

# CS4620 Homework 1 Solution

1. What is the screen resolution and refresh rate of your display? What is the data rate flowing across the cable to the display? State any assumptions you had to make to answer.

Answer: Some typical data rates:

$$1024 \times 768, 24\text{bpp}, 60\text{Hz}: 1.05\text{Gbps}$$

$$1200 \times 900, 24\text{bpp}, 60\text{Hz}: 1.49\text{Gbps}$$

$$1280 \times 800, 24\text{bpp}, 60\text{Hz}: 1.37\text{Gbps}$$

$$1440 \times 900, 24\text{bpp}, 60\text{Hz}: 1.74\text{Gbps}$$

2. Find out the transfer function of your display. You can find a gamma correction applet on the course website. The applet displays an area of alternating black and white horizontal stripes with a grey patch in the middle. A slider controls the pixel value of the grey patch. Look from a distance where the black and white stripes blend together, and slide the control until the patch can hardly be distinguished from the surrounding area. What is the pixel value of the grey patch? Assuming that your display's transfer function fits the standard gamma model, what is gamma for your display? (For simplicity, assume the pixel value reported by the applet is being sent directly to the display.)

Answer: Suppose we get a pixel value  $a$ , then we have  $\left(\frac{a}{255}\right)^\gamma = 0.5$ , thus  $\gamma = \frac{\log 0.5}{\log(a/255)}$ .

3. Consider an 8-bit display whose gamma is 2, with a maximum display intensity of  $I_{\max}$  and viewing flare of  $I_{\min}$ . This means you will observe an intensity of  $I_{\min}$  if the pixel value is 0, while the maximum total intensity is  $I_{\min} + I_{\max}$ . Assuming that a relative difference in intensity of  $\geq 2\%$  is visually noticeable, then which of the 255 steps from one pixel value to the next are noticeable under the following conditions:

- (a)  $I_{\min} = 0$  (no viewing flare);
- (b)  $I_{\min} = 0.01I_{\max}$  (1% viewing flare);
- (c) do (a) and (b) for a non gamma corrected display ( $\gamma = 1$ ).

For instance, if pixel value 123 produces intensity  $x$ , then the step between 123 and 124 is visible if and only if pixel value 124 produces intensity at least  $1.02x$ .

Answer: Let  $I(a)$  denote the observed intensity when pixel value is  $a$ .

- (a)  $I(a) = I_{\max} \cdot \left(\frac{a}{255}\right)^2$ , let  $\frac{I(a+1)}{I(a)} = \left(\frac{a+1}{a}\right)^2 \geq 1.02$ , we have  $a \leq 100.5$ . So steps  $0 \rightarrow 1, 1 \rightarrow 2, \dots, 100 \rightarrow 101$  are noticeable.
- (b)  $I(a) = I_{\max} \cdot \left[\left(\frac{a}{255}\right)^2 + 0.01\right]$ , let  $\frac{I(a+1)}{I(a)} \geq 1.02$ , we have  $6.4 \leq a \leq 93.6$ . So steps  $7 \rightarrow 8, 8 \rightarrow 9, \dots, 93 \rightarrow 94$  are noticeable.
- (c)  $I(a) = I_{\max} \cdot \frac{a}{255}$ , let  $\frac{I(a+1)}{I(a)} = \frac{a+1}{a} \geq 1.02$ , we have  $a \leq 50$ . So steps  $0 \rightarrow 1, 1 \rightarrow 2, \dots, 50 \rightarrow 51$  are noticeable. It is OK if we assume  $\frac{I(a+1)}{I(a)} > 1.02$  and get  $a < 50$ .

(d)  $I(a) = I_{\max} \cdot \left(\frac{a}{255} + 0.01\right)$ , let  $\frac{I(a+1)}{I(a)} \geq 1.02$ , we have  $a \leq 47.45$ . So steps  $0 \rightarrow 1$ ,  $1 \rightarrow 2, \dots, 47 \rightarrow 48$  are noticeable.

4. Linear quantization vs. gamma correction. For a given quantization method, we define the “precision” of the method to be the maximum relative difference between two sequential intensity levels, paying attention only to the levels that are at least 1% of the maximum intensity. For this problem, disregard viewing flare in the computations.

(a) What is the “precision” of the 8-bit gamma-2.2 encoding?

(b) How many bits per pixel do we need, with a linear quantization, to be as “precise” as the above gamma encoding?

Answer: Let  $I(a)$  denote the intensity when pixel value is  $a$ , the relative difference between levels  $a$  and  $a + 1$  is  $\frac{I(a+1)}{I(a)} - 1 = \left(\frac{a+1}{a}\right)^\gamma - 1$ , in which  $\gamma = 2.2$  for (a) and  $\gamma = 1$  for (b). For either of the cases, the relative difference decreases when  $a$  increases, so the maximum relative difference will always be found between the first two steps above  $0.01I_{\max}$ .

(a) Let  $I(a) = \left(\frac{a}{255}\right)^{2.2} \cdot I_{\max} \geq 0.01I_{\max}$ , we have  $a \geq 31.4$ . Thus the first two steps above  $0.01I_{\max}$  are 32 and 33. Consequently the “precision” is  $\frac{I(33)}{I(32)} - 1 = 7\%$ .

(b) Suppose  $a$  and  $a + 1$  are the first two steps above  $0.01I_{\max}$ . To make it more “precise” than the gamma encoding, we need  $I(a + 1) \leq 1.07I(a)$ , which means the step size  $I(a + 1) - I(a) \leq 0.07I(a)$ . Note  $I(a)$  is the first step above  $0.01I_{\max}$ , so it is slightly larger than  $0.01I_{\max}$ . As a result, we can roughly say the step size  $I(a+1) - I(a) \leq 0.07 \cdot 0.01I_{\max} = 0.0007I_{\max}$ . Since it is linear quantization, the number of steps is at least  $\frac{I_{\max}}{0.0007I_{\max}} = 1428$ . Finally we would say 11 bits is required because  $2^{10} < 1428 < 2^{11}$ .