

### Relational Calculus

Chapter 4, Part B

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### Relational Calculus

- Comes in two flavors: <u>Tuple relational calculus</u> (TRC) and <u>Domain relational calculus</u> (DRC).
- Calculus has variables, constants, comparison ops, logical connectives and quantifiers.
  - TRC: Variables range over (i.e., get bound to) *tuples*.
  - <u>DRC</u>: Variables range over *domain elements* (= field values).
  - Both TRC and DRC are simple subsets of first-order logic.
- Expressions in the calculus are called *formulas*. An answer tuple is essentially an assignment of constants to variables that make the formula evaluate to *true*.

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### Domain Relational Calculus

\* Query has the form:

$$\langle x1, x2, ..., xn \rangle | p(\langle x1, x2, ..., xn \rangle) \rangle$$

- \* *Answer* includes all tuples  $\langle x1, x2, ..., xn \rangle$  that make the *formula*  $p(\langle x1, x2, ..., xn \rangle)$  be *true*.
- Formula is recursively defined, starting with simple atomic formulas (getting tuples from relations or making comparisons of values), and building bigger and better formulas using the logical connectives.

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- $\langle x1, x2, ..., xn \rangle \in Rname$ , or  $X \circ p Y$ , or  $X \circ p$  constant
- op is one of  $<,>,=,\leq,\geq,\neq$
- \* Formula:
  - an atomic formula, or
  - $\neg p, p \land q, p \lor q$ , where p and q are formulas, or
  - $\exists X(p(X))$ , where variable X is *free* in p(X), or
  - $\forall X(p(X))$ , where variable X is *free* in p(X)
- ❖ The use of quantifiers  $\exists X$  and  $\forall X$  is said to  $\underline{bind}$  X.
  - A variable that is not bound is free.

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#### Free and Bound Variables

- ❖ The use of quantifiers  $\exists X$  and  $\forall X$  in a formula is said to  $\underline{bind}$  X.
  - A variable that is **not bound** is **free**.
- \* Let us revisit the definition of a query:

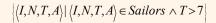
$$\langle x1, x2, ..., xn \rangle | p(\langle x1, x2, ..., xn \rangle)$$

There is an important restriction: the variables x1, ..., xn that appear to the left of `|' must be the *only* free variables in the formula p(...).

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# Find all sailors with a rating above 7



- ❖ The condition  $\langle I, N, T, A \rangle$  ∈ *Sailors* ensures that the domain variables *I*, *N*, *T* and *A* are bound to fields of the same Sailors tuple.
- ❖ The term ⟨I,N,T,A⟩ to the left of `|' (which should be read as *such that*) says that every tuple ⟨I,N,T,A⟩ that satisfies T>7 is in the answer.
- Modify this query to answer:
  - Find sailors who are older than 18 or have a rating under 9, and are called 'Joe'.

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Find sailors rated > 7 who have reserved boat #103



$$\langle I, N, T, A \rangle | \langle I, N, T, A \rangle \in Sailors \land T > 7 \land$$
  
 $\exists Ir, Br, D | \langle Ir, Br, D \rangle \in Reserves \land Ir = I \land Br = 103 |$ 

- \* We have used  $\exists Ir, Br, D (...)$  as a shorthand for  $\exists Ir (\exists Br (\exists D(...)))$
- ❖ Note the use of ∃ to find a tuple in Reserves that 'joins with' the Sailors tuple under consideration.

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Find sailors rated > 7 who've reserved a



red boat

$$\langle I, N, T, A \rangle | \langle I, N, T, A \rangle \in Sailors \land T > 7 \land$$

$$\exists Ir, Br, D | \langle Ir, Br, D \rangle \in Reserves \land Ir = I \land$$

$$\exists B, BN, C | \langle B, BN, C \rangle \in Boats \land B = Br \land C = 'red' | | |$$

- \* Observe how the parentheses control the scope of each quantifier's binding.
- This may look cumbersome, but with a good user interface, it is very intuitive. (MS Access, QBE)

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### Find sailors who've reserved all boats



 $\{\langle I, N, T, A \rangle | \langle I, N, T, A \rangle \in Sailors \land$  $\forall B,BN,C \mid \neg \mid \langle B,BN,C \rangle \in Boats \mid \lor$  $\exists Ir, Br, D \ (\langle Ir, Br, D \rangle \in \text{Reserves} \land I = Ir \land Br = B)$ 

\* Find all sailors *I* such that for each 3-tuple  $\langle B,BN,C\rangle$ either it is not a tuple in Boats or there is a tuple in Reserves showing that sailor *I* has reserved it.

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# Find sailors who've reserved all boats (again!)



$$\begin{aligned} & \langle (I, N, T, A) | \langle I, N, T, A \rangle \in Sailors \land \\ & \forall \langle B, BN, C \rangle \in Boats \\ & \left[ \exists \langle Ir, Br, D \rangle \in \text{Reserves}[I = Ir \land Br = B] \right] \end{aligned}$$

- \* Simpler notation, same query. (Much clearer!)
- To find sailors who've reserved all red boats:

.... 
$$(C \neq 'red' \vee \exists \langle Ir, Br, D \rangle \in Reserves[I = Ir \wedge Br = B])$$

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# Unsafe Queries, Expressive Power



- It is possible to write syntactically correct calculus queries that have an infinite number of answers! Such queries are called <u>unsafe</u>.
  - e.g.,  $\{S \mid \neg \{S \in Sailors\}\}$
- It is known that every query that can be expressed in relational algebra can be expressed as a safe query in DRC / TRC; the converse is also true.
- <u>Relational Completeness</u>: Query language (e.g., SQL) can express every query that is expressible in relational algebra/calculus.

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# Summary



- Relational calculus is non-operational, and users define queries in terms of what they want, not in terms of how to compute it. (Declarativeness.)
- Algebra and safe calculus have same expressive power, leading to the notion of relational completeness.

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