CS 4620 Homework 3: Transformations

out: Sunday 14 September 2008
due: Friday 17 September 2008

1. A math joke (from http://haha.nu/funny/funny-math/, where you can find more jokes):

> After explaining to a student through various lessons and examples that:
>
> $$\lim_{x \to 8} \frac{1}{x-8} = \infty$$
>
> I tried to check if the student really understood that, so I gave a different example. This was the result:
>
> $$\lim_{x \to 5} \frac{1}{x-5} = \rotatebox{90}{5}$$

From the view today the student actually learnt to perform a 2D transformation. Let's explore the transformation in the joke. In the following questions, you are required to express all the tranformations by 3x3 homogeneous transformation matrices.

(a) Suppose the characters "8" and "∞" are both symmetric in the particular font (Fig. 1), there are four different transformations that can transform from figure 1(a) to 1(b). Compute these transformations.

(b) Among the four transformations, only one can transform from figure 2(a) to 2(b). Point out which one.

(c) Choosing different origins will change the transformation matrix. Compute the transformation matrix for figure 2, if we place the new origin at the point $(0.8, 0)$.

2. Explore the commutativity of transformations. For two 3D transformations $\mathbf{A}$ and $\mathbf{B}$, we say $\mathbf{A}$ and $\mathbf{B}$ are commutative if their compositions $\mathbf{AB} = \mathbf{BA}$. Now define the following three 3D transformations:

   i. Rotation $\mathbf{R}$ which transforms $+x$ axis to $+y$, $+y$ to $+z$, and $+z$ to $+x$;

   ii. Scale $\mathbf{S}$ which scales along x, y and z axes by factors 2, 3 and 4, respectively;

   iii. Translation $\mathbf{T}$ which moves along x, y and z axes by 3, 2 and 1 units, repsectively.

Questions:

(a) Write $\mathbf{R}$, $\mathbf{S}$ and $\mathbf{T}$ in 4x4 homogeneous matrices;

(b) Compute the compositions $\mathbf{RS}$, $\mathbf{SR}$, $\mathbf{ST}$, $\mathbf{TS}$, $\mathbf{TR}$ and $\mathbf{RT}$. Is $\mathbf{R}$ and $\mathbf{S}$ commutative? How about $\mathbf{S}$ and $\mathbf{T}$, $\mathbf{T}$ and $\mathbf{R}$?

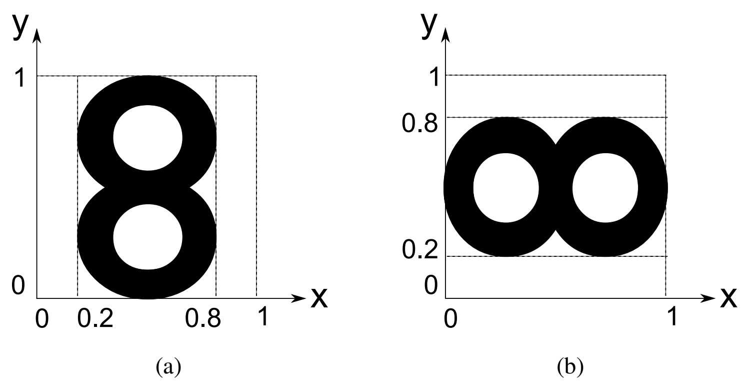

Figure 1: Characters “8” and “∞”

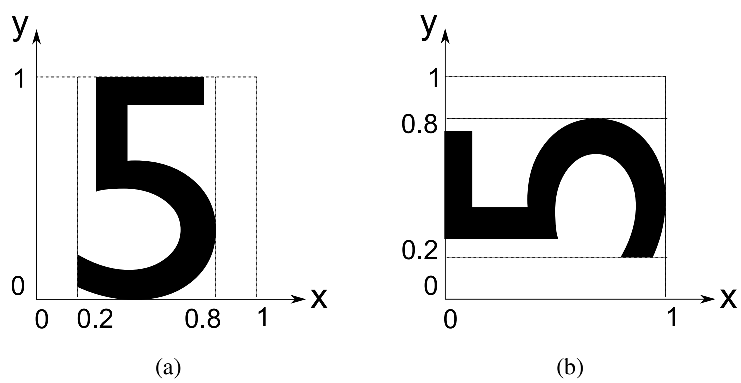

Figure 2: Characters “5” and “[illegible]”

(c) Fill in the following table with commutativity of transformations in general. For instance, if any rotation is commutative with any translation, fill row 1 col 3 with $\surd$, otherwise $\times$.

	Rotation	Scale	Translation
Rotation			
Scale			
Translation			

(d)* For all $\times$ cells in the table, find out the sufficient and necessary conditions under which the corresponding types of transformations becomes commutative.