CS 465 Homework 4 Solution

1&2 Since (u_i, v_i) is the image of (x_i, y_i), we have

$$M \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix}$$

Note the points are linearly independent, the matrix on the left hand side is invertible. Thus

$$M = \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix}^{-1}$$

For question 1, the answer is

$$M = \begin{bmatrix} 0.17 & 0.06 & -0.44 \\ -0.02 & 0.09 & 0.59 \\ 0 & 0 & 1 \end{bmatrix}$$

3&4* Let $\vec{p}_i$ denote the column vector $(x_i, y_i, 1)^T$, and $\vec{q}_i$ denote $(u_i, v_i, 1)^T$, we have

$$M \begin{bmatrix} | & | & | & | \\ \vec{p}_1 & \vec{p}_2 & \vec{p}_3 & \vec{p}_4 \\ | & | & | & | \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ \lambda_1\vec{q}_1 & \lambda_2\vec{q}_2 & \lambda_3\vec{q}_3 & \lambda_4\vec{q}_4 \\ | & | & | & | \end{bmatrix}$$

for some scalers λ_i. Since λM also satisfies the above equation, we don't have to insist on the constraint that $m_{33} = 1$.

First of all, starting from four intermediate points (in homogeneous coordinates) $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$, $(1, 1, 1)$, we find a transformation, M', which transforms those intermediate points to $\vec{q}_i$'s, thus we have

$$M' \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} | & | & | & | \\ \mu_1\vec{q}_1 & \mu_2\vec{q}_2 & \mu_3\vec{q}_3 & \mu_4\vec{q}_4 \\ | & | & | & | \end{bmatrix}$$

Note the first 3 column of the matrix in the left hand side is an identical matrix, we have

$$M' = \begin{bmatrix} | & | & | \\ \mu_1\vec{q}_1 & \mu_2\vec{q}_2 & \mu_3\vec{q}_3 \\ | & | & | \end{bmatrix}$$

and for the 4th column,

$$\begin{bmatrix} | & | & | \\ \mu_1\vec{q}_1 & \mu_2\vec{q}_2 & \mu_3\vec{q}_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} | \\ \mu_4\vec{q}_4 \\ | \end{bmatrix}$$

which is equivalant to

$$\begin{bmatrix} | & | & | \\ \vec{q}_1 & \vec{q}_2 & \vec{q}_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} = \mu_4 \begin{bmatrix} | \\ \vec{q}_4 \\ | \end{bmatrix}$$

Let $\mu_4 = 1$ we have $(\mu_1, \mu_2, \mu_3)^T = \begin{bmatrix} \vec{q}_1 & \vec{q}_2 & \vec{q}_3 \end{bmatrix}^{-1} \vec{q}_4$, and

$$M' = \begin{bmatrix} | & | & | \\ \vec{q}_1 & \vec{q}_2 & \vec{q}_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} \mu_1 & & \\ & \mu_2 & \\ & & \mu_3 \end{bmatrix}$$

By changing $\vec{q}_i$'s to $\vec{p}_i$'s, we can similarly find another matrix M'' that transforms from those intermediate points to $\vec{p}_i$, so the final matrix is $M = M'M''^{-1}$.

For question 3, by applying the above method, and divid the matrix by the m_{33}, which makes the right bottom corner of the matrix to be 1, then we get

$$M = \begin{bmatrix} 0.0960 & -0.0472 & 0.0354 \\ -0.0159 & -0.0232 & 0.6244 \\ -0.0177 & -0.1024 & 1 \end{bmatrix}$$