Relational Calculus

Chapter 4, Part B

Relational Calculus

- Comes in two flavors: *Tuple relational calculus* (TRC) and *Domain relational calculus* (DRC).
- Calculus has *variables, constants, comparison ops*, *logical connectives* and *quantifiers*.
  - *TRC*: Variables range over (i.e., get bound to) *tuples*.
  - *DRC*: Variables range over *domain elements* (= field values).
  - Both TRC and DRC are simple subsets of first-order logic.
- Expressions in the calculus are called *formulas*. An answer tuple is essentially an assignment of constants to variables that make the formula evaluate to *true*.

Domain Relational Calculus

- *Query* has the form:

$$\left\{\left\langle x1,x2,...,xn\right\rangle \mid p\left(\left\langle x1,x2,...,xn\right\rangle\right)\right\}$$

- *Answer* includes all tuples $\left\langle x1,x2,...,xn\right\rangle$ that make the *formula* $p\left(\left\langle x1,x2,...,xn\right\rangle\right)$ be *true*.
- *Formula* is recursively defined, starting with simple *atomic formulas* (getting tuples from relations or making comparisons of values), and building bigger and better formulas using the *logical connectives*.

DRC Formulas

- *Atomic formula:*
  - $\langle x1, x2, ..., xn \rangle \in Rname$, or X *op* Y, or X *op* constant
  - *op* is one of $<, >, =, \leq, \geq, \neq$
- *Formula:*
  - an atomic formula, or
  - $\neg p, p \wedge q, p \vee q$, where p and q are formulas, or
  - $\exists X(p(X))$, where variable X is *free* in p(X), or
  - $\forall X(p(X))$, where variable X is *free* in p(X)
- The use of quantifiers $\exists X$ and $\forall X$ is said to *bind* X.
  - A variable that is not bound is free.

Free and Bound Variables

- The use of quantifiers $\exists X$ and $\forall X$ in a formula is said to *bind* X.
  - A variable that is not bound is free.
- Let us revisit the definition of a query:

$$\{\langle x1, x2, ..., xn \rangle \mid p(\langle x1, x2, ..., xn \rangle)\}$$

- There is an important restriction: the variables x1, ..., xn that appear to the left of \`|' must be the *only* free variables in the formula p(...).

Find all sailors with a rating above 7

$$\{\langle I, N, T, A \rangle \mid \langle I, N, T, A \rangle \in Sailors \wedge T > 7\}$$

- The condition $\langle I, N, T, A \rangle \in Sailors$ ensures that the domain variables *I*, *N*, *T* and *A* are bound to fields of the same Sailors tuple.
- The term $\langle I, N, T, A \rangle$ to the left of \`|' (which should be read as *such that*) says that every tuple $\langle I, N, T, A \rangle$ that satisfies *T*>7 is in the answer.
- Modify this query to answer:
  - Find sailors who are older than 18 or have a rating under 9, and are called 'Joe'.

Find sailors rated > 7 who have reserved boat #103

$$\{\langle I,N,T,A\rangle \mid \langle I,N,T,A\rangle \in Sailors \wedge T>7 \wedge$$
$$\exists\ Ir,Br,D\ (\langle Ir,Br,D\rangle \in \text{Re}serves \wedge Ir=I \wedge Br=103)\}$$

- We have used $\exists\ Ir,\ Br,\ D\ (\ldots)$ as a shorthand for $\exists\ Ir\,(\exists\ Br\,(\exists\ D\,(\ldots)))$
- Note the use of $\exists$ to find a tuple in Reserves that `joins with' the Sailors tuple under consideration.

Find sailors rated > 7 who've reserved a red boat

$$\{\langle I,N,T,A\rangle \mid \langle I,N,T,A\rangle \in Sailors \wedge T>7 \wedge$$
$$\exists\ Ir,Br,D\,(\langle Ir,Br,D\rangle \in \text{Re}serves \wedge Ir=I \wedge$$
$$\exists\ B,BN,C\,(\langle B,BN,C\rangle \in Boats \wedge B=Br \wedge C='red'))\}$$

- Observe how the parentheses control the scope of each quantifier's binding.
- This may look cumbersome, but with a good user interface, it is very intuitive. (MS Access, QBE)

Find sailors who've reserved all boats

$$\{\langle I,N,T,A\rangle \mid \langle I,N,T,A\rangle \in Sailors \wedge$$
$$\forall\ B,BN,C\,(\neg(\langle B,BN,C\rangle \in Boats) \vee$$
$$(\exists\ Ir,Br,D\,(\langle Ir,Br,D\rangle \in \text{Re}serves \wedge I=Ir \wedge Br=B)))\}$$

- Find all sailors *I* such that for each 3-tuple $\langle B,BN,C\rangle$ either it is not a tuple in Boats or there is a tuple in Reserves showing that sailor *I* has reserved it.

Find sailors who've reserved all boats (again!)

$$\{\langle I,N,T,A\rangle \mid \langle I,N,T,A\rangle \in Sailors \wedge$$
$$\forall \langle B,BN,C\rangle \in Boats$$
$$(\exists \langle Ir,Br,D\rangle \in \mathrm{Re}serves(I = Ir \wedge Br = B))\}$$

- Simpler notation, same query. (Much clearer!)
- To find sailors who've reserved all red boats:

$$\ldots\ldots \quad (C \neq 'red' \vee \exists \langle Ir,Br,D\rangle \in \mathrm{Re}serves(I = Ir \wedge Br = B))$$

Unsafe Queries, Expressive Power

- It is possible to write syntactically correct calculus queries that have an infinite number of answers! Such queries are called *unsafe*.
  - e.g., $\{S \mid \neg(S \in Sailors)\}$
- It is known that every query that can be expressed in relational algebra can be expressed as a safe query in DRC / TRC; the converse is also true.
- *Relational Completeness*: Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus.

Summary

- Relational calculus is non-operational, and users define queries in terms of what they want, not in terms of how to compute it. (Declarativeness.)
- Algebra and safe calculus have same expressive power, leading to the notion of relational completeness.