

# CS4450 Problem Set #2

## 1 CSMA/CD: Random Access

Let  $A$  and  $B$  be two stations attempting to transmit on an Ethernet. Each has a steady queue of frames ready to send;  $A$ 's frames are denoted  $A_1, A_2, \dots$ , and  $B$ 's are defined similarly.

Recall the random access protocol discussed in class. In case of a collision,  $A$  and  $B$  back off for  $d \times T$  time where  $T$  is the back off unit time and  $d \in D = \{0, \dots, 2^k - 1\}$ , where  $k$  is the number of collisions so far. You can think of selecting a  $d$  from  $D$  as choosing a time slot to transmit the packet from  $2^k$  future slots.

Suppose  $A$  and  $B$  simultaneously attempt to send their first frame, collide, and happen to choose back off times of  $0 \times T$  and  $1 \times T$ , respectively, meaning  $A$  wins the race and transmits  $A_1$  while  $B$  waits.

- a) At the end of the first transmission,  $B$  will attempt to retransmit  $B_1$  while  $A$  will attempt to transmit  $A_2$ . These attempts will collide. Now  $A$  will choose a waiting time in  $\{0 \times T, 1 \times T\}$ , while  $B$  will choose a waiting time in  $\{0 \times T, \dots, 3 \times T\}$ . What is the probability that  $A$  wins this second back off race?
- b) Suppose  $A$  wins the second back off race in (a).  $A$  transmits  $A_2$ , and when it is finished,  $A$  and  $B$  collide again as  $A$  tries to transmit  $A_3$  and  $B$  tries once more to transmit  $B_1$ . What is the probability that  $A$  wins this third back off race?
- c) Given that  $A$  wins the first three back off races, what is a lower bound for the probability that  $A$  wins all of the remaining back off races?
- d) In the case that (c) holds, what happens to the frame  $B_1$ ?

## 2 CSMA/CD: Random Access

Let  $A$  and  $B$  be two stations attempting to transmit a single packet on an Ethernet.

Recall that in case of the  $k$ th collision,  $A$  and  $B$  choose a  $d \in D = \{0, \dots, 2^k - 1\}$  and wait for  $d \times T$  time. Here,  $d$  is chosen randomly from the set  $D$ , where the probability of selecting any element in  $D$  is distributed uniformly.

- a) Let  $P_k$  be the probability of success after the  $k^{th}$  collision in the  $(k+1)^{th}$  attempt. Write  $P_k$  in terms of  $k$ .
- b) Let  $S_k$  be the probability of success in  $(k+1)$  attempts given there is a collision to start with. Write  $S_k$  in terms of  $k$ .
- c) Let  $S$  be the probability of success after  $k$  collisions, at some point in the future. Calculate  $S$ .

Now, we will consider when the probability of selecting elements from  $D$ , i.e. selecting a time slot, is not distributed uniformly.

Specifically, let

$$D = \{0, 1, 2, \dots, 2^k - 1\} \text{ and } P = \{p, 2p, 3p, \dots, 2^k p\}$$

where  $p$  is the solution to

$$p + 2p + 3p + \dots + 2^k p = 1.$$

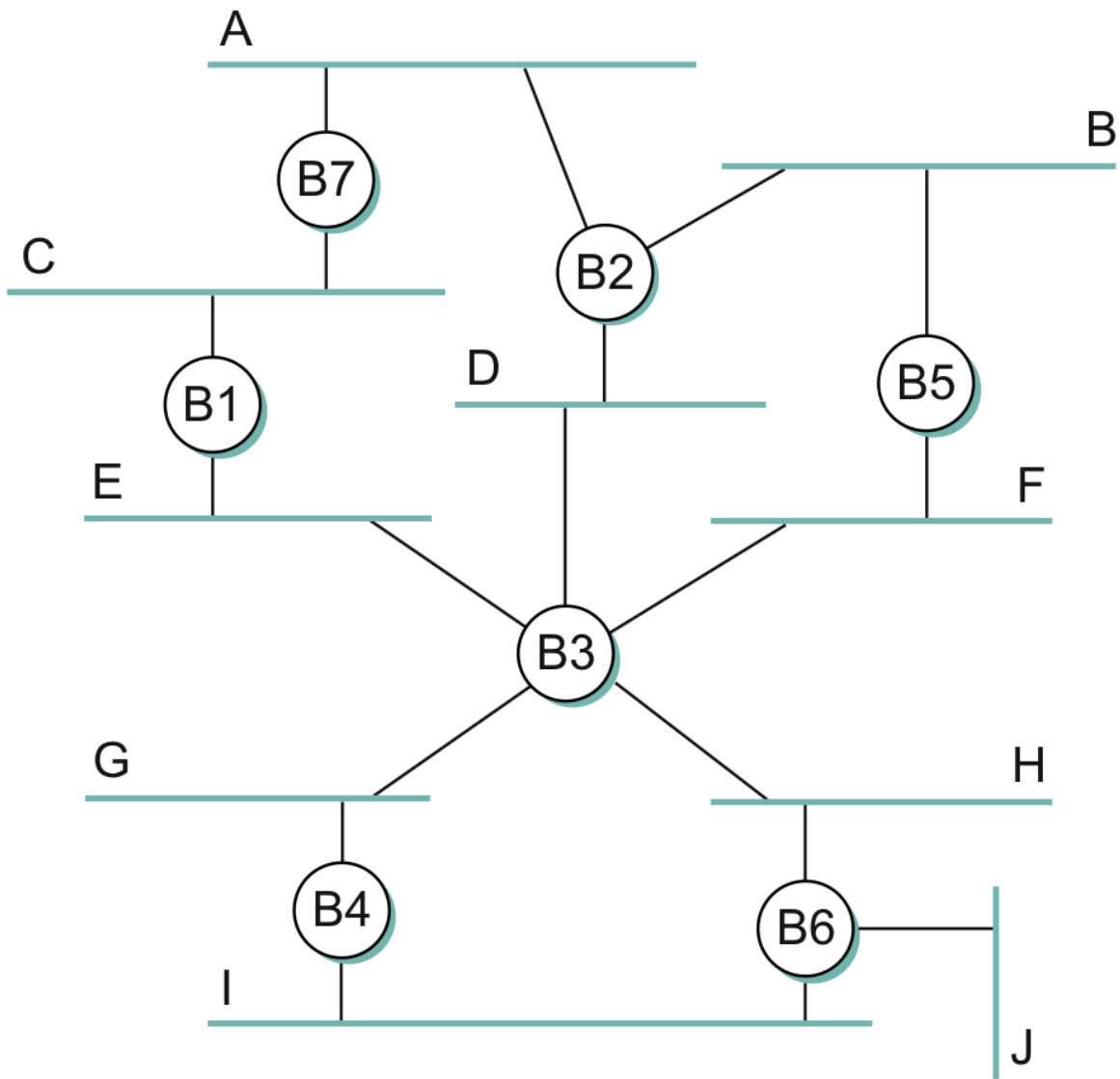
Let  $p_i$ , the  $i^{th}$  element of  $P$ , be the probability of choosing  $d_i$ , the  $i^{th}$  element of  $D$ . Let  $P_k$  and  $S_k$  be defined as before.

- d) Given the probability distribution above, calculate the probability of success in the second attempt, i.e.  $P_1$ .
- e) Calculate the probability of success in the third attempt, i.e.  $P_2$ . Calculate  $S_2$ .
- f) Write  $P_k$  and  $S_k$  in terms of  $k$ .

Now, assume there are 3 stations,  $A$ ,  $B$  &  $C$ , and a uniform probability distribution in choosing slots.

- g) Can we use the same method as we used in (a) & (b) to calculate  $P_k$  and  $Q_k$ ? Why/why not?

### 3 The Spanning Tree Algorithm



Above an extended LAN and its corresponding network graph is given.

- Which ports are selected by the spanning tree algorithm?
- Assume that the bridge *B1* fails. Which ports are selected by the spanning tree algorithm after the recovery process and a new tree has been formed?

## 4 Programming

Suppose  $N$  stations are waiting for another packet to finish on an Ethernet. All transmit at once when the packet is finished and collide.

Write a program to implement the simulation of this case up until the point when one of the  $N$  waiting stations succeeds. Model time as an integer,  $T$ , in units of slot times and treat collisions as taking one slot time (e.g. a collision at time  $T$  followed by a backoff of  $k = 0$  should result in a retransmission attempt at time  $T + 1$ ).

- a) Find the average delay before one station transmits successfully, for  $N=5$ ,  $N=10$ ,  $N = 20$ ,  $N = 40$ , and  $N = 100$ .
- b) Plot the average delay against the number of stations. How is delay related to the number of stations?