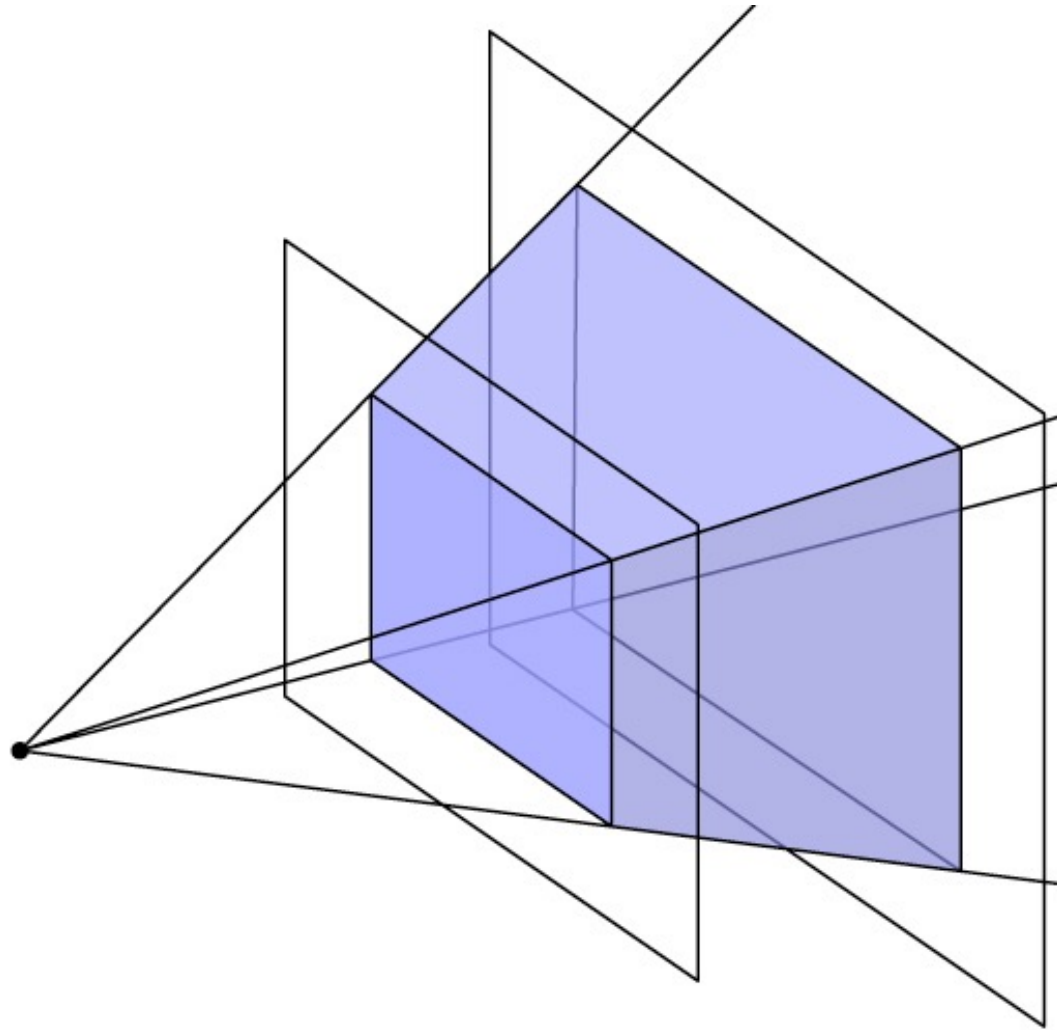
to implement perspective, just move z to w :

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -dx/z \\ -dy/z \\ 1 \end{bmatrix} \sim \begin{bmatrix} dx \\ dy \\ -z \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

View volume: perspective



View volume: perspective (clipped)



Carrying depth through perspective

- Perspective has a varying denominator—can't preserve depth!
- Compromise: preserve depth on near and far planes

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \sim \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ -z \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & a & b \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

–that is, choose a and b so that $z'(n) = n$ and $z'(f) = f$.

$$\tilde{z}(z) = az + b$$

$$z'(z) = \frac{\tilde{z}}{-z} = \frac{az + b}{-z}$$

want $z'(n) = n$ and $z'(f) = f$

result: $a = -(n + f)$ and $b = nf$ (try it)

Official perspective matrix

- Use near plane distance as the projection distance
– i.e., $d = -n$
- Scale by -1 to have fewer minus signs
– scaling the matrix does not change the projective transformation

$$\mathbf{P} = \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n + f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Perspective projection matrix

- Product of perspective matrix with orth. projection matrix

$$\mathbf{M}_{\text{per}} = \mathbf{M}_{\text{orth}} \mathbf{P}$$

$$= \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{r+l}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{t+b}{t-b} \\ 0 & 0 & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n+f & -fn \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Perspective transformation chain

- Transform into world coords (modeling transform, M_m)
- Transform into eye coords (camera xf., $M_{\text{cam}} = F_c^{-1}$)
- Perspective matrix, P
- Orthographic projection, M_{orth}
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Pipeline of transformations

- Standard sequence of transforms

