CS4620 Practice Problems (Prelim 1)

Problem 1: Transformations (2008fa)

Express the homogeneous 3D transformation defined by the matrix

$$\begin{bmatrix} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

as a sequence of transformations in the following ways:

1. A rotation followed by a translation.
2. A translation followed by a rotation.

Express the 2D linear transformation defined by the matrix

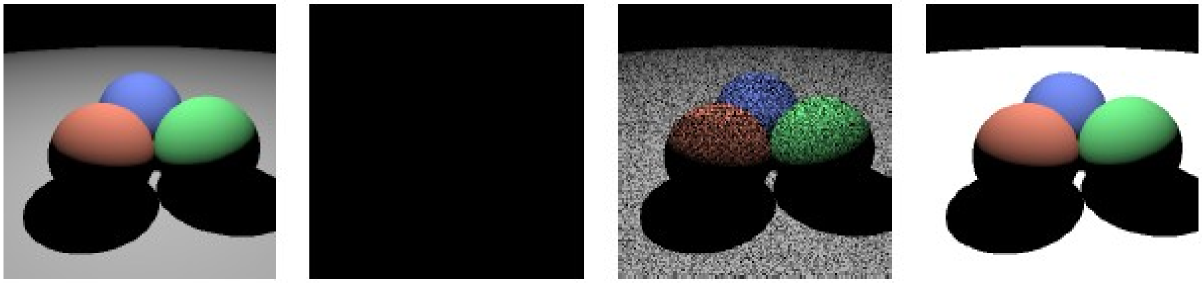
$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

as a sequence of transformations in the following ways:

3. A rotation followed by a nonuniform scale followed by a rotation.
4. A shear along the x axis followed by a nonuniform scale followed by a shear along the y axis.

Hint: For the 2D transformations, sometimes it might be easier to draw a picture and see how the given matrix transforms points in the plane (the points of the unit box for example)

Problem 2: Ray tracing 1 (2008fa)



correct 1 2 3

Figure 2: Four images from a ray tracer. The left image is correct, and the other three were produced by introducing single-statement bugs into the program.

Look at each of the three images in Figure ?? that were produced by a Ray I ray tracer with various bugs. For each one:

(a) Could it have been caused by a problem with ray generation?

(b) Could it have been caused by a problem with ray intersection?

(c) Could it have been caused by a problem with shading computations?

For each "yes" answer, back it up with an example of an error that would cause the observed symptoms. There is no right or wrong explanation; only plausible and implausible ones. But when there is a clearly plausible cause, very far-fetched explanations (which are roughly equivalent to a "no" answer) won't make full credit.

Shadow computations count as part of shading. Computing surface normals counts as part of ray intersection.

Use **"yes", "possibly"** and **"not likely"** as answers
and give plausible explanation/example for each on of them

Problem 3: Ray tracing 2 (2008fa)

The following pseudocode intersects a ray with some geometric shape.

```
function surfaceIntersect(Ray r)
   Point3 p = r.origin;
   Vector3 d = r.direction;
   p.x = 2 * p.x;
   d.x = 2 * d.x;
   [r1, r2] = quadraticRoots(d.x*d.x + d.y*d.y,
                            2*(p.x*d.x + p.y*d.y),
                            p.x*p.x + p.y*p.y - 1);
   t1 = -p.z/d.z;
   t2 = (1 - p.z)/d.z;
   tmin = max(min(r1, r2), min(t1, t2));
   tmax = min(max(r1, r2), max(t1, t2));
   if (tmin < tmax) {
      if (tmin > 0) return tmin;
      if (tmax > 0) return tmax;
      }
   return INFINITY;
```

The function `quadraticRoots` returns the two roots of a quadratic with the given coefficients, if there are two roots.

1. What is the shape? Give a detailed and precise definition, including all dimensions.
2. When there are no roots, what values could `quadraticRoots` return for `r1` and `r2` that will make this code work correctly without changes?
3. Give an example of a hit with `d.z == 0` and explain what values the variables in this function take on and how the return value arises.

Problem 4: Transformation matrices (2008fa)

Classify each of the following 2D homogeneous matrices as follows: (a) rotation, (b) mirror reflection, (c) uniform scale, (d) nonuniform scale, (e) translation, (f) shear, or (g) combination.

Earlier categories take precedence over later categories if more than one applies. There is not a one-to-one mapping between the matrices and the categories.

1. $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

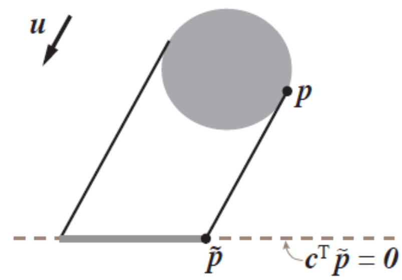
6. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Problem 5: Planar Shadows (2010fa)

Problem 2: Planar Shadows (15 pts)

Consider a directional light source (unit direction, $\boldsymbol{u}$), and the planar shadow created by literally projecting each mesh vertex position, $\boldsymbol{p}$, to its shadow point $\tilde{\boldsymbol{p}}$ on the 3D plane specified (in homogeneous coordinates) by $\boldsymbol{c}^T\tilde{\boldsymbol{p}} = 0$ where $\boldsymbol{c} \in \mathbb{R}^4$. Derive a formula for the 4x4 projection matrix, $\mathbf{A}$, that maps a homogeneous object point, $\boldsymbol{p} = (x, y, z, 1)^T$, to its shadow point, $\tilde{\boldsymbol{p}} = \mathbf{A}\boldsymbol{p}$. *(Hint: Consider the ray $\mathbf{p} + t\mathbf{u}$.)*



Problem 6: View Frustum Culling (2009fa)

"View frustum culling" is a technique to avoid drawing (or cull) geometry which is outside the view frustum. To assist with culling, assume that *each object has a bounding sphere* with object-frame center position, $\mathbf{c}_o = (c_x, c_y, c_z, 1)^T$, and radius R_o. Imagine that you know you have an $[l, r] \times [b, t] \times [f, n]$ orthographic viewing volume, and you know each of the matrices ($\mathbf{M}_{vp}$, $\mathbf{M}_{orth}$, $\mathbf{M}_{cam}$, $\mathbf{M}_m$) used to construct the orthographic view transformation which maps points from *object space* to *screen space*:

$$\mathbf{p}_s = \begin{pmatrix} x_s \\ y_s \\ z_c \\ 1 \end{pmatrix} = \mathbf{M}_{vp}\,\mathbf{M}_{orth}\,\mathbf{M}_{cam}\,\mathbf{M}_m\,\mathbf{p}_o = \mathbf{M} \begin{pmatrix} x_o \\ y_o \\ z_o \\ 1 \end{pmatrix}.$$

Derive a simple mathematical test to determine if an object is safely "off screen."

Assume that $\mathbf{M}_m$ is a rigid-body transformation

Problem 7: Camera matrix (2003fa)

Write the 4x4 matrix for this affine transformation:

3. (7 pts) A viewing transformation for a camera at the position $(0, 3, 4)$ looking at the origin with up vector $(0, 1, 0)$. It is OK to use a matrix inverse in your answer.

Hint: Try solving the problem, without explicitly computing cross products, but just considering the right-hand rule.