Perspective-Correct Textures (11.3.1)

- Linear interpolation of positions OK in screen space
- Leads to foreshortening effect in other attributes
  - texture coordinates, colors, etc.
- **Hyperbolic Interpolation**
  - Observe that 1/h is interpolated in screen space w/o distortion
  - Therefore interpolate (1/h,u/h, v/h) in screen space then **divide** interpolated (u/h,v/h) by interpolated (1/h) value.
  - Also known as **rational linear interpolation**
  - Linear interpolation can be done with a DDA as before
  - Early implementations attempt to reduce division costs
    - Quake: divide only every 16 pixels of scanline, & lin. interp.

Antialiasing & Compositing

CS4620 Lecture 15

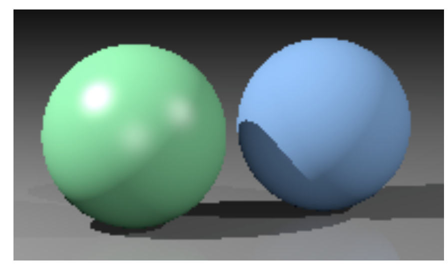
Pixel coverage

- Antialiasing and compositing both deal with questions of pixels that contain unresolved detail
- Antialiasing: how to carefully throw away the detail
- Compositing: how to account for the detail when combining images

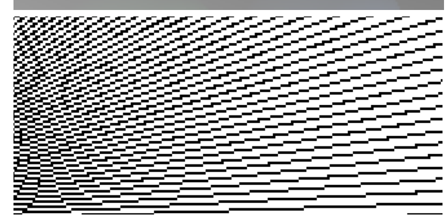
Aliasing

point sampling a continuous image:

continuous image defined by ray tracing procedure

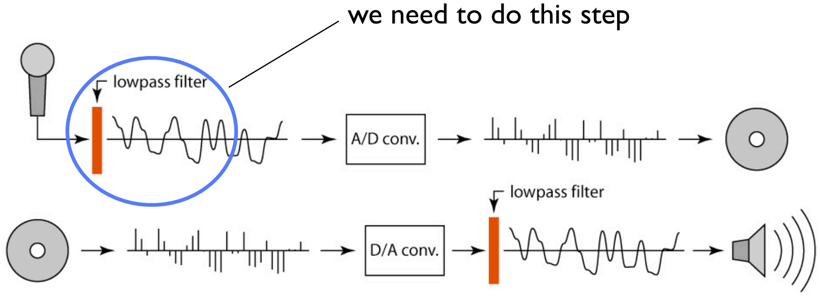


continuous image defined by a bunch of black rectangles



Signal processing view

- Recall this picture:

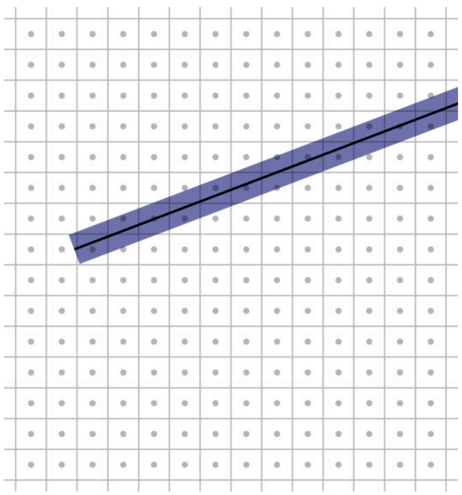


Antialiasing

- A name for techniques to prevent aliasing
- In image generation, we need to lowpass filter
  - Sampling the convolution of filter & image
  - Boils down to averaging the image over an area
  - Weight by a filter
- Methods depend on source of image
  - Rasterization (lines and polygons)
  - Point sampling (e.g. ray tracing)
  - Texture mapping

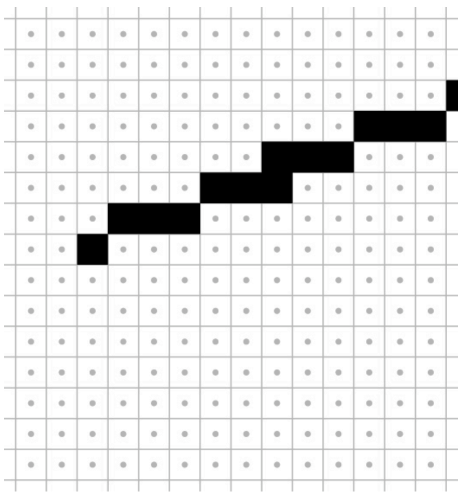
Rasterizing lines

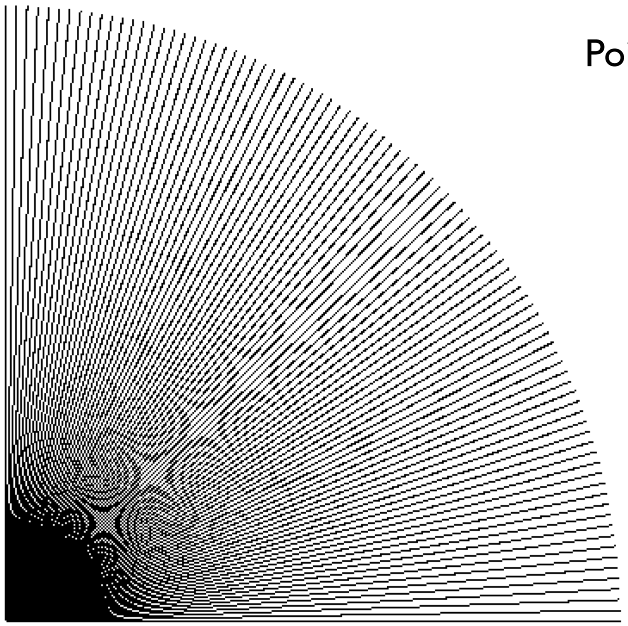
- Define line as a rectangle
- Specify by two endpoints
- Ideal image: black inside, white outside



Point sampling

- Approximate rectangle by drawing all pixels whose centers fall within the line
- Problem: all-or-nothing leads to jaggies
  - this is sampling with no filter (aka. point sampling)

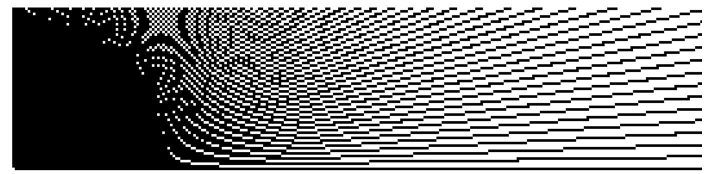




Point sampling
in action

Aliasing

- Point sampling is fast and simple
- But the lines have stair steps and variations in width
- This is an aliasing phenomenon
  - Sharp edges of line contain high frequencies
- Introduces features to image that are not supposed to be there!

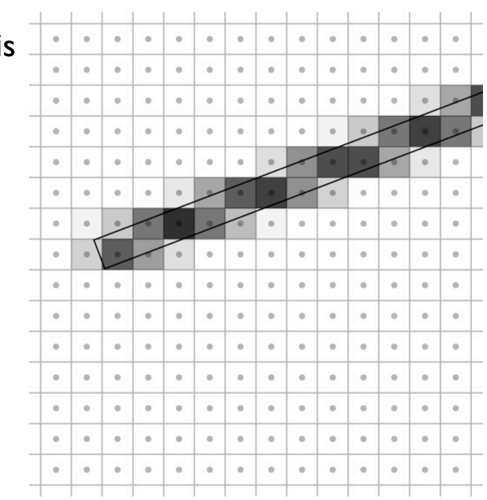


Antialiasing

- Point sampling makes an all-or-nothing choice in each pixel
  - therefore steps are inevitable when the choice changes
  - yet another example where discontinuities are bad
- On bitmap devices this is necessary
  - hence high resolutions required
  - 600+ dpi in laser printers to make aliasing invisible
- On continuous-tone devices we can do better

Antialiasing

- Basic idea: replace “is the image black at the pixel center?” with “how much is pixel covered by black?”
- Replace yes/no question with quantitative question.

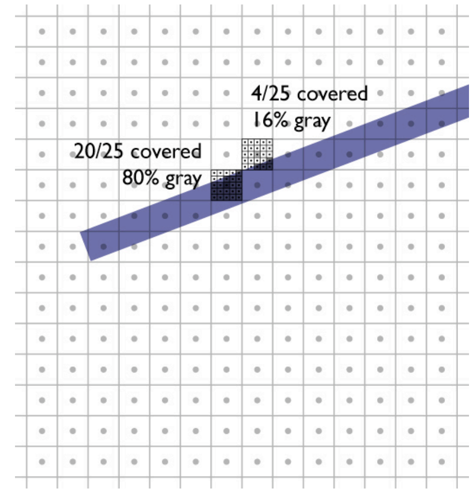


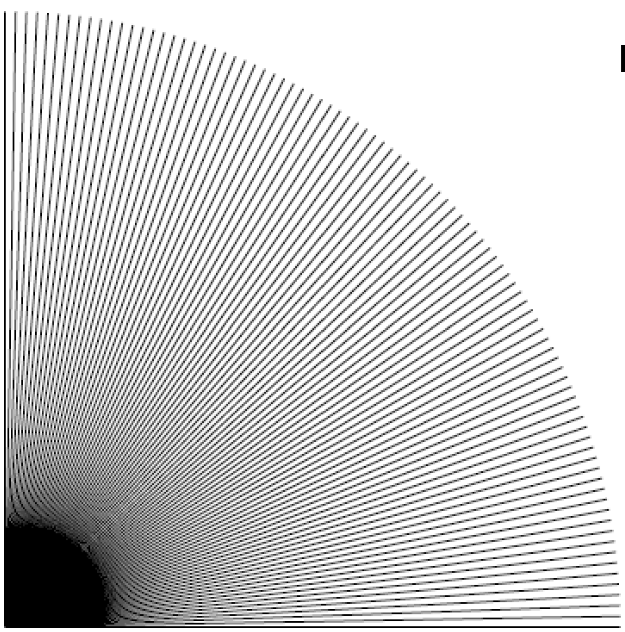
Box filtering

- Pixel intensity is proportional to area of overlap with square pixel area
- Also called “unweighted area averaging”

Box filtering by supersampling

- Compute coverage fraction by counting subpixels
- Simple, accurate
- But slow





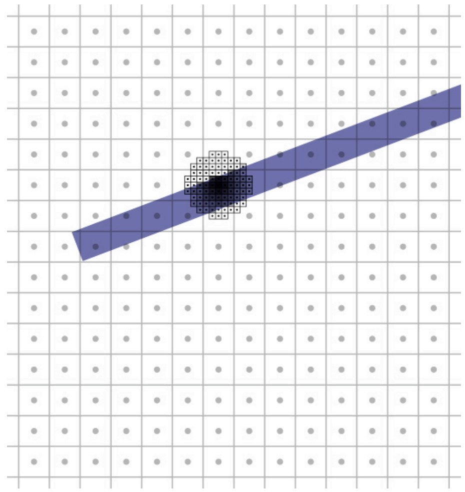
Box filtering in action

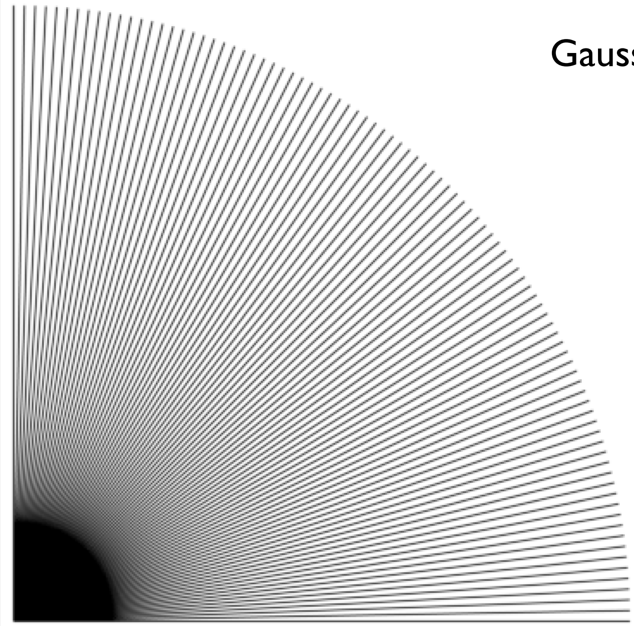
Weighted filtering

- Box filtering problem: treats area near edge same as area near center
  - results in pixel turning on “too abruptly”
- Alternative: weight area by a smoother filter
  - unweighted averaging corresponds to using a box function
  - sharp edges mean high frequencies
    - so want a filter with good extinction for higher freqs.
  - a Gaussian is a popular choice of smooth filter
  - important property: normalization (unit integral)

Weighted filtering by supersampling

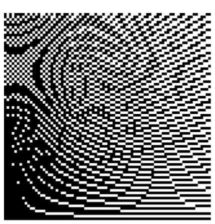
- Compute filtering integral by summing filter values for covered subpixels
- Simple, accurate
- But really slow



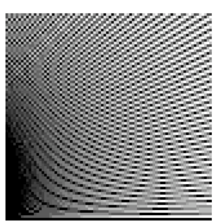


Gaussian filtering in action

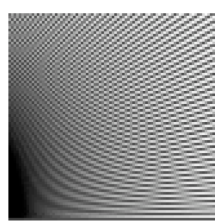
Filter comparison



Point sampling



Box filtering



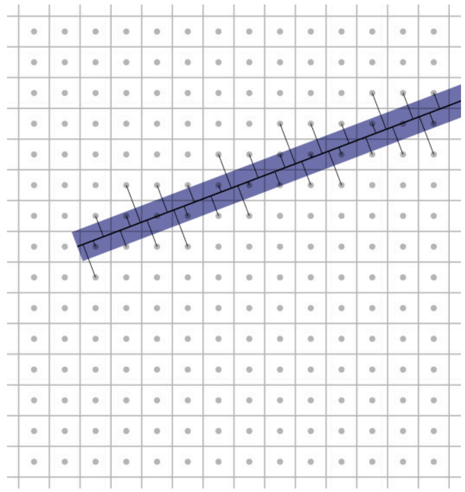
Gaussian filtering

Antialiasing and resampling

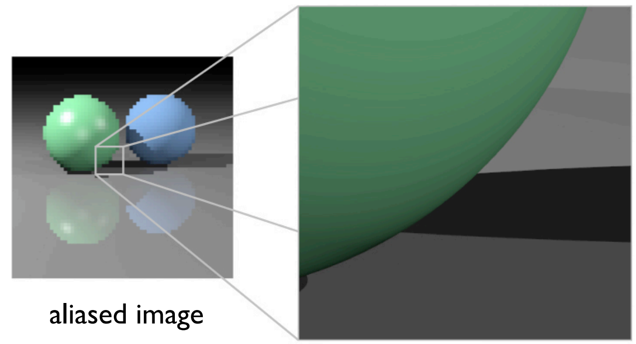
- Antialiasing by regular supersampling is *the same as* rendering a larger image and then resampling it to a smaller size
- Convolution of filter with high-res image produces an estimate of the area of the primitive in the pixel.
- So we can re-think this
  – one way: we're computing area of pixel covered by primitive
  – another way: we're computing average color of pixel
    - this way generalizes easily to arbitrary filters, arbitrary images

More efficient antialiased lines

- Filter integral is the same for pixels the same distance from the center line
- Just look up in precomputed table based on distance
  - Gupta-Sproull
- Does not handle ends…

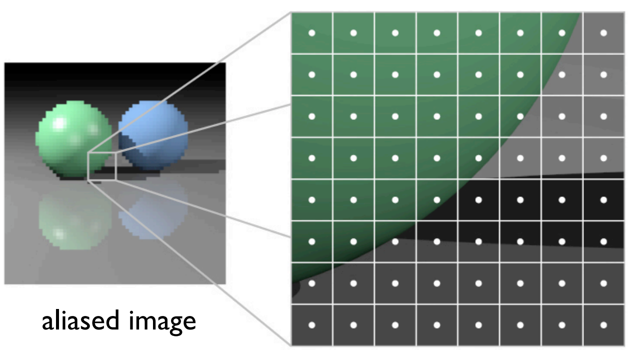


Antialiasing in ray tracing



aliased image

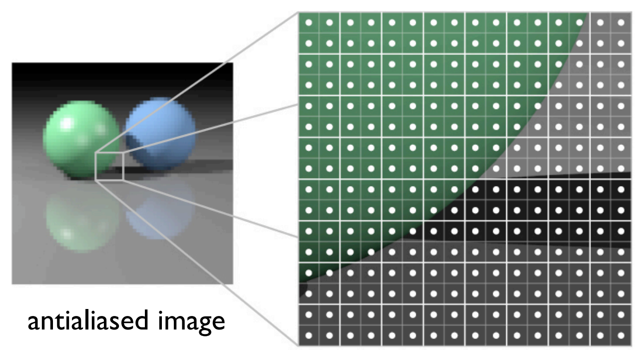
Antialiasing in ray tracing



aliased image

one sample per pixel

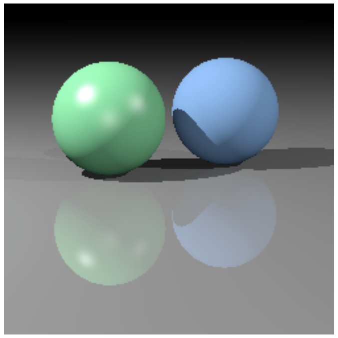
Antialiasing in ray tracing



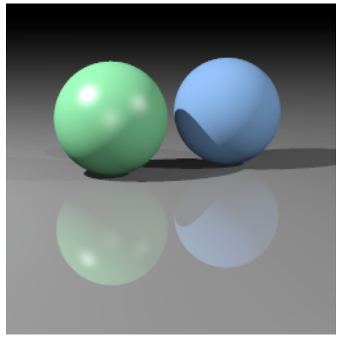
antialiased image

four samples per pixel

Antialiasing in ray tracing



one sample/pixel

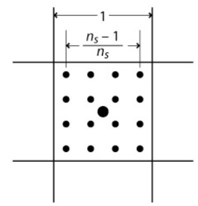


9 samples/pixel

Details of supersampling

- For image coordinates with integer pixel centers:

```
// one sample per pixel
for iy = 0 to (ny–1) by 1
  for ix = 0 to (nx–1) by 1 {
    ray = camera.getRay(ix, iy);
    image.set(ix, iy, trace(ray));
  }
```




```
// ns^2 samples per pixel
for iy = 0 to (ny–1) by 1
  for ix = 0 to (nx–1) by 1 {
    Color sum = 0;
    for dx = –(ns–1)/2 to (ns–1)/2 by 1
      for dy = –(ns–1)/2 to (ns–1)/2 by 1 {
        x = ix + dx / ns;
        y = iy + dy / ns;
        ray = camera.getRay(x, y);
        sum += trace(ray);
      }
    image.set(ix, iy, sum / (ns*ns));
  }
```


Details of supersampling

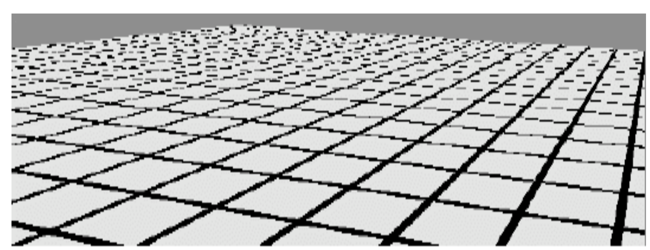
- For image coordinates in unit square

```
// one sample per pixel
for iy = 0 to (ny–1) by 1
  for ix = 0 to (nx–1) by 1 {
    double x = (ix + 0.5) / nx;
    double y = (iy + 0.5) / ny;
    ray = camera.getRay(x, y);
    image.set(ix, iy, trace(ray));
  }
```

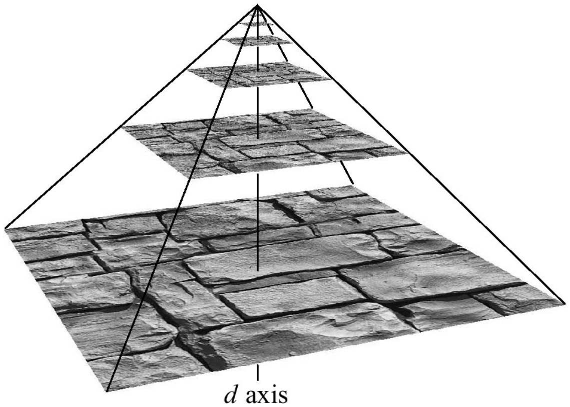
```
// ns^2 samples per pixel
for iy = 0 to (ny–1) by 1
  for ix = 0 to (nx–1) by 1 {
    Color sum = 0;
    for dx = 0 to (ns–1) by 1
      for dy = 0 to (ns–1) by 1 {
        x = (ix + (dx + 0.5) / ns) / nx;
        y = (iy + (dy + 0.5) / ns) / ny;
        ray = camera.getRay(x, y);
        sum += trace(ray);
      }
    image.set(ix, iy, sum / (ns*ns));
  }
```


Antialiasing in textures

- Would like to render textures with one (or few) s/p
- Need to filter first!
  - perspective produces very high image frequencies

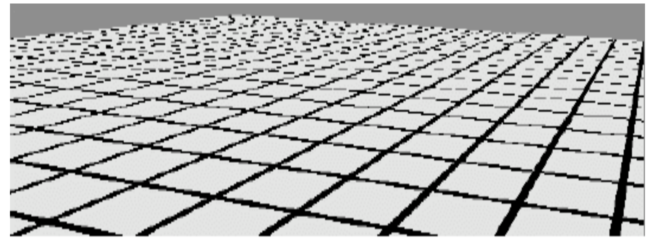


Mipmap image pyramid

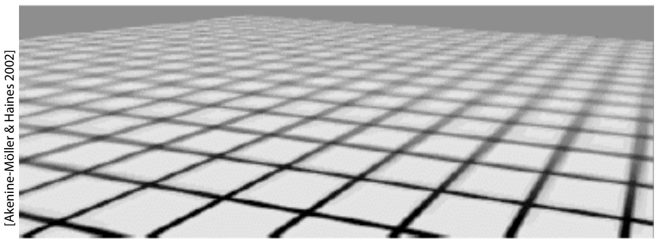


[Akenine-Möller & Haines 2002]

Texture minification



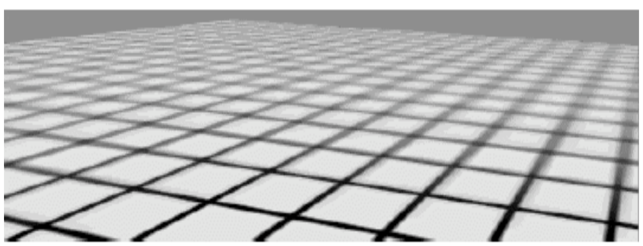
point sampled minification



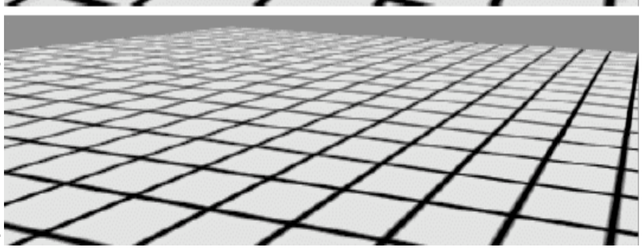
mipmap minification

[Akenine-Möller & Haines 2002]

Texture minification



mipmap minification



higher quality minification

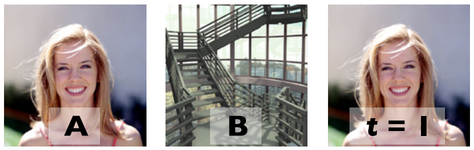
[Akenine-Möller & Haines 2002]

Compositing

[*Titanic* ; DigitalDomain; vfxhq.com]

Combining images

- Often useful combine elements of several images
- Trivial example: video crossfade
  - smooth transition from one scene to another



$$r_C = t r_A + (1-t) r_B$$
$$g_C = t g_A + (1-t) g_B$$
$$b_C = t b_A + (1-t) b_B$$

  - note: weights sum to 1.0
    - no unexpected brightening or darkening
    - no out-of-range results
  - this is *linear interpolation*

Foreground and background

- In many cases just adding is not enough
- Example: compositing in film production
  - shoot foreground and background separately
  - also include CG elements
  - this kind of thing has been done in analog for decades
  - how should we do it digitally?

Foreground and background

- How we compute new image varies with position

[Chuang et al. / Corel]

- Therefore, need to store some kind of tag to say what parts of the image are of interest

Binary image mask

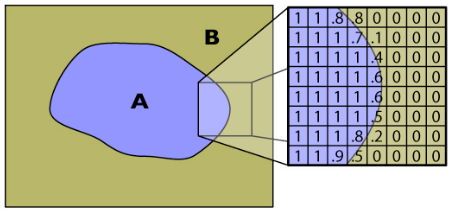
- First idea: store one bit per pixel
  - answers question "is this pixel part of the foreground?"

[Chuang et al. / Corel]

  - causes jaggies similar to point-sampled rasterization
  - same problem, same solution: intermediate values

Partial pixel coverage

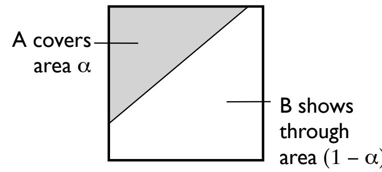
- The problem: pixels near boundary are not strictly foreground or background



  – how to represent this simply?
  – interpolate boundary pixels between the fg. and bg. colors

Alpha compositing

- Formalized in 1984 by Porter & Duff
- Store fraction of pixel covered, called α



$$C = A \textbf{ over } B$$

$$\begin{aligned} r_C &= \alpha_A r_A + (1 - \alpha_A) r_B \\ g_C &= \alpha_A g_A + (1 - \alpha_A) g_B \\ b_C &= \alpha_A b_A + (1 - \alpha_A) b_B \end{aligned}$$

  – this exactly like a spatially varying crossfade
- Convenient implementation
  – 8 more bits makes 32
  – 2 multiplies + 1 add per pixel for compositing

Alpha compositing—example

[Chuang et al./ Corel]

Compositing composites

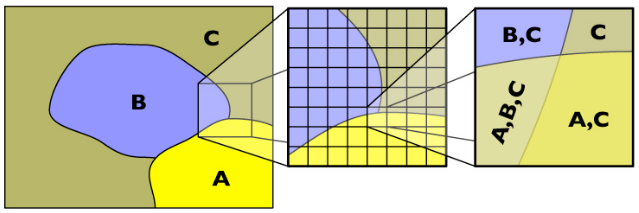
- so far have only considered single fg. over single bg.
- in real applications we have *n* layers
  – *Titanic* example
  – compositing foregrounds to create new foregrounds
    - what to do with α?
- desirable property: associativity

$$A \textbf{ over } (B \textbf{ over } C) = (A \textbf{ over } B) \textbf{ over } C$$

  – to make this work we need to be careful about how α is computed

Compositing composites

- Some pixels are partly covered in more than one layer



– in D = A **over** (B **over** C) what will be the result?

$$c_D = \alpha_A c_A + (1 - \alpha_A)[\alpha_B c_B + (1 - \alpha_B) c_C]$$
$$= \alpha_A c_A + (1 - \alpha_A)\alpha_B c_B + (1 - \alpha_A)(1 - \alpha_B) c_C$$

Fraction covered by neither A nor B

Associativity?

- What does this imply about (A **over** B)?
  – Coverage has to be

$$\alpha_{(A \textbf{ over } B)} = 1 - (1 - \alpha_A)(1 - \alpha_B)$$
$$= \alpha_A + (1 - \alpha_A)\alpha_B$$

  – …but the color values then don't come out nicely in D = (A **over** B) **over** C:

$$c_D = \alpha_A c_A + (1 - \alpha_A)\alpha_B c_B + (1 - \alpha_A)(1 - \alpha_B) c_C$$
$$= \alpha_{(A \textbf{ over } B)}(\cdots) + (1 - \alpha_{(A \textbf{ over } B)}) c_C$$

An optimization

- Compositing equation again

$$c_C = \alpha_A c_A + (1 - \alpha_A) c_B$$

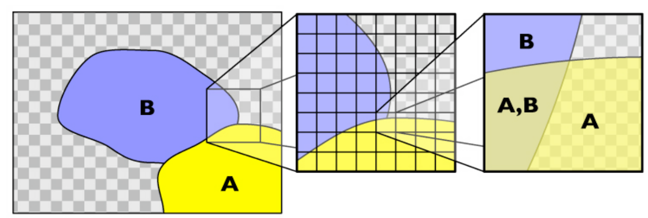
- Note c_A appears only in the product $\alpha_A c_A$
  – so why not do the multiplication ahead of time?
- Leads to ***premultiplied alpha***:
  – store pixel value (r', g', b', α) where $c' = \alpha c$
  – C = A **over** B becomes

$$c'_C = c'_A + (1 - \alpha_A) c'_B$$

  – Turns out to be more than an optimization…
  – Hint: so far the background has been opaque!

Compositing composites

- What about just C = A **over** B (with B transparent)?

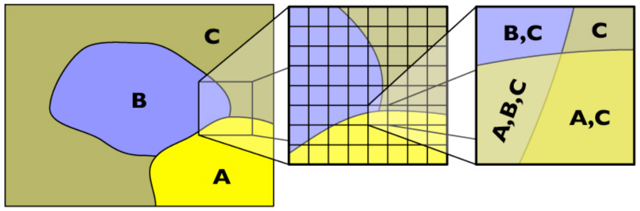


  – in premultiplied alpha, the result

$$\alpha_C = \alpha_A + (1 - \alpha_A)\alpha_B$$

looks just like blending colors, and it leads to associativity.

Associativity!

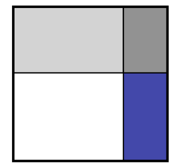


$$\begin{aligned} c_D &= c'_A + (1-\alpha_A)[c'_B + (1-\alpha_B)c'_C] \\ &= [c'_A + (1-\alpha_A)c'_B] + (1-\alpha_A)(1-\alpha_B)c'_C \\ &= c'_{(A \textbf{ over } B)} + (1-\alpha_{(A \textbf{ over } B)})c'_C \end{aligned}$$

– This is another good reason to premultiply

Independent coverage assumption

- Why is it reasonable to blend α like a color?
- Simplifying assumption: covered areas are independent
  – that is, uncorrelated in the statistical sense

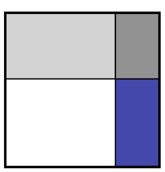


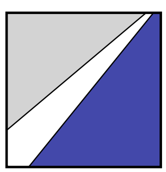
description	area
$\bar{A} \cap \bar{B}$	$(1-\alpha_A)(1-\alpha_B)$
$A \cap \bar{B}$	$\alpha_A(1-\alpha_B)$
$\bar{A} \cap B$	$(1-\alpha_A)\alpha_B$
$A \cap B$	$\alpha_A \alpha_B$

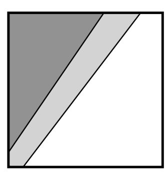
[Porter & Duff 84]

Independent coverage assumption

- Holds in most but not all cases


this


not this

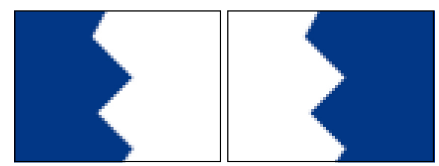

or this

- This will cause artifacts
  – but we'll carry on anyway because it is simple and usually works…

Alpha compositing—failures

[Chuang et al./ Corel] [Cornell PCG]


positive correlation:
too much foreground



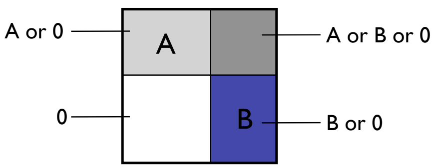

negative correlation:
too little foreground

Other compositing operations

- Generalized form of compositing equation:

$$\alpha C = A \ \mathbf{op} \ B$$

$$c = F_A a + F_B b$$



1 x 2 x 3 x 2 = 12 reasonable choices

operation	quadruple	diagram	F_A	F_B
clear	(0,0,0,0)		0	0
A	(0,A,0,A)		1	0
B	(0,0,B,B)		0	1
A **over** B	(0,A,B,A)		1	$1-\alpha_A$
B **over** A	(0,A,B,B)		$1-\alpha_B$	1
A **in** B	(0,0,0,A)		α_B	0
B **in** A	(0,0,0,B)		0	α_A
A **out** B	(0,A,0,0)		$1-\alpha_B$	0
B **out** A	(0,0,B,0)		0	$1-\alpha_A$
A **atop** B	(0,0,B,A)		α_B	$1-\alpha_A$
B **atop** A	(0,A,0,B)		$1-\alpha_B$	α_A
A **xor** B	(0,A,B,0)		$1-\alpha_B$	$1-\alpha_A$

[Porter & Duff 84]

