

## Schema Refinement and Normal Forms

### Chapter 15

## The Evils of Redundancy

- ❖ **Redundancy** is at the root of several problems associated with relational schemas:
  - **redundant storage, insert/delete/update anomalies**
- ❖ Integrity constraints, in particular **functional dependencies**, can be used to identify schemas with such problems and to suggest refinements.
- ❖ Main refinement technique: **decomposition** (replacing ABCD with, say, AB and BCD, or ACD and ABD).
- ❖ Decomposition should be used judiciously:
  - Is there reason to decompose a relation?
  - What problems (if any) does the decomposition cause?

## Functional Dependencies (FDs)

- ❖ A **functional dependency**  $X \rightarrow Y$  holds over relation R if, for every allowable instance  $r$  of R:
  - $t1 \in r, t2 \in r, \pi_X(t1) = \pi_X(t2)$  implies  $\pi_Y(t1) = \pi_Y(t2)$
  - i.e., given two tuples in  $r$ , if the X values agree, then the Y values must also agree. (X and Y are sets of attributes.)
- ❖ An FD is a statement about **all** allowable relations.
  - Must be identified based on semantics of application.
  - Given some allowable instance  $r1$  of R, we can check if it violates some FD  $f$ , but we cannot tell if  $f$  holds over R!
- ❖ K is a candidate key for R means that  $K \rightarrow R$ 
  - However,  $K \rightarrow R$  does not require K to be **minimal**!

## Example: Constraints on Entity Set

- ❖ Consider relation obtained from Hourly\_Emps:
  - Hourly\_Emps (ssn, name, lot, rating, hrly\_wages, hrs\_worked)
- ❖ **Notation:** We will denote this relation schema by listing the attributes: SNLRWH
  - This is really the set of attributes {S,N,L,R,W,H}.
  - Sometimes, we will refer to all attributes of a relation by using the relation name. (e.g., Hourly\_Emps for SNLRWH)
- ❖ Some FDs on Hourly\_Emps:
  - ssn is the key:  $S \rightarrow \text{SNLRWH}$
  - rating determines hrly\_wages:  $R \rightarrow W$

## Example (Contd.)

- ❖ Problems due to  $R \rightarrow W$ :
  - **Update anomaly:** Can we change W in just the 1st tuple of SNLRWH?
  - **Insertion anomaly:** What if we want to insert an employee and don't know the hourly wage for his rating?
  - **Deletion anomaly:** If we delete all employees with rating 5, we lose the information about the wage for rating 5!

S	N	L	R	W	H
123-22-3666	Attishoo	48	8	10	40
231-31-5368	Smiley	22	8	10	30
131-24-3650	Smethurst	35	5	7	30
434-26-3751	Guldu	35	5	7	32
612-67-4134	Madayan	35	8	10	40

S	N	L	R	H
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Hourly\_Emps2

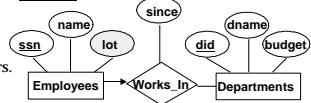
R	W
8	10
5	7

Wages

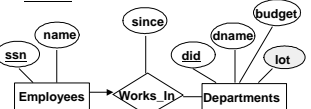
## Refining an ER Diagram

- ❖ 1st diagram translated:
  - Workers(S,N,L,D,S)
  - Departments(D,M,B)
  - Lots associated with workers.
- ❖ Suppose all workers in a dept are assigned the same lot:  $D \rightarrow L$
- ❖ Redundancy: fixed by:
  - Workers2(S,N,D,S)
  - Dept\_Lots(D,L)
- ❖ Can fine-tune this:
  - Workers2(S,N,D,S)
  - Departments(D,M,B,L)

Before:



After:



## Reasoning About FDs

- ❖ Given some FDs, we can usually infer additional FDs:
  - $ssn \rightarrow did, did \rightarrow lot$  implies  $ssn \rightarrow lot$
- ❖ An FD  $f$  is *implied by* a set of FDs  $F$  if  $f$  holds whenever all FDs in  $F$  hold.
  - $F^+$  = closure of  $F$  is the set of all FDs that are implied by  $F$ .
- ❖ Armstrong's Axioms ( $X, Y, Z$  are sets of attributes):
  - Reflexivity: If  $X \subseteq Y$ , then  $X \rightarrow Y$
  - Augmentation: If  $X \rightarrow Y$ , then  $XZ \rightarrow YZ$  for any  $Z$
  - Transitivity: If  $X \rightarrow Y$  and  $Y \rightarrow Z$ , then  $X \rightarrow Z$
- ❖ These are *sound* and *complete* inference rules for FDs!

## Reasoning About FDs (Contd.)

- ❖ Couple of additional rules (that follow from AA):
  - Union: If  $X \rightarrow Y$  and  $X \rightarrow Z$ , then  $X \rightarrow YZ$
  - Decomposition: If  $X \rightarrow YZ$ , then  $X \rightarrow Y$  and  $X \rightarrow Z$
- ❖ Example: Contracts(*cid, sid, jid, did, pid, qty, value*), and:
  - $C$  is the key:  $C \rightarrow CSJDPQV$
  - Project purchases each part using single contract:  $JP \rightarrow C$
  - Dept purchases at most one part from a supplier:  $SD \rightarrow P$
- ❖  $JP \rightarrow C, C \rightarrow CSJDPQV$  imply  $JP \rightarrow CSJDPQV$
- ❖  $SD \rightarrow P$  implies  $SDJ \rightarrow JP$
- ❖  $SDJ \rightarrow JP, JP \rightarrow CSJDPQV$  imply  $SDJ \rightarrow CSJDPQV$

## Reasoning About FDs (Contd.)

- ❖ Computing the closure of a set of FDs can be expensive. (Size of closure is exponential in # attrsl!)
- ❖ Typically, we just want to check if a given FD  $X \rightarrow Y$  is in the closure of a set of FDs  $F$ . An efficient check:
  - Compute attribute closure of  $X$  (denoted  $X^+$ ) wrt  $F$ :
    - ♦ Set of all attributes  $A$  such that  $X \rightarrow A$  is in  $F^+$
    - ♦ There is a linear time algorithm to compute this.
  - Check if  $Y$  is in  $X^+$
- ❖ Does  $F = \{A \rightarrow B, B \rightarrow C, CD \rightarrow E\}$  imply  $A \rightarrow E$ ?
  - i.e., is  $A \rightarrow E$  in the closure  $F^+$ ? Equivalently, is  $E$  in  $A^+$ ?

## Normal Forms

- ❖ Returning to the issue of schema refinement, the first question to ask is whether any refinement is needed!
- ❖ If a relation is in a certain *normal form* (BCNF, 3NF etc.), it is known that certain kinds of problems are avoided/minimized. This can be used to help us decide whether decomposing the relation will help.
- ❖ Role of FDs in detecting redundancy:
  - Consider a relation  $R$  with 3 attributes, ABC.
    - ♦ No FDs hold: There is no redundancy here.
    - ♦ Given  $A \rightarrow B$ : Several tuples could have the same  $A$  value, and if so, they'll all have the same  $B$  value!

## Boyce-Codd Normal Form (BCNF)

- ❖ Reln  $R$  with FDs  $F$  is in BCNF if, for all  $X \rightarrow A$  in  $F^+$ 
  - $A \in X$  (called a *trivial* FD), or
  - $X$  contains a key for  $R$ .
- ❖ In other words,  $R$  is in BCNF if the only non-trivial FDs that hold over  $R$  are key constraints.
  - No dependency in  $R$  that can be predicted using FDs alone.
  - If we are shown two tuples that agree upon the  $X$  value, we cannot infer the  $A$  value in one tuple from the  $A$  value in the other.
  - If example relation is in BCNF, the 2 tuples must be identical (since  $X$  is a key).

X	Y	A
x	y1	a
x	y2	?

## Third Normal Form (3NF)

- ❖ Reln  $R$  with FDs  $F$  is in 3NF if, for all  $X \rightarrow A$  in  $F^+$ 
  - $A \in X$  (called a *trivial* FD), or
  - $X$  contains a key for  $R$ , or
  - $A$  is part of some key for  $R$ .
- ❖ *Minimality* of a key is crucial in third condition above!
- ❖ If  $R$  is in BCNF, obviously in 3NF.
- ❖ If  $R$  is in 3NF, some redundancy is possible. It is a compromise, used when BCNF not achievable (e.g., no "good" decomp, or performance considerations).
  - *Lossless-join, dependency-preserving decomposition of  $R$  into a collection of 3NF relations always possible.*

## What Does 3NF Achieve?

- ❖ If 3NF violated by  $X \rightarrow A$ , one of the following holds:
  - $X$  is a subset of some key  $K$ 
    - ◆ We store  $(X, A)$  pairs redundantly.
  - $X$  is not a proper subset of any key.
    - ◆ There is a chain of FDs  $K \rightarrow X \rightarrow A$ , which means that we cannot associate an  $X$  value with a  $K$  value unless we also associate an  $A$  value with an  $X$  value.
- ❖ But: even if reln is in 3NF, these problems could arise.
  - e.g., Reserves SBDC,  $S \rightarrow C$ ,  $C \rightarrow S$  is in 3NF, but for each reservation of sailor  $S$ , same  $(S, C)$  pair is stored.
- ❖ Thus, 3NF is indeed a compromise relative to BCNF.

## Decomposition of a Relation Scheme

- ❖ Suppose that relation  $R$  contains attributes  $A_1 \dots A_n$ . A **decomposition** of  $R$  consists of replacing  $R$  by two or more relations such that:
  - Each new relation scheme contains a subset of the attributes of  $R$  (and no attributes that do not appear in  $R$ ), and
  - Every attribute of  $R$  appears as an attribute of one of the new relations.
- ❖ Intuitively, decomposing  $R$  means we will store instances of the relation schemes produced by the decomposition, instead of instances of  $R$ .
- ❖ E.g., Can decompose SNLRWH into SNLRH and RW.

## Example Decomposition

- ❖ Decompositions should be used only when needed.
  - SNLRWH has FDs  $S \rightarrow W$  and  $R \rightarrow W$
  - Second FD causes violation of 3NF:  $W$  values repeatedly associated with  $R$  values. Easiest way to fix this is to create a relation  $RW$  to store these associations, and to remove  $W$  from the main schema:
    - ◆ i.e., we decompose SNLRWH into SNLRH and  $RW$
- ❖ The information to be stored consists of SNLRWH tuples. If we just store the projections of these tuples onto SNLRH and  $RW$ , are there any potential problems that we should be aware of?

## Problems with Decompositions

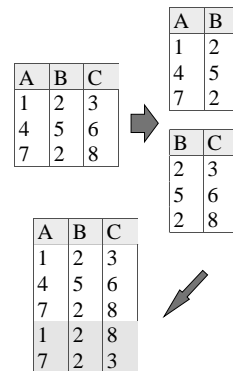
- ❖ There are three potential problems to consider:
  - ❶ Some queries become more expensive.
    - ◆ e.g., How much did sailor Joe earn? (salary =  $W * H$ )
  - ❷ Given instances of the decomposed relations, we may not be able to reconstruct the corresponding instance of the original relation!
    - ◆ Fortunately, not in the SNLRWH example.
  - ❸ Checking some dependencies may require joining the instances of the decomposed relations.
    - ◆ Fortunately, not in the SNLRWH example.
- ❖ **Tradeoff:** Must consider these issues vs. redundancy.

## Lossless Join Decompositions

- ❖ Decomposition of  $R$  into  $X$  and  $Y$  is **lossless-join** w.r.t. a set of FDs  $F$  if, for every instance  $r$  that satisfies  $F$ :
  - $\pi_X(r) \bowtie \pi_Y(r) = r$
- ❖ It is always true that  $r \subseteq \pi_X(r) \bowtie \pi_Y(r)$ 
  - In general, the other direction does not hold! If it does, the decomposition is lossless-join.
- ❖ Definition extended to decomposition into 3 or more relations in a straightforward way.
- ❖ It is essential that all decompositions used to deal with redundancy be lossless! (Avoids Problem (2).)

## More on Lossless Join

- ❖ The decomposition of  $R$  into  $X$  and  $Y$  is lossless-join wrt  $F$  if and only if the closure of  $F$  contains:
  - $X \cap Y \rightarrow X$ , or
  - $X \cap Y \rightarrow Y$
- ❖ In particular, the decomposition of  $R$  into  $UV$  and  $R - V$  is lossless-join if  $U \rightarrow V$  holds over  $R$ .



## Dependency Preserving Decomposition

- ❖ Consider CSJDPQV, C is key,  $JP \rightarrow C$  and  $SD \rightarrow P$ .
  - BCNF decomposition: CSJDQV and SDP
  - Problem: Checking  $JP \rightarrow C$  requires a join!
- ❖ Dependency preserving decomposition (Intuitive):
  - If R is decomposed into X, Y and Z, and we enforce the FDs that hold on X, on Y and on Z, then all FDs that were given to hold on R must also hold. (*Avoids Problem (3).*)
- ❖ Projection of set of FDs F: If R is decomposed into X, ... projection of F onto X (denoted  $F_X$ ) is the set of FDs  $U \rightarrow V$  in  $F^+$  (closure of F) such that U, V are in X.

## Dependency Preserving Decompositions (Contd.)

- ❖ Decomposition of R into X and Y is dependency preserving if  $(F_X \text{ union } F_Y)^+ = F^+$ 
  - i.e., if we consider only dependencies in the closure  $F^+$  that can be checked in X without considering Y, and in Y without considering X, these imply all dependencies in  $F^+$ .
- ❖ Important to consider  $F^+$ , not F, in this definition:
  - ABC,  $A \rightarrow B$ ,  $B \rightarrow C$ ,  $C \rightarrow A$ , decomposed into AB and BC.
  - Is this dependency preserving? Is  $C \rightarrow A$  preserved????
- ❖ Dependency preserving does not imply lossless join:
  - ABC,  $A \rightarrow B$ , decomposed into AB and BC.
- ❖ And vice-versa! (Example?)

## Decomposition into BCNF

- ❖ Consider relation R with FDs F. If  $X \rightarrow Y$  violates BCNF, decompose R into R - Y and XY.
  - Repeated application of this idea will give us a collection of relations that are in BCNF; lossless join decomposition, and guaranteed to terminate.
  - e.g., CSJDPQV, key C,  $JP \rightarrow C$ ,  $SD \rightarrow P$ ,  $J \rightarrow S$
  - To deal with  $SD \rightarrow P$ , decompose into SDP, CSJDQV.
  - To deal with  $J \rightarrow S$ , decompose CSJDQV into JS and CJDQV
- ❖ In general, several dependencies may cause violation of BCNF. The order in which we "deal with" them could lead to very different sets of relations!

## BCNF and Dependency Preservation

- ❖ In general, there may not be a dependency preserving decomposition into BCNF.
  - e.g., CSZ,  $CS \rightarrow Z$ ,  $Z \rightarrow C$
  - Can't decompose while preserving 1st FD; not in BCNF.
- ❖ Similarly, decomposition of CSJDPQV into SDP, JS and CJDQV is not dependency preserving (w.r.t. the FDs  $JP \rightarrow C$ ,  $SD \rightarrow P$  and  $J \rightarrow S$ ).
  - However, it is a lossless join decomposition.
  - In this case, adding JPC to the collection of relations gives us a dependency preserving decomposition.
  - ♦ JPC tuples stored only for checking FD! (*Redundancy!*)

## Decomposition into 3NF

- ❖ Obviously, the algorithm for lossless join decomp into BCNF can be used to obtain a lossless join decomp into 3NF (typically, can stop earlier).
- ❖ To ensure dependency preservation, one idea:
  - If  $X \rightarrow Y$  is not preserved, add relation XY.
  - Problem is that XY may violate 3NF! e.g., consider the addition of CJP to 'preserve'  $JP \rightarrow C$ . What if we also have  $J \rightarrow C$ ?
- ❖ Refinement: Instead of the given set of FDs F, use a *minimal cover* for F.

## Minimal Cover for a Set of FDs

- ❖ Minimal cover G for a set of FDs F:
  - Closure of F = closure of G.
  - Right hand side of each FD in G is a single attribute.
  - If we modify G by deleting an FD or by deleting attributes from an FD in G, the closure changes.
- ❖ Intuitively, every FD in G is needed, and "as small as possible" in order to get the same closure as F.
- ❖ e.g.,  $A \rightarrow B$ ,  $ABCD \rightarrow E$ ,  $EF \rightarrow GH$ ,  $ACDF \rightarrow EG$  has the following minimal cover:
  - $A \rightarrow B$ ,  $ACD \rightarrow E$ ,  $EF \rightarrow G$  and  $EF \rightarrow H$
- ❖ M.C.  $\rightarrow$  Lossless-Join, Dep. Pres. Decomp!!! (in book)



## *Summary of Schema Refinement*

- ❖ If a relation is in BCNF, it is free of redundancies that can be detected using FDs. Thus, trying to ensure that all relations are in BCNF is a good heuristic.
- ❖ If a relation is not in BCNF, we can try to decompose it into a collection of BCNF relations.
  - Must consider whether all FDs are preserved. If a lossless-join, dependency preserving decomposition into BCNF is not possible (or unsuitable, given typical queries), should consider decomposition into 3NF.
  - Decompositions should be carried out and/or re-examined while keeping *performance requirements* in mind.