CS 4620 Homework 7: Convoluted with Splines

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Problem 1: Convolution Theory (*Shirley and Marschner (3rd ed.)*, Ch.9, Question 1)

(a) Show that discrete convolution is commutative and associative.
(b) Do the same for continuous convolution.

Problem 2: Separable-Kernel Convolution (*Shirley and Marschner (3rd ed.)*, Ch.9.3, page 209)

Given the 5-by-5 image, I,

0	1	0	1	0
1	0	1	0	3
0	1	0	1	0
1	0	1	0	1
0	3	0	1	3

apply the 2D box filter of "radius" $r = 1$—the filter mask is 3-by-3. Exploit its separable nature by performing the two-pass filter algorithm. Boundary cases should be handled by ignoring missing values and renomalizing the filter, i.e., so that weights for available values sum to one. Show the intermediate matrix, $I^{(1)}$, resulting from the first pass. (Note that the image matrix is symmetric, so that you can transform in either x first or y first, with the difference in $I^{(1)}$ being only a transpose.)

Problem 3: Quintic Hermite Splines with Acceleration Control

You are familiar with the matrix form of the cubic Hermite curve

$$\mathbf{p}(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{v}_0 \\ \mathbf{v}_1 \end{bmatrix}$$

with end-point "position" and "velocity" data,

$$\mathbf{p}(0) = \mathbf{p}_0, \quad \mathbf{p}(1) = \mathbf{p}_1, \quad \mathbf{p}'(0) = \mathbf{v}_0, \quad \mathbf{p}'(1) = \mathbf{v}_1.$$

Derive the matrix $\mathbf{M}$ in the matrix form of a quintic Hermite curve,

$$\mathbf{p}(t) = \begin{bmatrix} t^5 & t^4 & t^3 & t^2 & t & 1 \end{bmatrix} \mathbf{M} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{v}_0 \\ \mathbf{v}_1 \\ \mathbf{a}_0 \\ \mathbf{a}_1 \end{bmatrix}$$

which also allows specification of end-point accelerations,

$$\mathbf{p}''(0) = \mathbf{a}_0, \quad \mathbf{p}''(1) = \mathbf{a}_1.$$