CS 4620 Homework 2: Blobby and Platonic Shadows

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Problem 1: Blobby Shadows (An exercise in tracing rays)

In this question you will determine if a point on a plane associated with a viewing ray is in an ellipsoidal object's shadow. The scene consists of three things:

- a point light located at position $\mathbf{l}$;
- an ellipsoidal surface given by the implicit function

$$0 = f(\mathbf{x}) = s_x(x - x_e)^2 + s_y(y - y_e)^2 + s_z(z - z_e)^2 - R^2 \quad (1)$$

  where $\mathbf{x} = (x, y, z)^T$, and s_x, s_y, s_z, and R are all positive values; and
- a planar surface given by the implicit function

$$0 = g(\mathbf{x}) = ax + by + cz + h. \quad (2)$$

  For simplicity you can assume the plane is oriented with the plane's normal (a, b, c) pointing into the halfspace containing the ellipsoid, the light, and the viewer's eye.

Consider a perspective viewing ray, $\mathbf{r}(t) = \mathbf{e} + t\mathbf{d}$ with $t > 0$, that is entering the scene:

1. Draw a labeled diagram of the scene, and include the viewing ray, and intersection points of the following steps.
2. Determine a test to see if the viewing ray hits the plane, and, if so, at what $t = t_{hit}$ and $\mathbf{r} = \mathbf{r}_{hit}$ values.
3. Determine a test to see if the point $\mathbf{r}_{hit}$ is in the ellipsoid's shadow. *(Hint: you will need to generate a shadow ray first.)*

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Problem 2: Platonic Shadows (... because blobs are so 90's)

Platonic solids are convex polyhedrons that are regular (see inset)—note that that the Utah teapot is not platonic, we just sometimes forget. In this question, we will replace the ellipsoid in Question #1 by a platonic solid. You will exploit the fact that each convex shape can be described as the intersection of a set of N half-spaces: $N=4$ for the tetrahedron, $N=6$ for the cube, etc.

Let $\mathbf{c}$ denote the center of a given platonic solid, and let the N half-spaces be represented by

$$\mathcal{H}_k = \{\mathbf{x} \mid \mathbf{n}_k^T(\mathbf{x}-\mathbf{c}) \le r_N\}, \quad k = 1 \ldots N, \tag{3}$$

where $\mathbf{n}_k$ are the outward-facing normals of each face, and $r_N > 0$ depends on the solid (N) used. The platonic solid $\mathcal{S}$ is the intersection of the N half-spaces:

$$\mathcal{S} = \bigcap_{k=1}^{N} \mathcal{H}_k. \tag{4}$$

Repeat the last step of question #1: Determine a test to see if the point $\mathbf{r}_{hit}$ is in the shadow of a platonic solid. Similar to the ray-cube intersection considered in class where ray-slab intersections were used, decompose the ray-PlatonicSolid test into simpler ray–half-space tests. Devise an expression to evaluate the ray's intersecting t range by taking the intersection of ray–half-space t ranges. *What is the simple test to see if the point is in the shadow? Draw a diagram to support your reasoning.*